CS 3630 Lecture Notes on Pose and Orientation

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1 Kinematics of a Differential Drive robot

1.1 Twists

Before talking about the differential drive robot, let's define the concept of a 2D twist, which is simply the derivative of the 2D rigid transformation from robot frame R to world frame W:

$$\dot{\xi}_R^W = \begin{bmatrix} \dot{t}_x \\ \dot{t}_y \\ \dot{\theta} \end{bmatrix}$$

The first two components make up the linear velocity v, and the last component is the angular velocity ω. Hence, we also frequently write $\dot{\xi}_R^W = (v, \omega)$. **Note that these velocities are expressed in the robot frame R.**

On the next page I will answer the question what happens if a robot undergoes a twist for a finite amount of time T. Can you guess or work out the answer without skipping ahead?

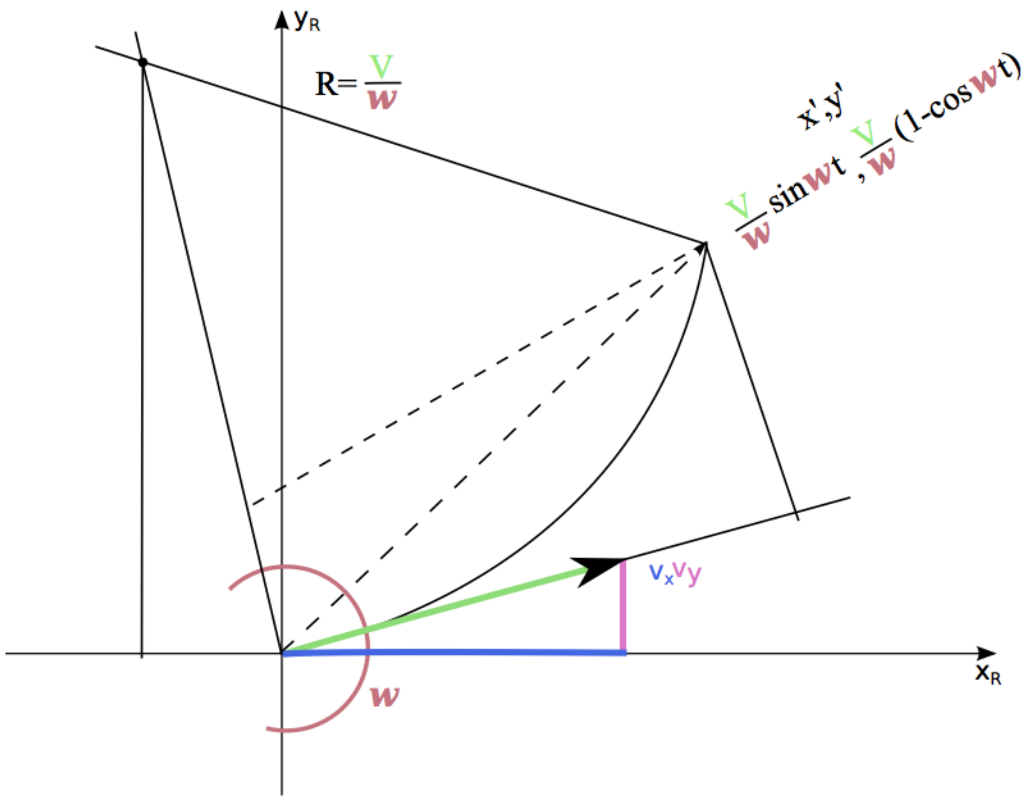

Figure 1: Integrating a constant twist forward traces out a circular trajectory.

A robot undergoing a constant twist, expressed in the robot frame, traces out a circular trajectory with radius $R = v/\omega$, as shown in Figure 1. Starting from the origin, after some time T we obtain

$$\xi(T) = \left(\begin{bmatrix} \cos\omega T & -\sin\omega T \\ \sin\omega T & \cos\omega T \end{bmatrix}, \frac{1}{\omega} \begin{bmatrix} 1-\cos\omega T & \sin\omega T \\ -\sin\omega T & 1-\cos\omega T \end{bmatrix} \begin{bmatrix} -v_y \\ v_x \end{bmatrix} \right)$$

A differential robot does not allow sideways velocity, however, so $v_y = 0$ and we have

$$\xi(T) = \left(\begin{bmatrix} \cos\omega T & -\sin\omega T \\ \sin\omega T & \cos\omega T \end{bmatrix}, \begin{bmatrix} \sin\omega T \\ 1-\cos\omega T \end{bmatrix} \frac{v_x}{\omega} \right) = \begin{bmatrix} R\sin\theta \\ R - R\cos\theta \\ \omega T \end{bmatrix}$$

which is easily seen to be a circular arc.

1.2 Inverse Kinematics

A question that is of interest for a robot is: if we know the robot's instantaneous twist $\dot{\xi}$, what are the wheel speeds? Wheel velocity $\dot{\varphi}$, the derivative of the wheel angle φ, can be calculated by dividing the instantaneous forward velocity v of the wheel by the **wheel radius** r:

$$\dot{\varphi} = \frac{\mathrm{v}}{\mathrm{r}}$$

Hence, we need to figure out at what velocities the wheels will move, in response to a twist $\dot{\xi}$. For a differential drive robot, the answer is easy, if we reason about two cases separately:

- Just forward motion with velocity v_x: the wheel velocities are then

$$\dot{\varphi}_R = \frac{\mathrm{v_x}}{\mathrm{r}}$$

$$\dot{\varphi}_L = \frac{\mathrm{v_x}}{\mathrm{r}}$$

- Just angular velocity ω: the wheel velocities then depend on the **length** L **of wheel axis**:

$$\dot{\varphi}_R = \frac{\omega \mathrm{L}}{2\mathrm{r}}$$

$$\dot{\varphi}_L = -\frac{\omega L}{2r}$$

If we have both, we can simply add them:

$$\dot{\varphi}_R = \frac{\omega \mathrm{L}}{2\mathrm{r}} + \frac{\mathrm{v_x}}{\mathrm{r}} \tag{1}$$

$$\dot{\varphi}_L = -\frac{\omega L}{2r} + \frac{\mathrm{v_x}}{\mathrm{r}} \tag{2}$$

This is called **inverse kinematics**, because it calculates the wheels speeds given a velocity, rather than the other way around.

1.3 Forward Kinematics

In a simulation, or to predict where a robot will go, we ask the **forward kinematics** question: given rotational wheel velocities $(\dot{\varphi}_L, \dot{\varphi}_R)$, what twist $\dot{\xi}$ will be achieved. In the case of the differential drive robot the answer can be found simply by solving the two equations 1 and 2 for v_x and ω:

$$\dot{\xi}_R = \begin{bmatrix} \frac{r}{2}\left(\dot{\varphi}_R + \dot{\varphi}_L\right) \\ 0 \\ \frac{r}{L}\left(\dot{\varphi}_R - \dot{\varphi}_L\right) \end{bmatrix}$$

It is interesting that the inverse kinematics is easier to derive than the forward kinematics.