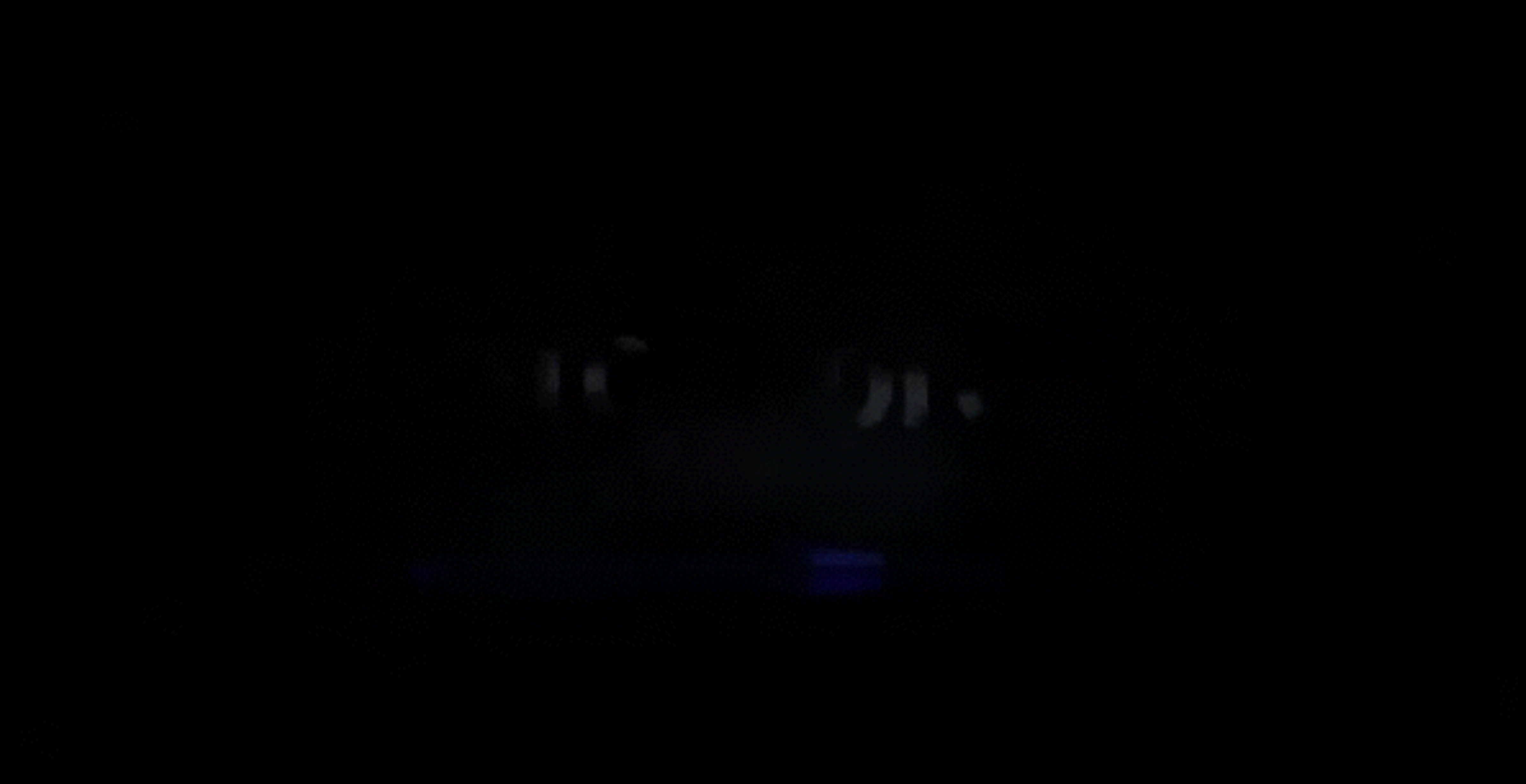
TIME AND MOTION

CS 3630 Introduction to Robotics and Perception

Frank Dellaert

BOT & DOLLY



Paths (Locus) vs. Trajectories (Locus + Time)

BOT & DOLLY

Paths (Locus) vs. Trajectories (Locus + Time)

QUINTICS

$$S\langle t\rangle = At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F$$

QUINTICS

$$S\langle t\rangle = At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F$$

$$\dot{S}\langle t\rangle = 5At^4 + 4Bt^3 + 3Ct^2 + 2Dt + E$$

$$\ddot{S}\langle t\rangle = 20At^3 + 12Bt^2 + 6Ct + 2D$$

QUINTICS

$$S\langle t\rangle = At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F$$

$$\dot{S}\langle t\rangle = 5At^4 + 4Bt^3 + 3Ct^2 + 2Dt + E$$

$$\ddot{S}\langle t\rangle = 20At^3 + 12Bt^2 + 6Ct + 2D$$

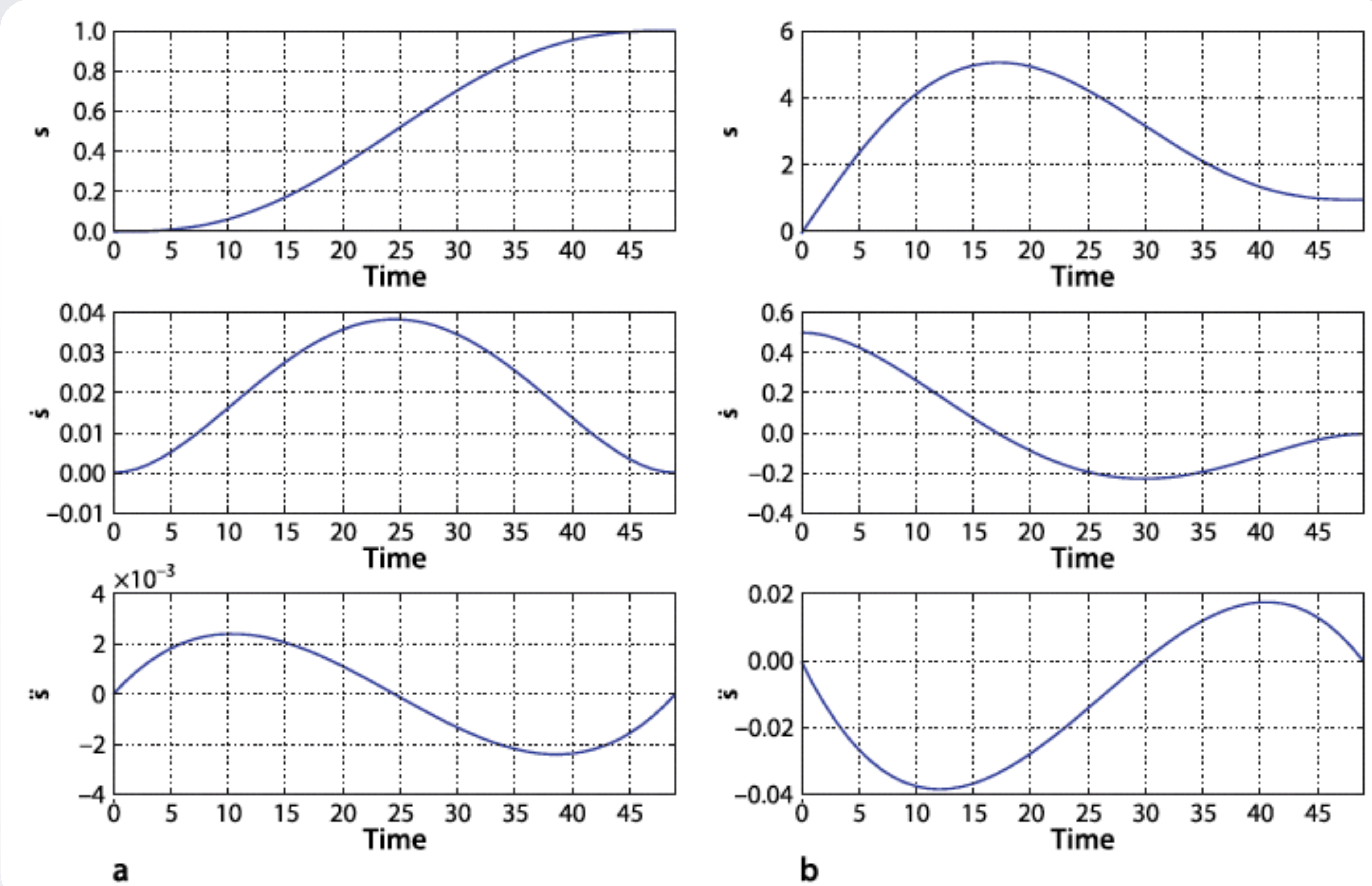
$$\begin{pmatrix} s_0 \\ s_T \\ \dot{s}_0 \\ \dot{s}_T \\ \ddot{s}_0 \\ \ddot{s}_T \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ T^5 & T^4 & T^3 & T^2 & T & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 5T^4 & 4T^3 & 3T^2 & 2T & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 20T^3 & 12T^2 & 6T & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \\ D \\ E \\ F \end{pmatrix}$$

QUINTICS

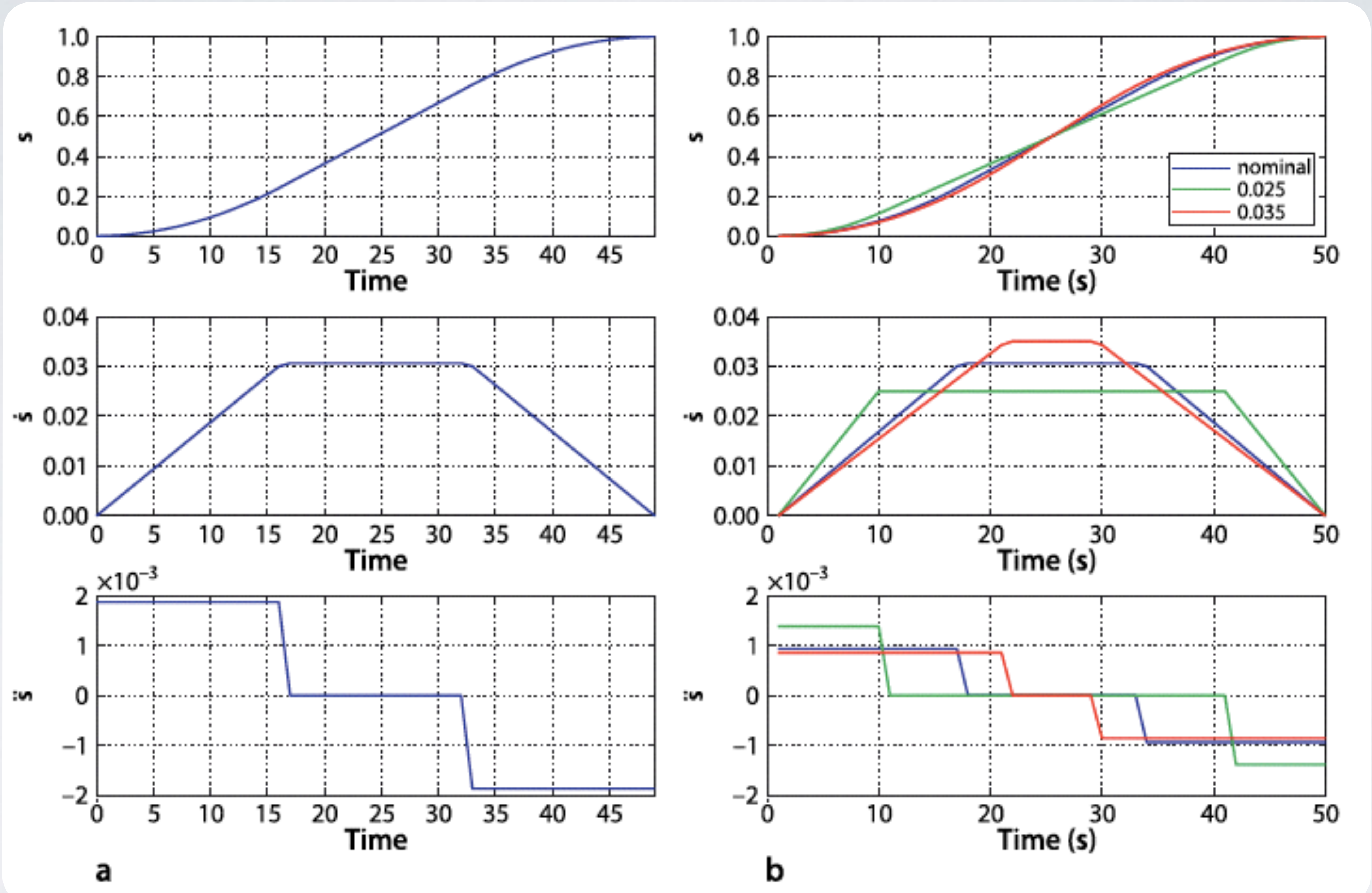
$$S\langle t\rangle = At^5 + Bt^4 + Ct^3 + Dt^2 + Et + F$$

$$\dot{S}\langle t\rangle = 5At^4 + 4Bt^3 + 3Ct^2 + 2Dt + E$$

$$\ddot{S}\langle t\rangle = 20At^3 + 12Bt^2 + 6Ct + 2D$$

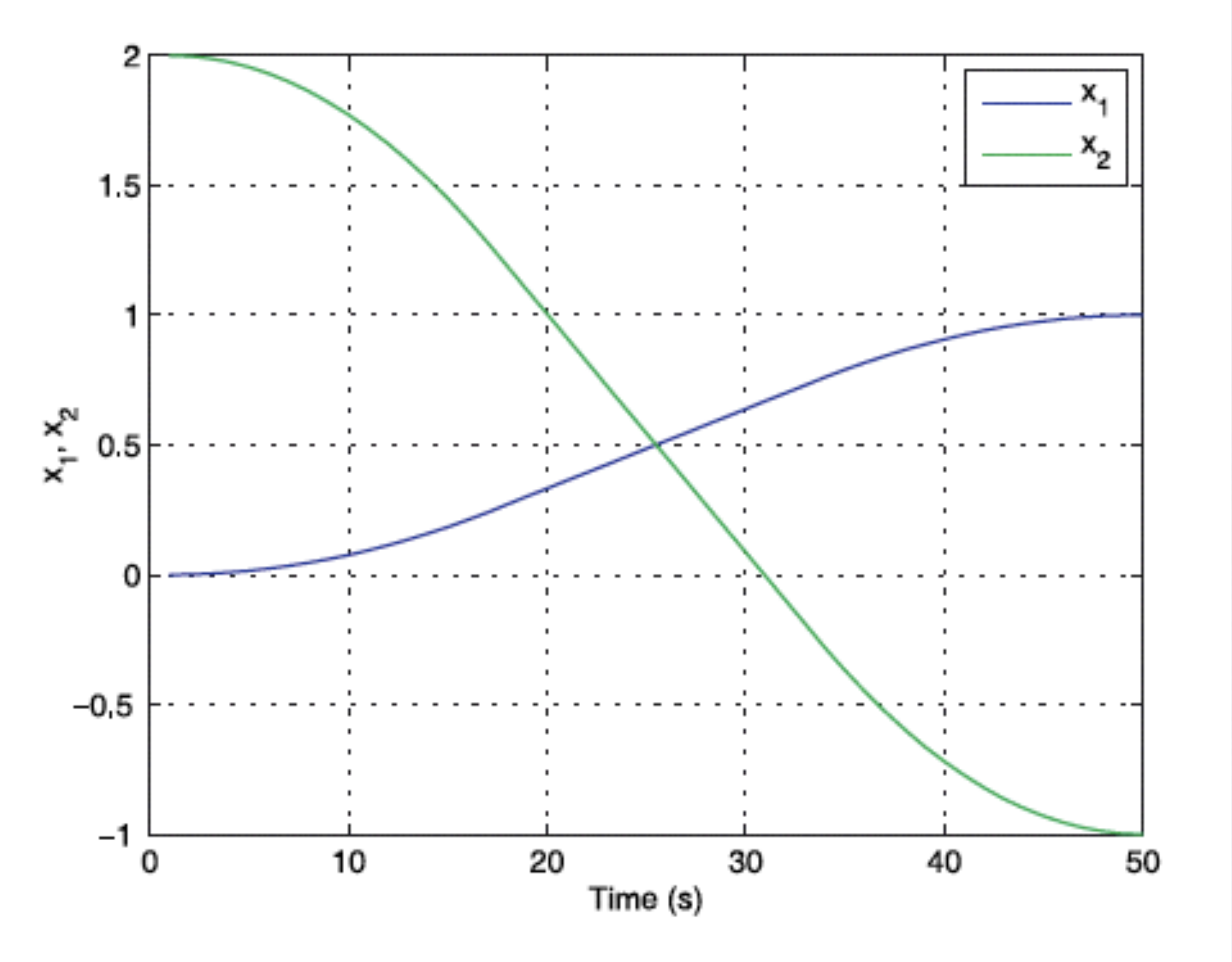


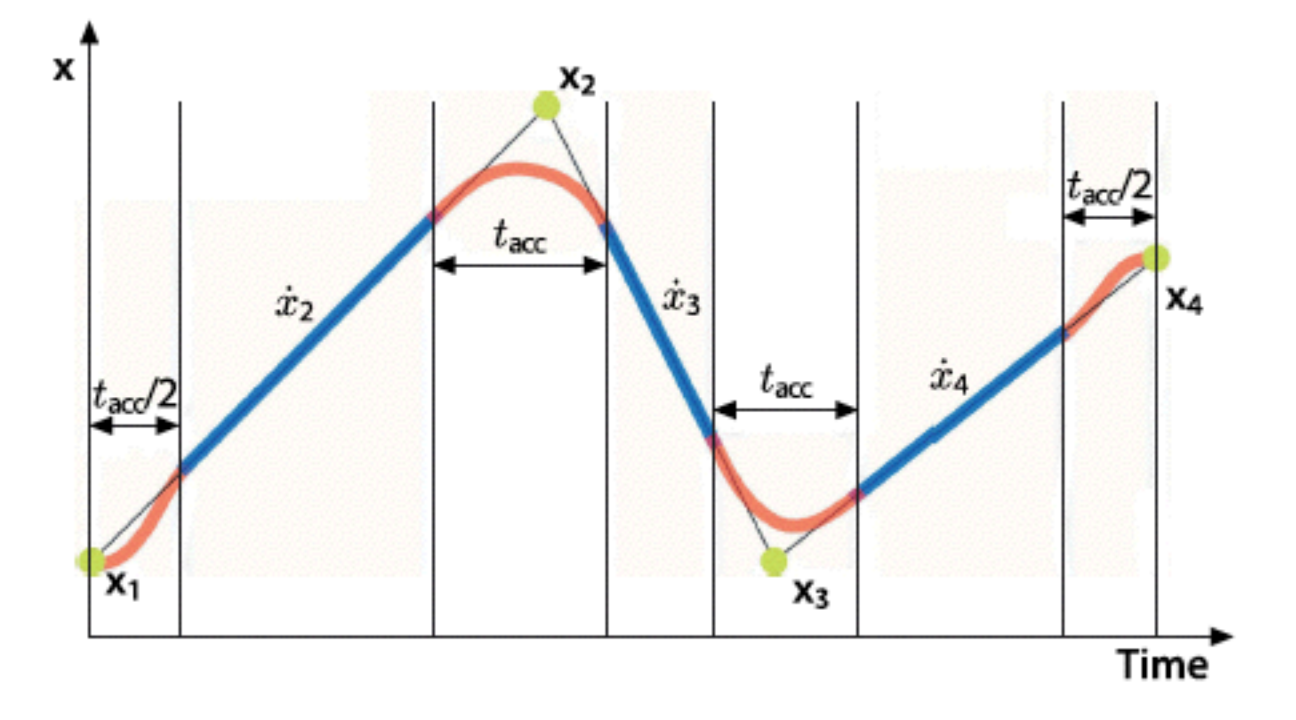
LINEAR SEGMENT WITH PARABOLIC BLEND



- No overshooting, reaches maximum velocity

MULTIPLE DIMENSIONS, MULTI-SEGMENT



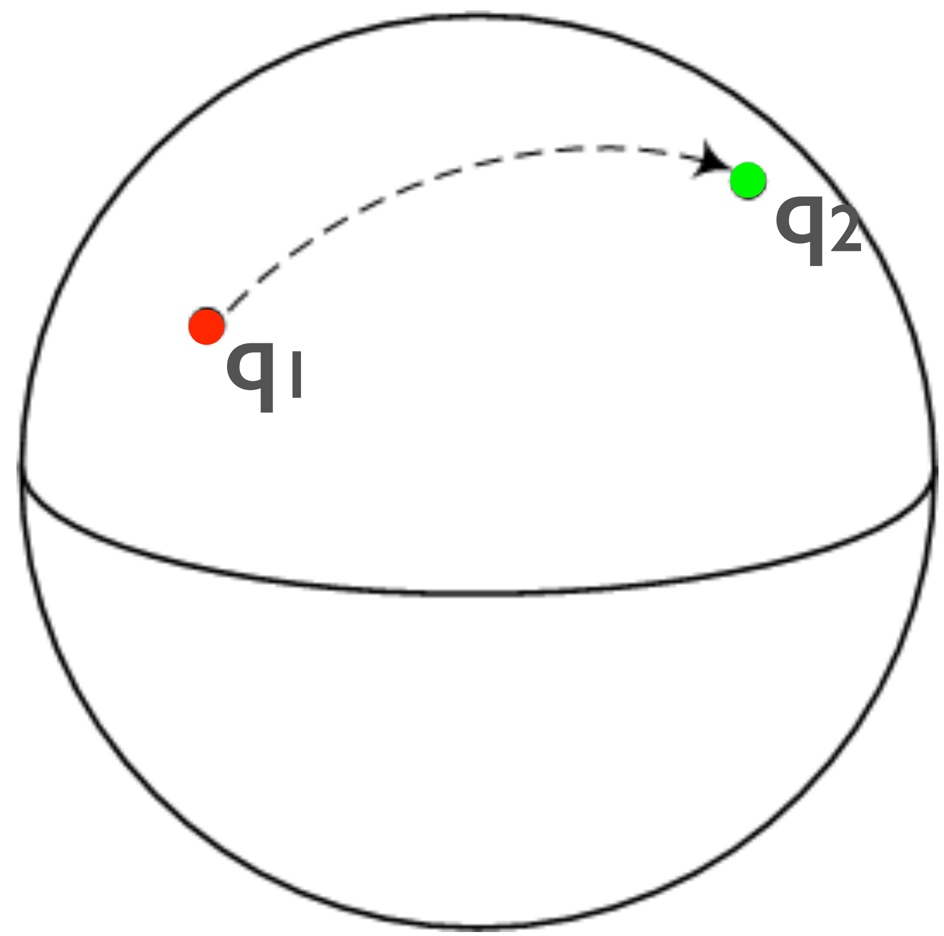


INTERPOLATION IN 3D

- Suppose $\delta = c(\theta/2) < s(\theta/2)\ \boldsymbol{v} >$ is rotation between q_1 and q_2

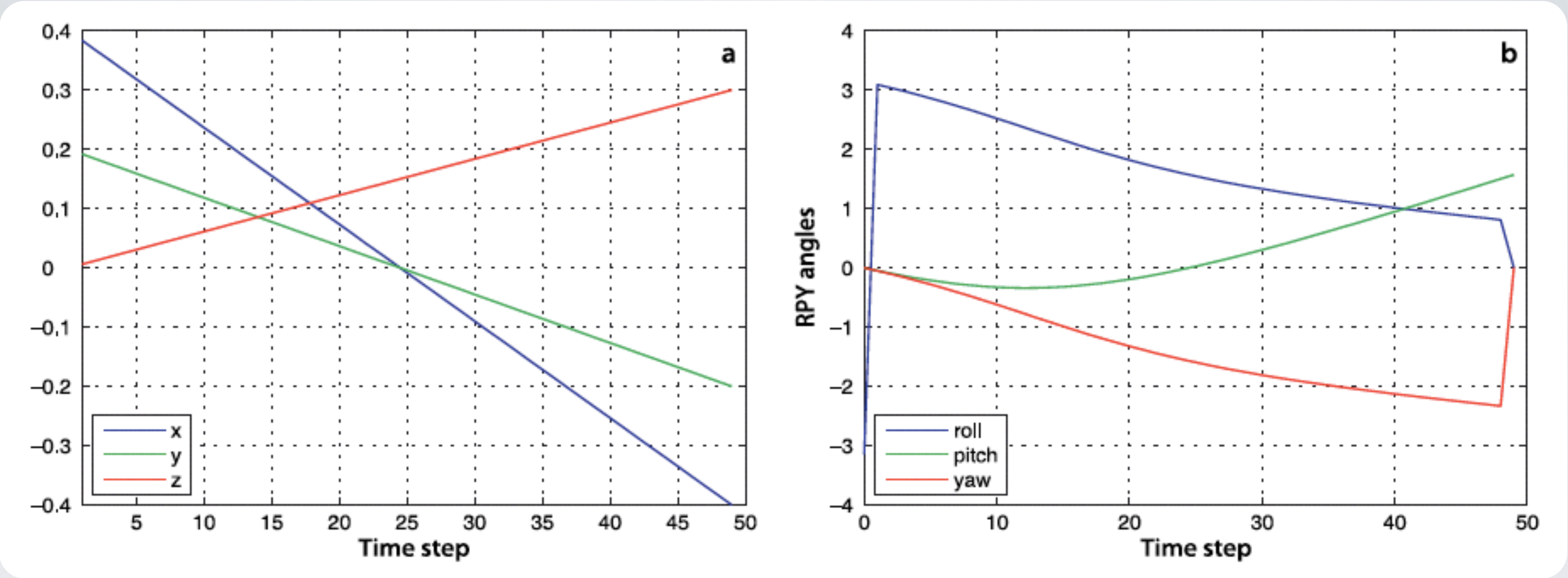
- Slerp(q_1, q_2, t) =

$$q_1 \oplus c(t\theta/2) < s(t\theta/2)\ \boldsymbol{v} >$$



- SLERP: Spherical Linear Interpolation

CARTESIAN MOTION



- Linear Interpolation for 3D translation
- Slerp for 3D rotation

INCREMENTAL MOTION

Remember:

$$\mathcal{R}_v^b(\phi,\theta,\psi) = \mathcal{R}_{v2}^b(\phi)\mathcal{R}_{v1}^{v2}(\theta)\mathcal{R}_v^{v1}(\psi) \tag{2.4}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{pmatrix}, \tag{2.5}$$

INCREMENTAL MOTION

Remember:

$$\mathcal{R}_v^b(\phi,\theta,\psi) = \mathcal{R}_{v2}^b(\phi)\mathcal{R}_{v1}^{v2}(\theta)\mathcal{R}_v^{v1}(\psi) \tag{2.4}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{pmatrix}, \tag{2.5}$$

When angles are small, $\omega\ \delta$t, we get

$$\boldsymbol{R}\langle t+\delta_t\rangle \approx \delta_t \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}\langle t\rangle + \boldsymbol{R}\langle t\rangle \approx \left(\delta_t \boldsymbol{S}(\boldsymbol{\omega}) + \boldsymbol{I}_{3\times 3}\right)\boldsymbol{R}\langle t\rangle$$

$$\boldsymbol{S}(\boldsymbol{\omega}) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$$

INCREMENTAL MOTION

Remember:

$$\mathcal{R}_v^b(\phi,\theta,\psi) = \mathcal{R}_{v2}^b(\phi)\mathcal{R}_{v1}^{v2}(\theta)\mathcal{R}_v^{v1}(\psi) \tag{2.4}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{pmatrix}, \tag{2.5}$$

When angles are small, $\omega\ \delta$t, we get

$$\boldsymbol{R}\langle t+\delta_t\rangle \approx \delta_t \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}\langle t\rangle + \boldsymbol{R}\langle t\rangle \approx \left(\delta_t \boldsymbol{S}(\boldsymbol{\omega}) + \boldsymbol{I}_{3\times 3}\right)\boldsymbol{R}\langle t\rangle$$

$$\boldsymbol{S}(\boldsymbol{\omega}) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$$

And hence:

$$\dot{\boldsymbol{R}}\langle t\rangle = \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}\langle t\rangle$$

INCREMENTAL MOTION

Remember:

$$\mathcal{R}_v^b(\phi,\theta,\psi) = \mathcal{R}_{v2}^b(\phi)\mathcal{R}_{v1}^{v2}(\theta)\mathcal{R}_v^{v1}(\psi) \tag{2.4}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ s_\phi s_\theta c_\psi - c_\phi s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi c_\theta \\ c_\phi s_\theta c_\psi + s_\phi s_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi c_\theta \end{pmatrix}, \tag{2.5}$$

When angles are small, $\omega\ \delta$t, we get

$$\boldsymbol{R}\langle t+\delta_t\rangle \approx \delta_t \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}\langle t\rangle + \boldsymbol{R}\langle t\rangle \approx \left(\delta_t \boldsymbol{S}(\boldsymbol{\omega}) + \boldsymbol{I}_{3\times 3}\right)\boldsymbol{R}\langle t\rangle$$

$$\boldsymbol{S}(\boldsymbol{\omega}) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix}$$

And hence:

$$\dot{\boldsymbol{R}}\langle t\rangle = \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}\langle t\rangle$$

Quaternions:

$$\dot{\mathring{q}} = \frac{1}{2}\mathring{q}(\omega)\mathring{q}$$

INERTIAL NAVIGATION

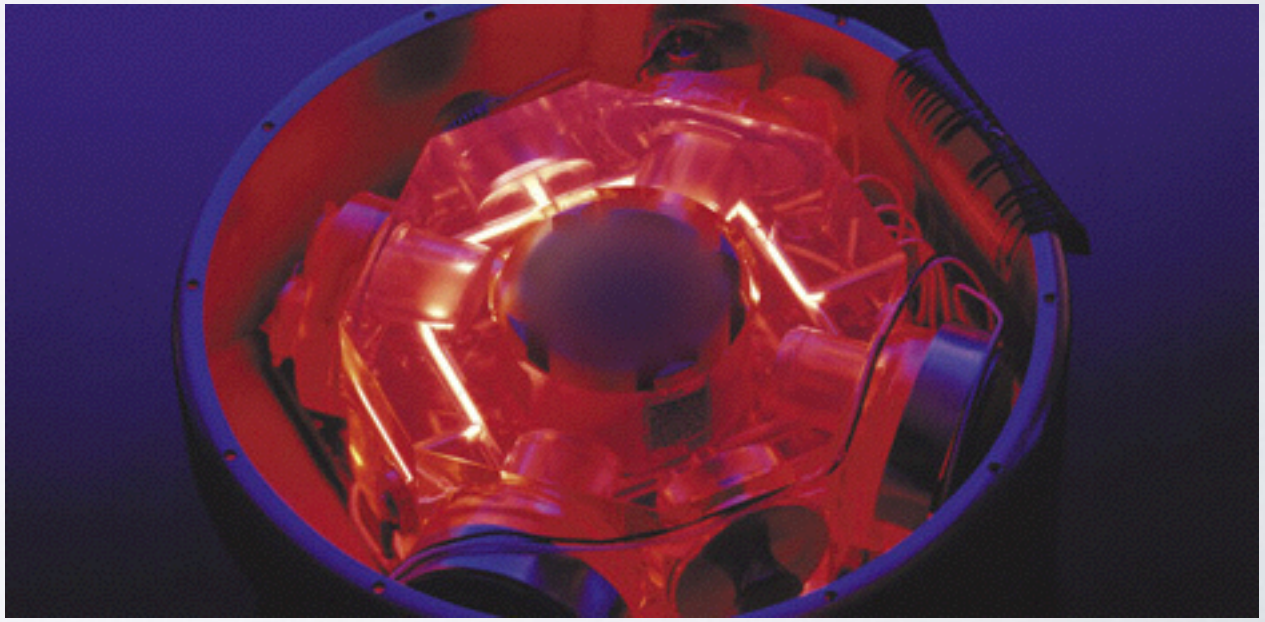
- Measure angular velocity with gyroscope, acceleration with accelerometer

- Integrate over time:

$$\boldsymbol{R}\langle k+1\rangle = \delta_t \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}\langle k\rangle + \boldsymbol{R}\langle k\rangle$$

$${}^{0}\boldsymbol{a} = {}^{0}\boldsymbol{R}_B \, {}^{B}\boldsymbol{a}$$

USS Alabama



Ring-laser Gyro

INERTIAL NAVIGATION

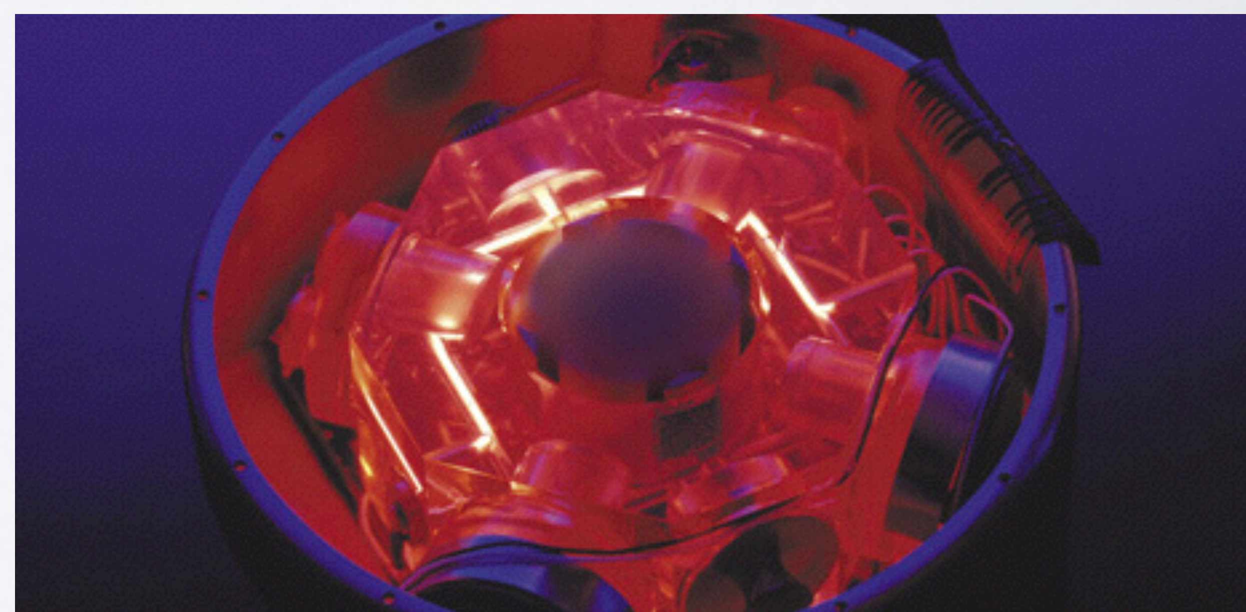
- Measure angular velocity with gyroscope, acceleration with accelerometer

- Integrate over time:

$$\boldsymbol{R}\langle k+1\rangle = \delta_t \boldsymbol{S}(\boldsymbol{\omega})\boldsymbol{R}\langle k\rangle + \boldsymbol{R}\langle k\rangle$$

$$^{0}\boldsymbol{a} = {}^{0}\boldsymbol{R}_B \, {}^{B}\boldsymbol{a}$$

USS Alabama



Ring-laser Gyro