Basic Integration for STA 6166

Integration simply put is finding the area under a function/curve. If the kernel, that is the function to be integrated, has simple graphical shape such as a rectangle or triangle, finding the area is a simple task. For STA 6166 all functions assumed are nonnegative, i.e. ≥ 0.

For constants a, b, c, t,and functions $f(\cdot), h(\cdot)$

- $\displaystyle\int_a^b (f(x) + h(x))\ dx = \int_a^b f(x)\ dx + \int_a^b h(x)\ dx$
- $\displaystyle\int_a^b cf(x)\ dx = c\int_a^b f(x)\ dx$
- Assume $b \in [a, c]$, then $\displaystyle\int_a^c f(x)\ dx = \int_a^b f(x)\ dx + \int_b^c f(x)\ dx$ (summing two areas)

Remark: The integration sign $\int$ is just the generalization of the summation sign $\sum$, so they behave similarly.

Integration for some basic functions

- $\displaystyle\int_a^b x^t\ dx = \left.\frac{x^{t+1}}{t+1}\right|_a^b = \frac{1}{t+1}(b^{t+1} - a^{t+1})$
- $\displaystyle\int_a^b \frac{1}{x}\ dx = \log(x)|_a^b = \log(b) - \log(a)$
- $\displaystyle\int_a^b \log(x)\ dx = \left.\frac{1}{x}\right|_a^b = (1/b) - (1/a)$
- $\displaystyle\int_a^b e^{tx}\ dx = \left.\frac{e^{tx}}{t}\right|_a^b = \frac{1}{t}(e^{tb} - e^{ta})$

Two useful techniques for integration (not necessary for this class) are

- Integration by parts
- Substitution