MATH 203: HOMEWORK 12
DUE BY 5PM ON FRIDAY, APRIL 25

1) (a) Let G be the group of transformations $x \to ax + b$ of $\mathbb{R}$ into $\mathbb{R}$, where $a \neq 0$, $a, b \in \mathbb{R}$, and let S be the group of all such transformations satisfying $a = 1$. Describe the right and left cosets of S in G.
(b) Let T be the subgroup of G consisting of all transformations with $b = 0$. Describe the right and left cosets of T in G.

2) Prove that the number of right cosets of any subgroup S of a finite group G equals the number of its left cosets. (Hint: Consider the map $x \to x^{-1}$.)

3) Let S be a subgroup of a group G, and let SaS denote the set of all products sas' for $s, s' \in S$. Prove that for any $a, b \in G$, either $SaS \cap SbS = \emptyset$ or $SaS = SbS$.

4) For a subgroup S of a group G, let $x \equiv y \bmod S$ mean $xy^{-1} \in S$.

(a) Prove the relation $\equiv$ is reflexive, symmetric, and transitive.
(b) Show that $x \equiv y \bmod S$ iff x and y lie in the same right coset of S in G.
(c) Show that $x \equiv y \bmod S$ implies $xa \equiv ya \bmod S$ for all $a \in G$.

5) Prove that a group of order p^m for p a prime number must contain a subgroup of order p.