MATH 203: HOMEWORK 11
DUE BY 5PM ON FRIDAY, APRIL 18

1)

(a) Prove that the additive group of Gaussian integers given by $m + n\sqrt{-1}$ for $m, n \in \mathbb{Z}$ is isomorphic to the multiplicative group of rational fractions of the form $2^n 3^m$ for $m, n \in \mathbb{Z}$.
(b) Show that both groups from part (a) are isomorphic to the group of all translations of a rectangular lattice in the plane.

2) Exhibit a group of transformations isomorphic with each of the following groups:

(a) The group of all real numbers under addition.
(b) The group of all nonzero real numbers under multiplication.
(c) The group of integers mod 8 under addition.

3) Show that a group G cannot be the union of two proper subgroups.

4) (a) If a cyclic group G is generated by a of order m, prove that the powers of a^k generate all of G iff $g.c.d\,(k, m) = 1$ (g.c.d denotes the greatest common division of k and m).
(b) How many different generators does a cyclic group of order 6 have?

Definition: Let G be a group, and let $a \in G$ be an element of G. If there exists some $n \in \mathbb{N}^*$ such that $a^n = e$, then a is said to be of **finite order**. Furthermore, the lowest positive integer $p \in \mathbb{N}^*$ such that $a^p = e$ is called the **order** of the element a.

5) Let G be an abelian group. Show that the elements of finite order in G form a subgroup. This subgroup is called the **torsion subgroup** of G.