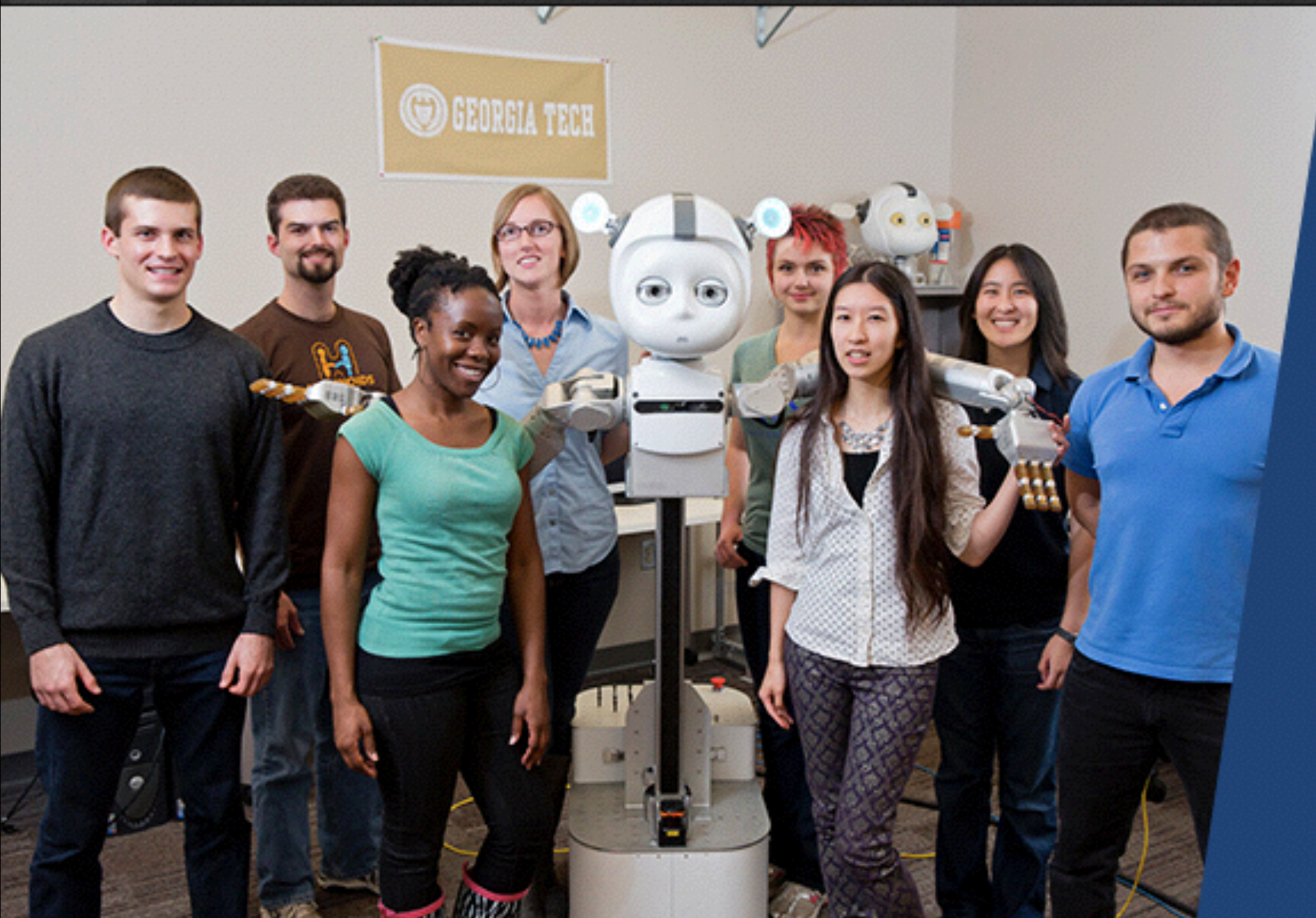




Optimization in Factor Graphs for Fast and Scalable 3D Reconstruction and Mapping

Frank Dellaert, Robotics & Intelligent Machines @ Georgia Tech





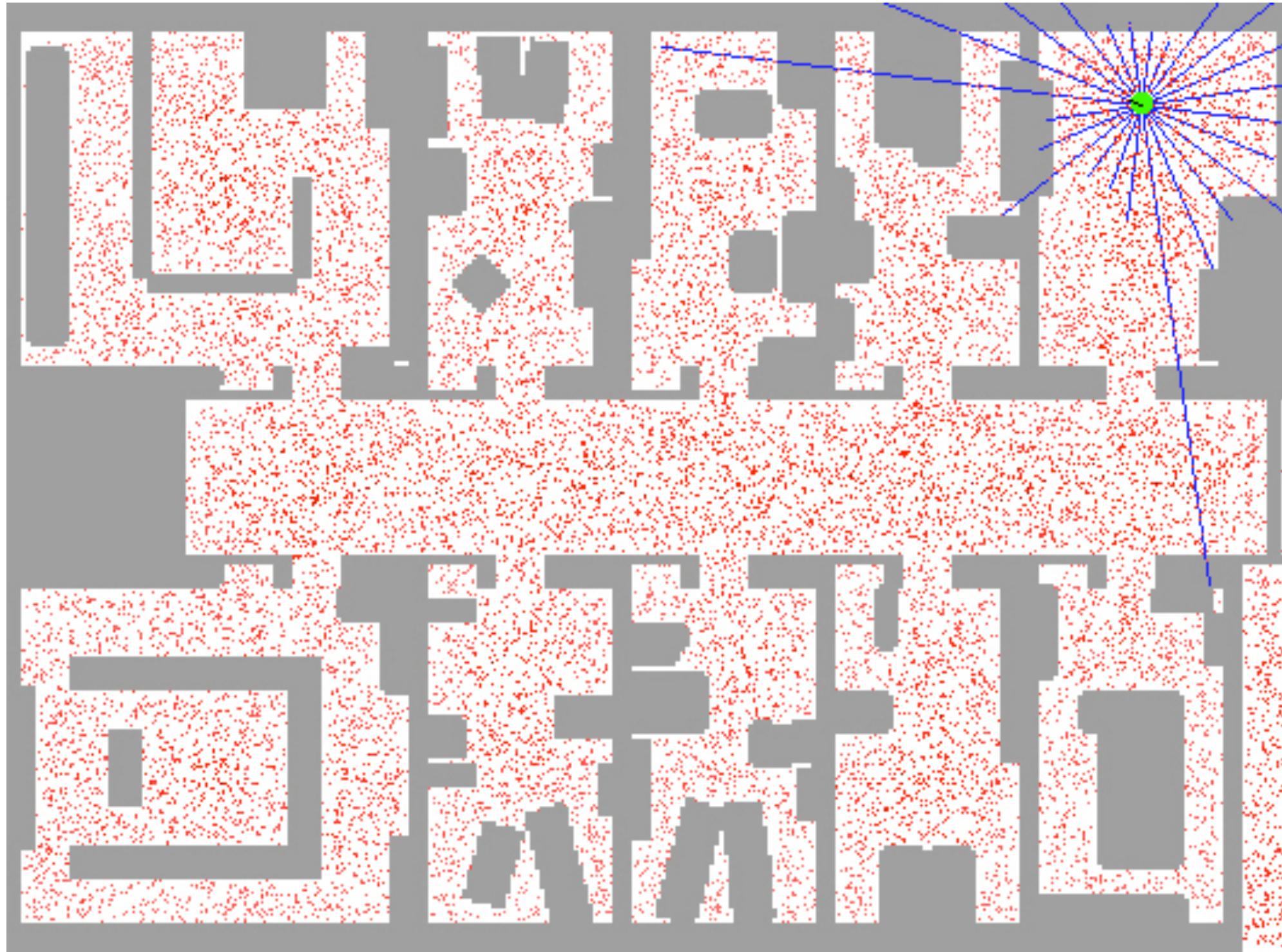
UNLIKE ANY OTHER

Offered jointly by the College of Computing and the College of Engineering, Georgia Tech's Ph.D. Program in Robotics is the first interdisciplinary robotics degree of its kind.



The next application deadline is December 15, 2014

Monte Carlo Localization

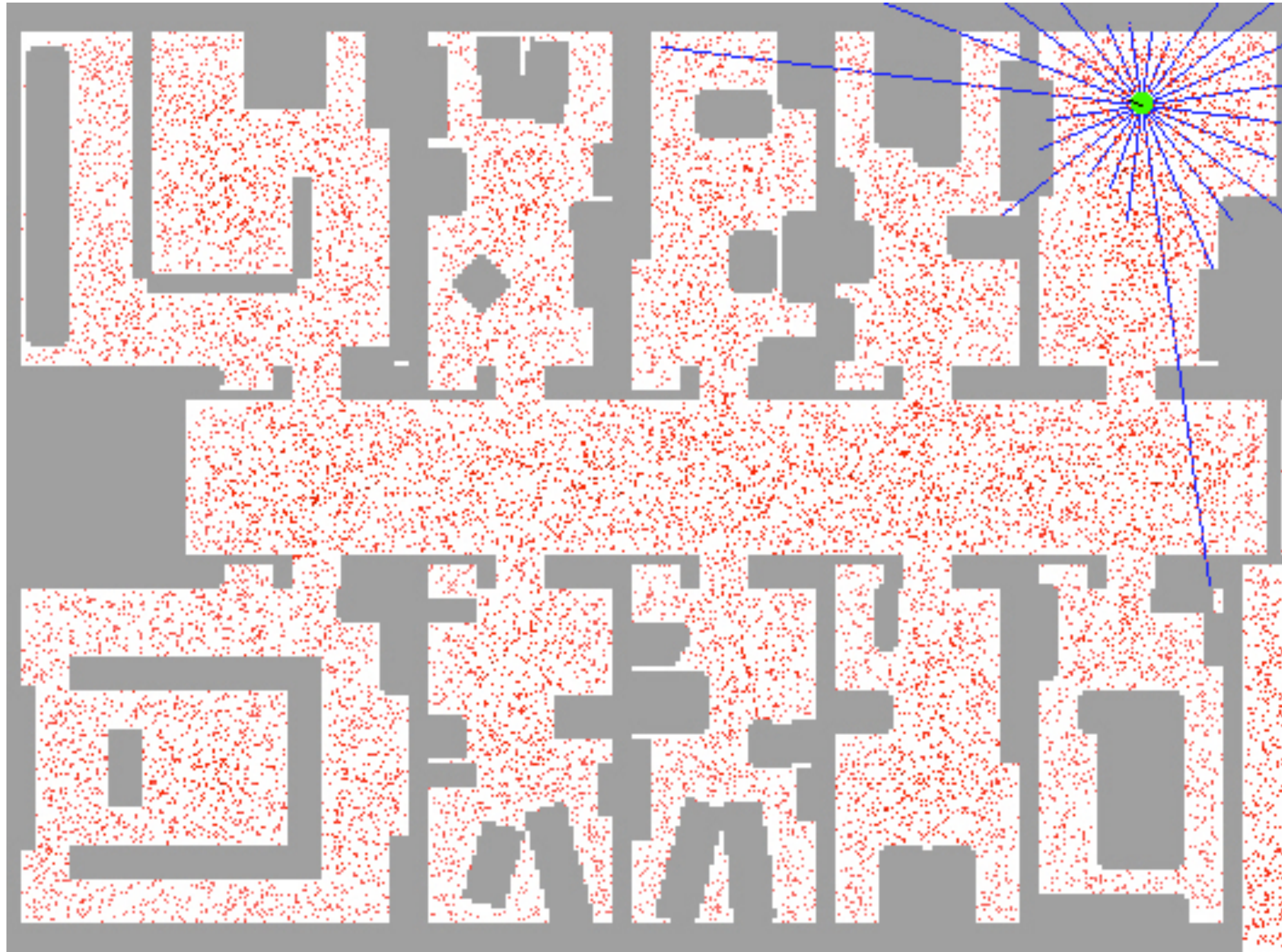


Dellaert, Fox, Burgard & Thrun, ICRA 1999



In the Smithsonian Institution's National Museum of American History and ON THIS WEB SITE!

Monte Carlo Localization



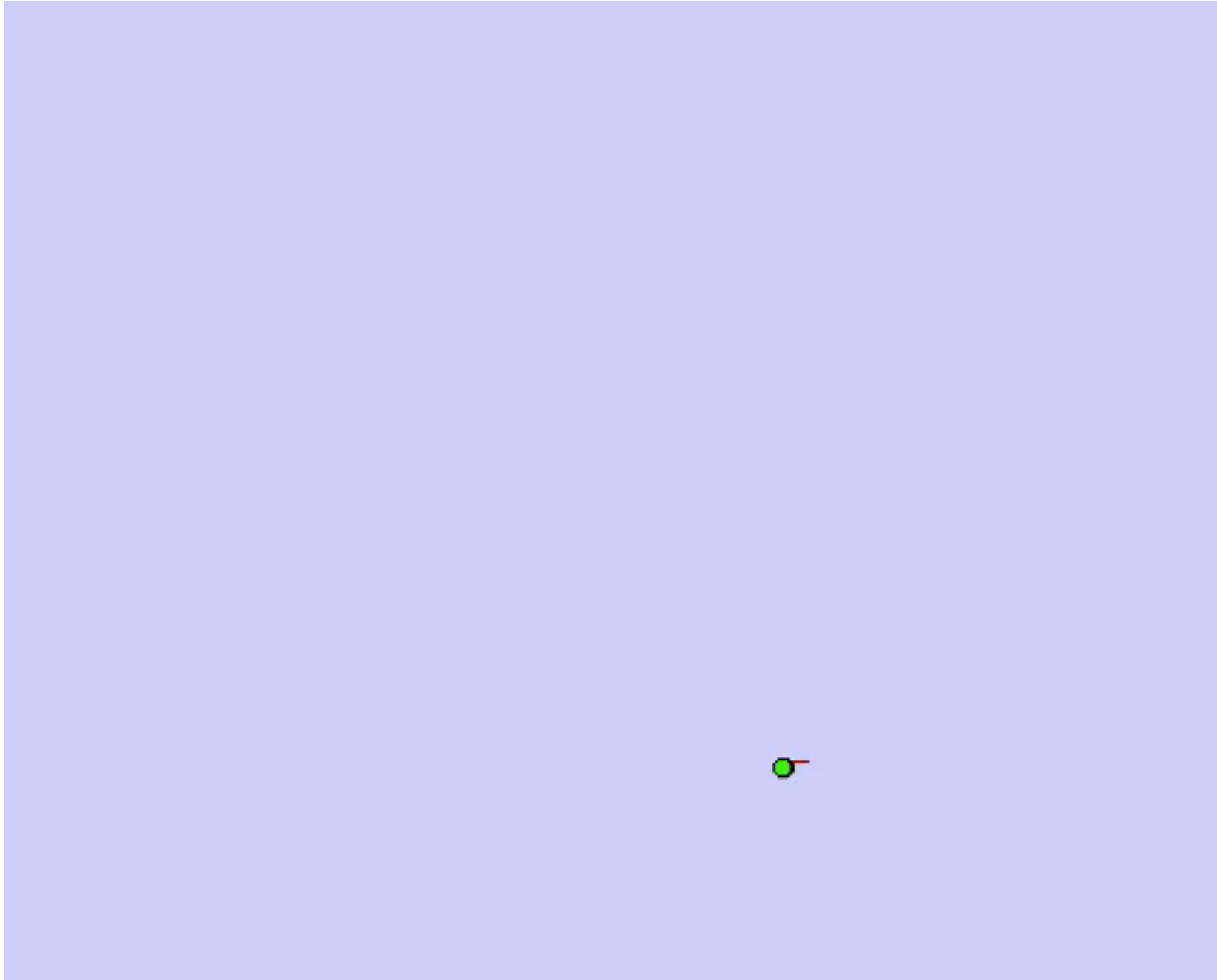
Dellaert, Fox, Burgard & Thrun, ICRA 1999



In the Smithsonian Institution's National Museum of American History and ON THIS WEB SITE!

SLAM = Simultaneous Localization and Mapping

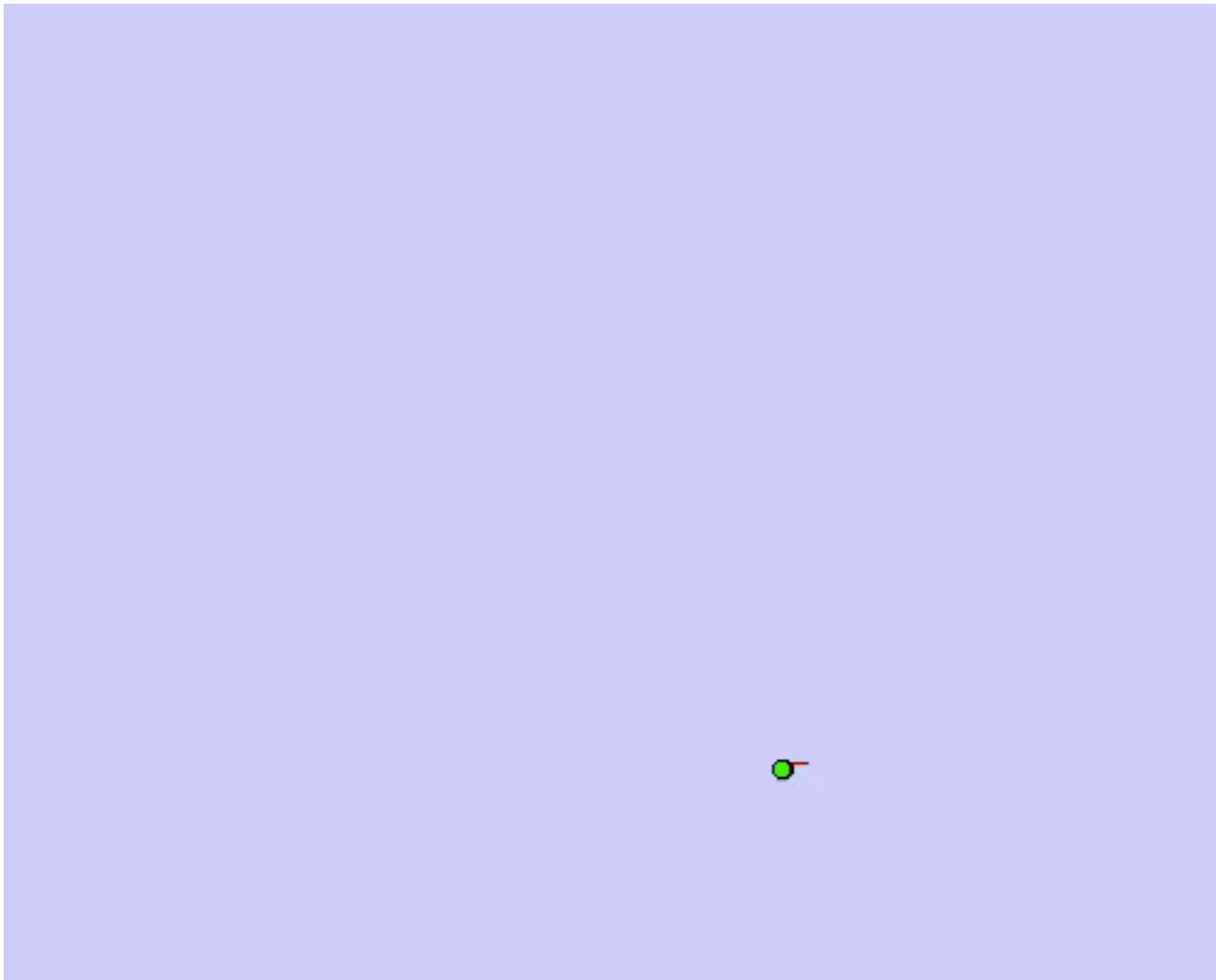
FastSLAM: Particle Filter on Trajectories



Montemerlo, Thrun, Koller, & Wegbreit, AAAI 2002

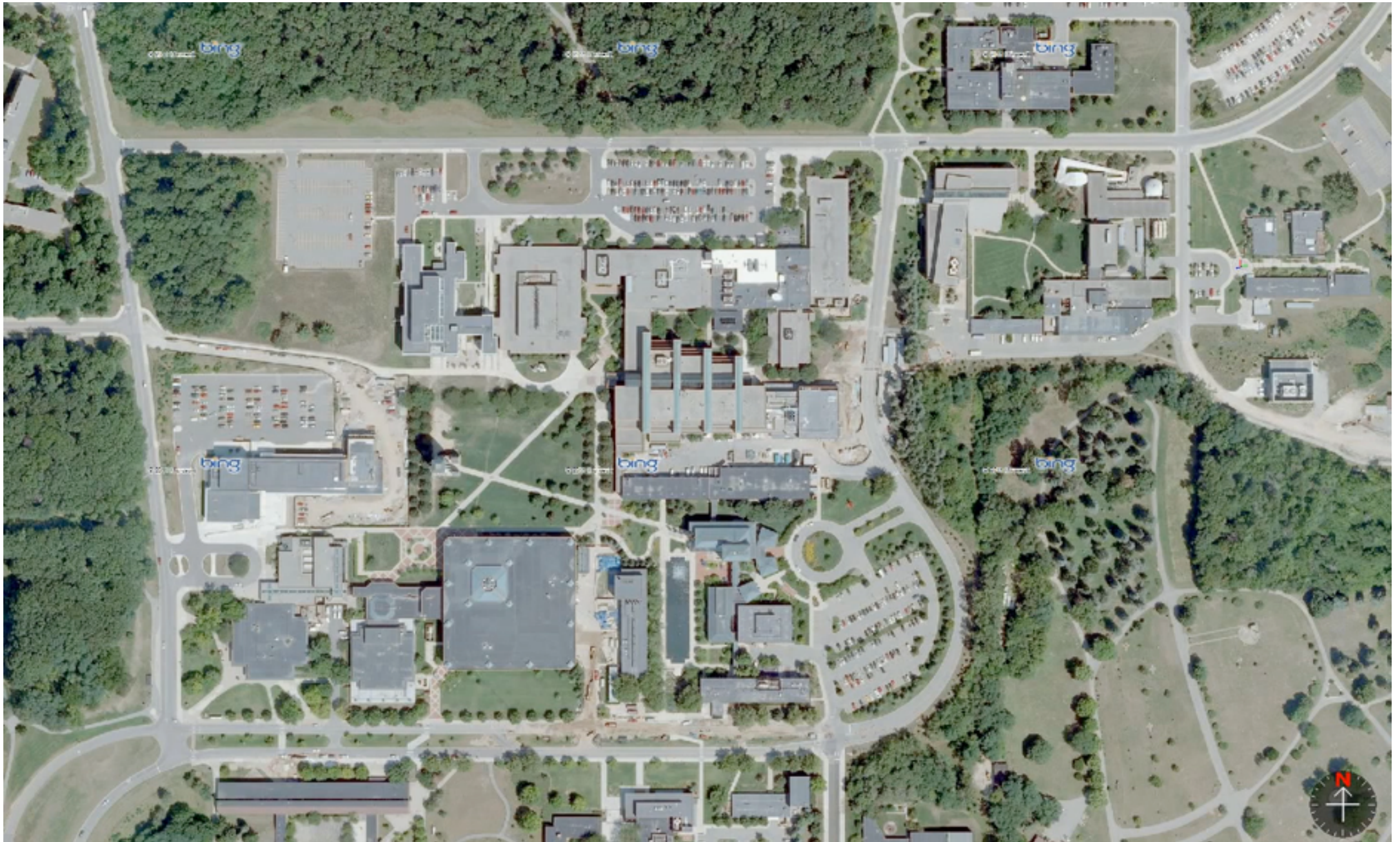
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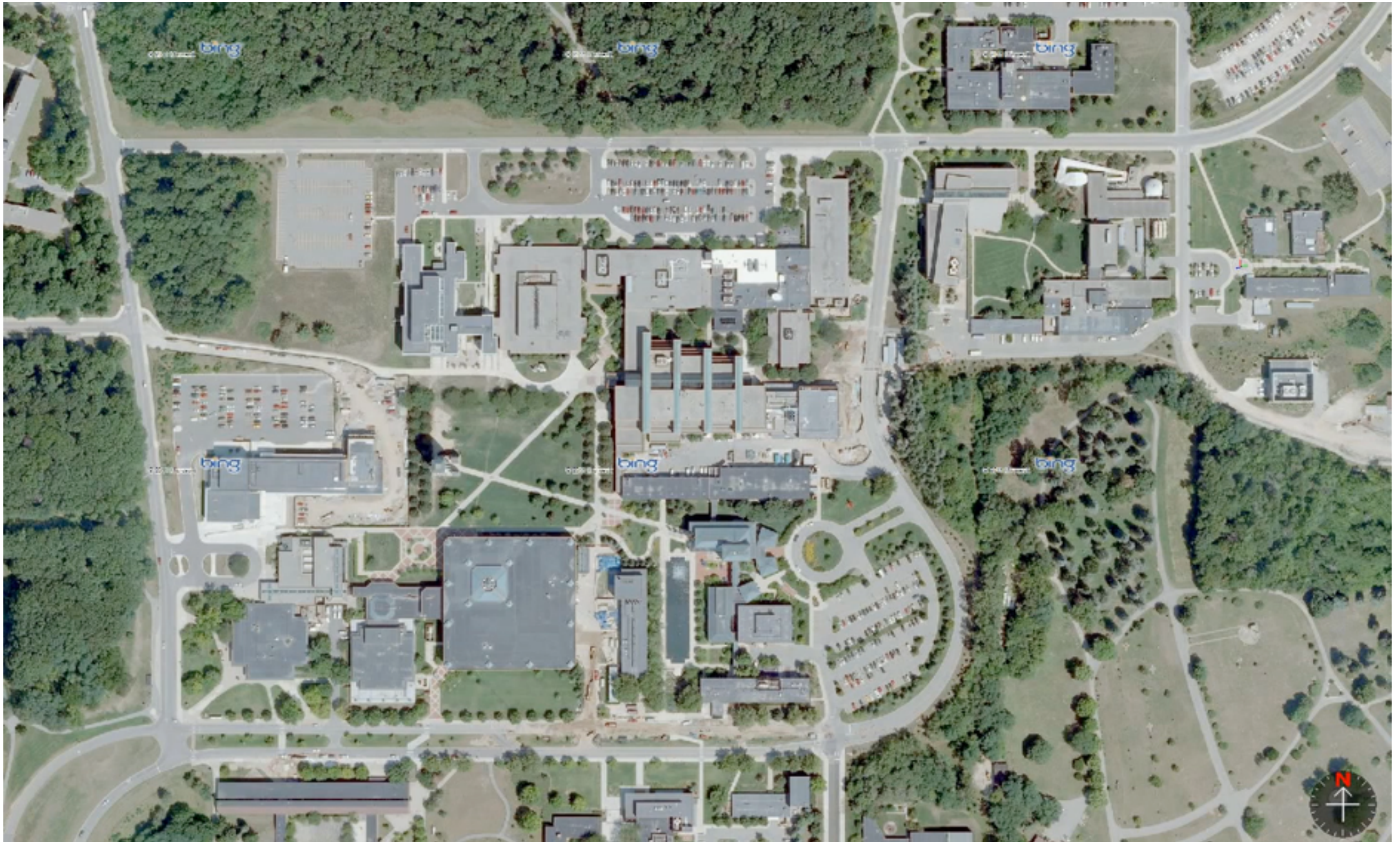
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2013: Full 3D LIDAR Mapping



Data/Movie by Nick Carlevaris-Bianco and Ryan Eustice (U. Michigan)

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Large-Scale Structure from Motion



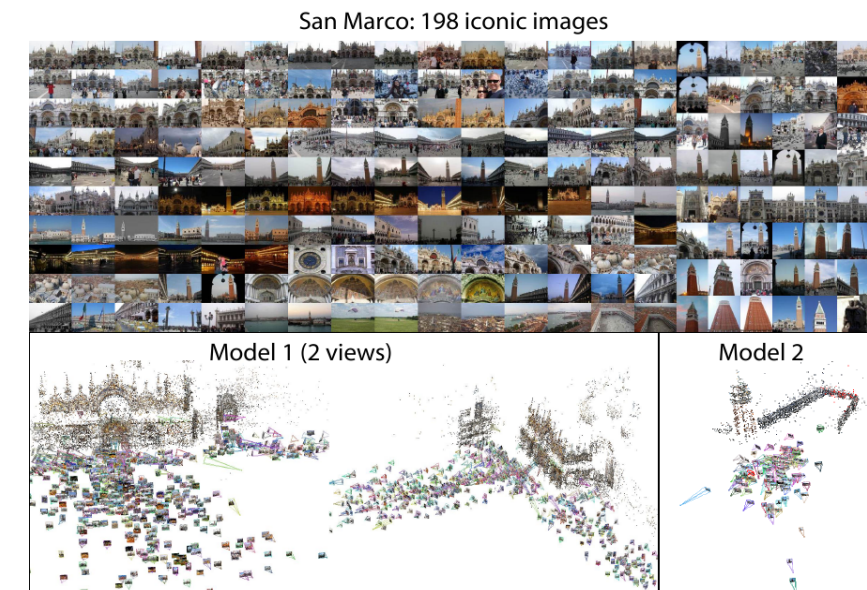
Photo Tourism

[Snavely et al. SIGGRAPH'06]



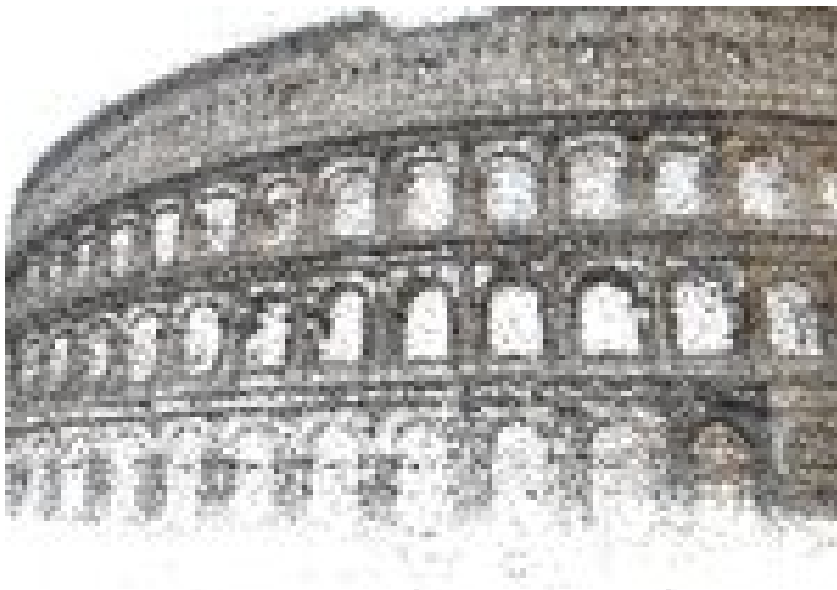
Photosynth

[Microsoft]



Iconic Scene Graphs

[Li et al. ECCV'08]



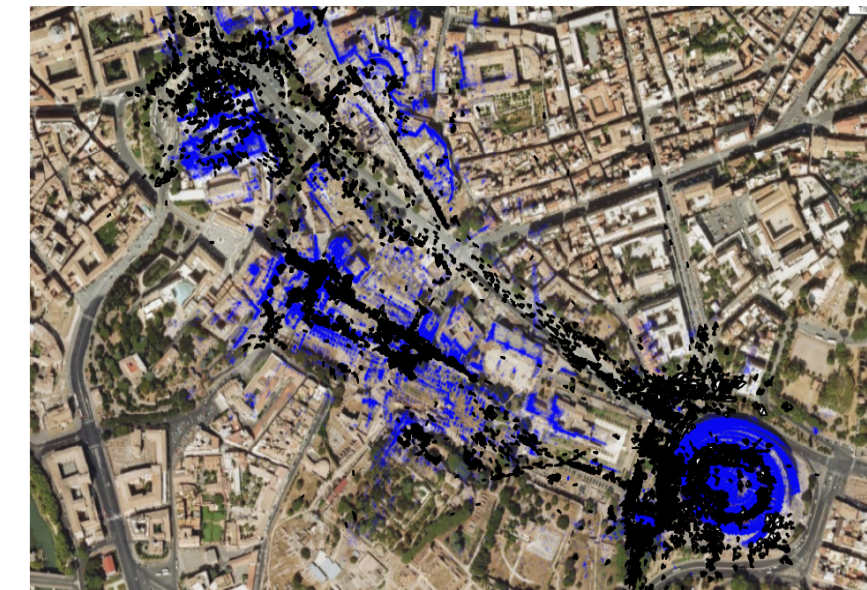
Build Rome in a Day

[Agarwal et al. ICCV'09]



Build Rome on a Cloudless Day

[Frahm et al. ECCV'10]

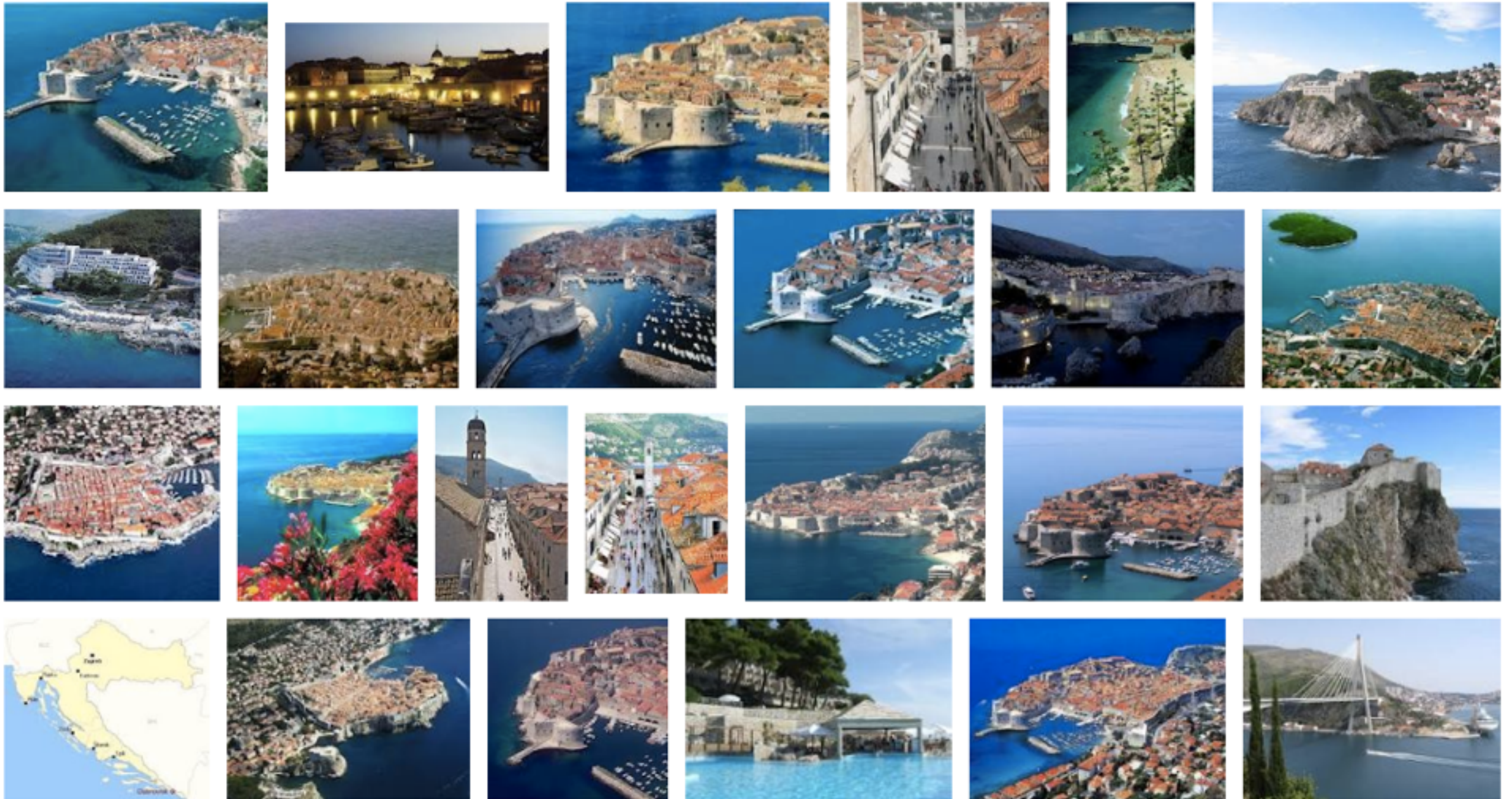


Discrete-Continuous Optim.

[Crandall et al. CVPR'11]

3D Models from Community Databases

- E.g., Google image search on “Dubrovnik”



3D Models from Community Databases

Agarwal, Snavely et al., University of Washington

<http://grail.cs.washington.edu/rome/>



5K images, 3.5M points, >10M factors

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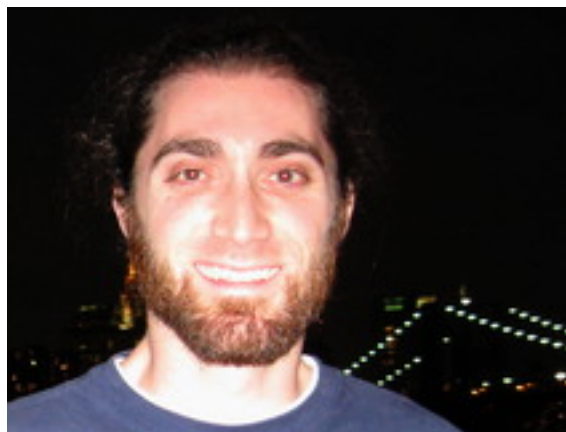


5K images, 3.5M points, >10M factors

4D Reconstruction

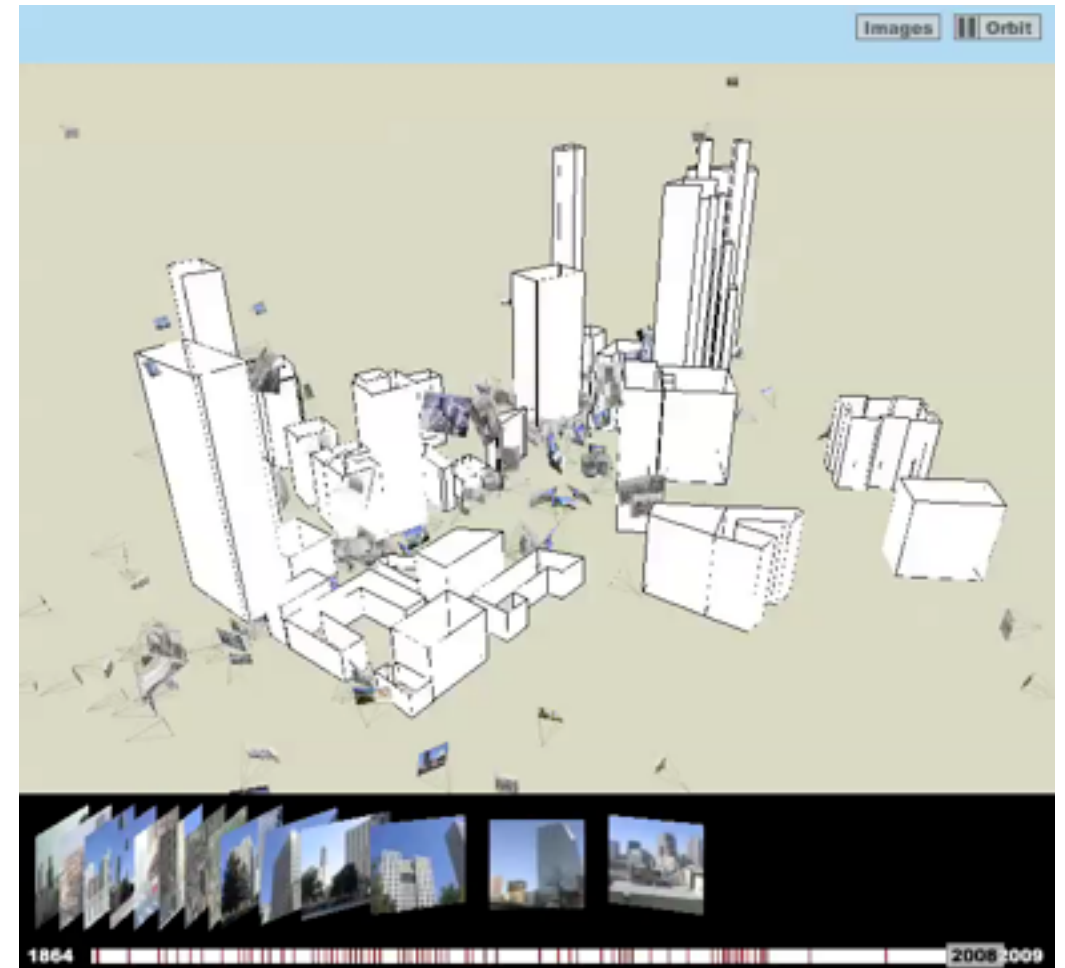


Historical Image Collection



Grant Schindler

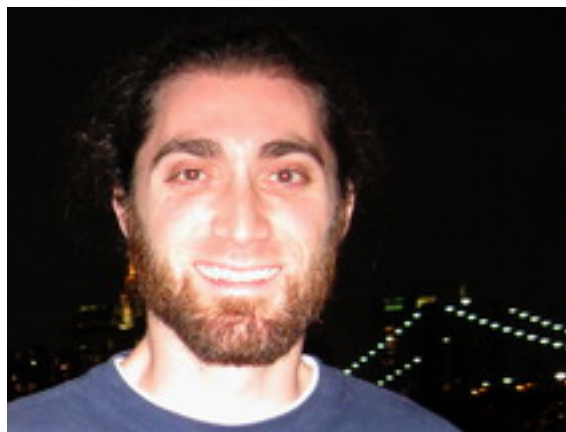
4D Cities: 3D + Time



4D Reconstruction

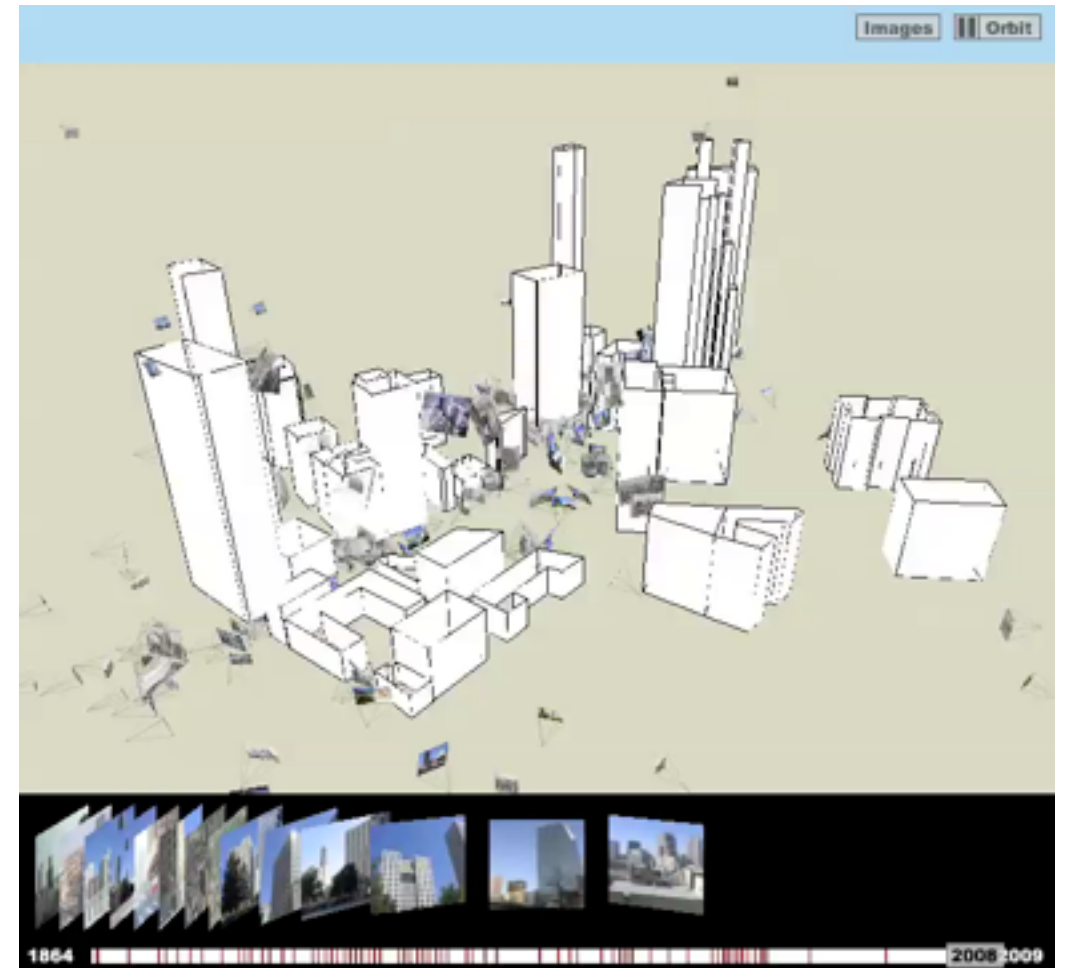


Historical Image Collection



Grant Schindler

4D Cities: 3D + Time

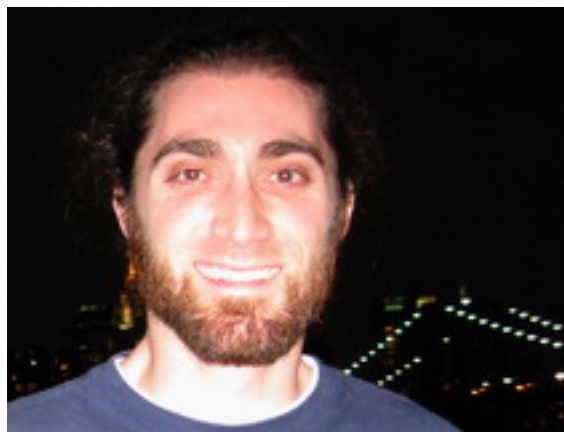
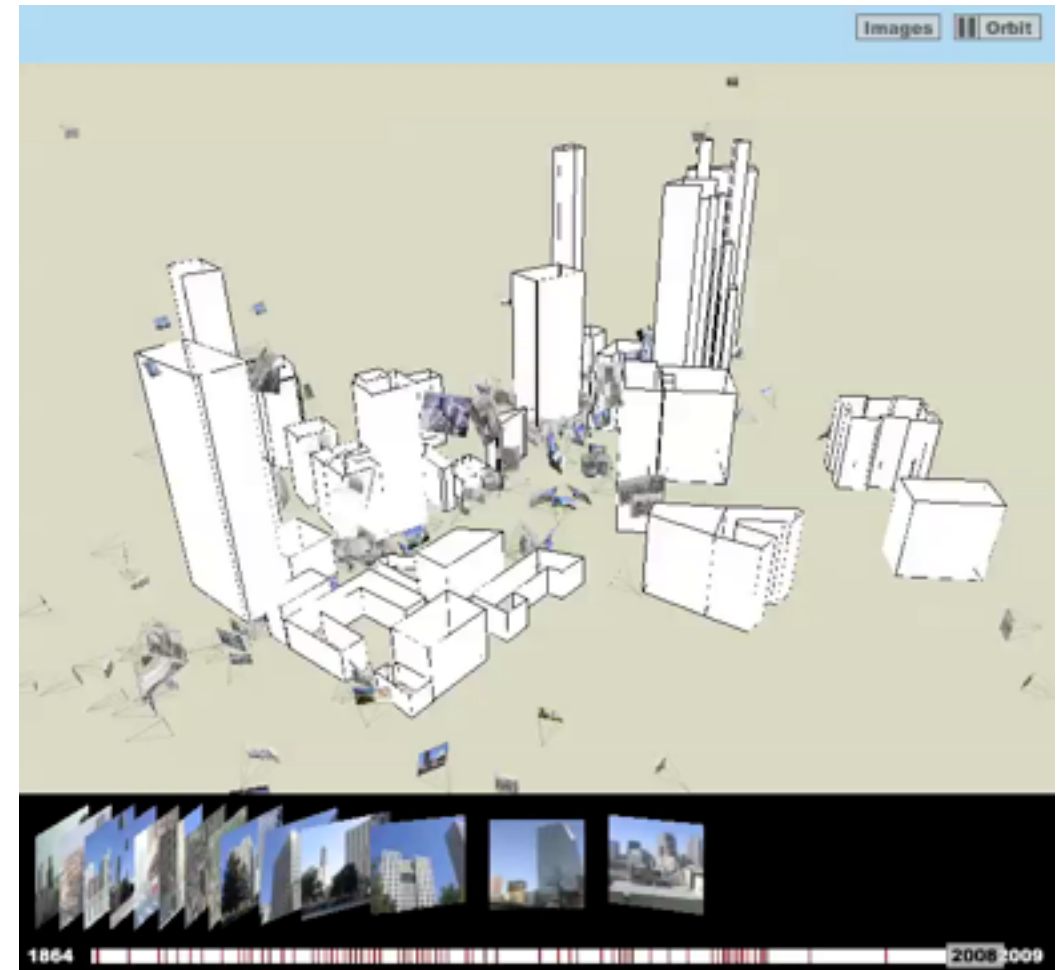
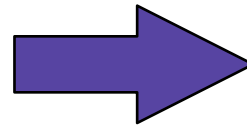


4D Reconstruction

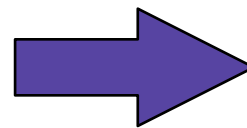
4D Cities: 3D + Time



Historical Image Collection



Grant Schindler



trimensional
3D Scanner for iPhone

3D Reconstruction



[Probabilistic Temporal Inference on Reconstructed 3D Scenes](#), G. Schindler and F. Dellaert,
IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), 2010.

3D Reconstruction



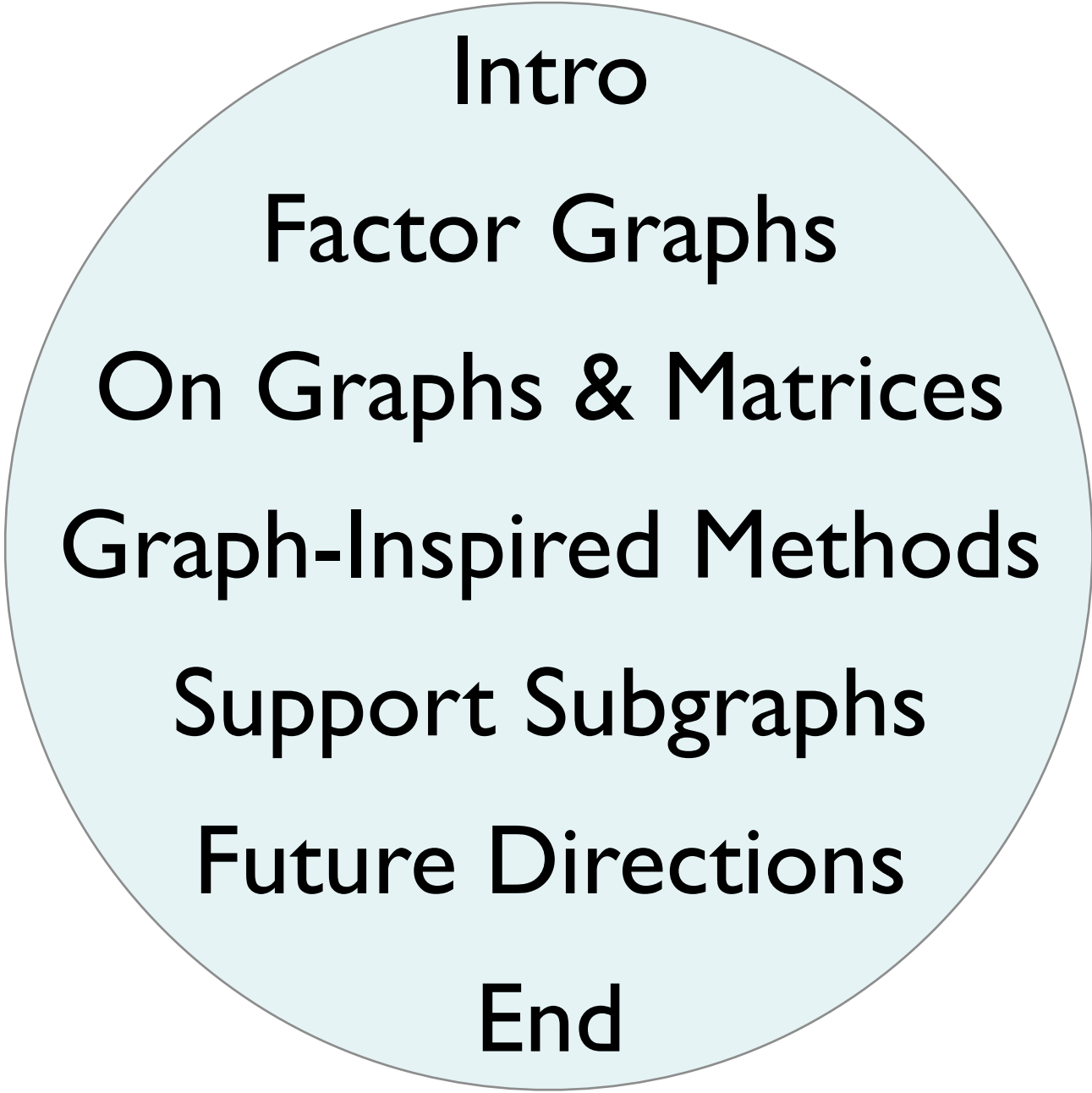
[Probabilistic Temporal Inference on Reconstructed 3D Scenes](#), G. Schindler and F. Dellaert,
IEEE Computer Society Conference on Computer Vision and Pattern Recognition (CVPR), 2010.

4D Structure over Time



4D Structure over Time





Intro
Factor Graphs
On Graphs & Matrices
Graph-Inspired Methods
Support Subgraphs
Future Directions
End

Acknowledgements

Ph.D. Students & Postdocs



Chris Beall



Alex Cunningham



Can Erdogan



Yong-Dian Jian



Abhijit Kundu



Andrew Melim



Manohar Paluri



Richard Roberts



Natesh Srinivasan



Duy-Nguyen Ta



Stephen Williams



Vadim Indelman

Alumni



Kai Ni



Ananth Ranganathan



Grant Schindler



Michael Kaess



Viorela Ila



Doru Balcan

- Provided Materials: Michael Kaess & John Leonard (MIT), Ryan Eustice & Ed Olson (Michigan), Tim Barfoot (Toronto), Pratik Agarwal (Freiburg)
- Sponsors: NSF, DARPA, ONR, ARO/ARL, Microsoft Research, Samsung, Intel
- Noah Snavely, Sameer Agarwal, and Steve Seitz for cool data/videos

Intro

Factor Graphs

On Graphs & Matrices

Graph-Inspired Methods

Support Subgraphs

Future Directions

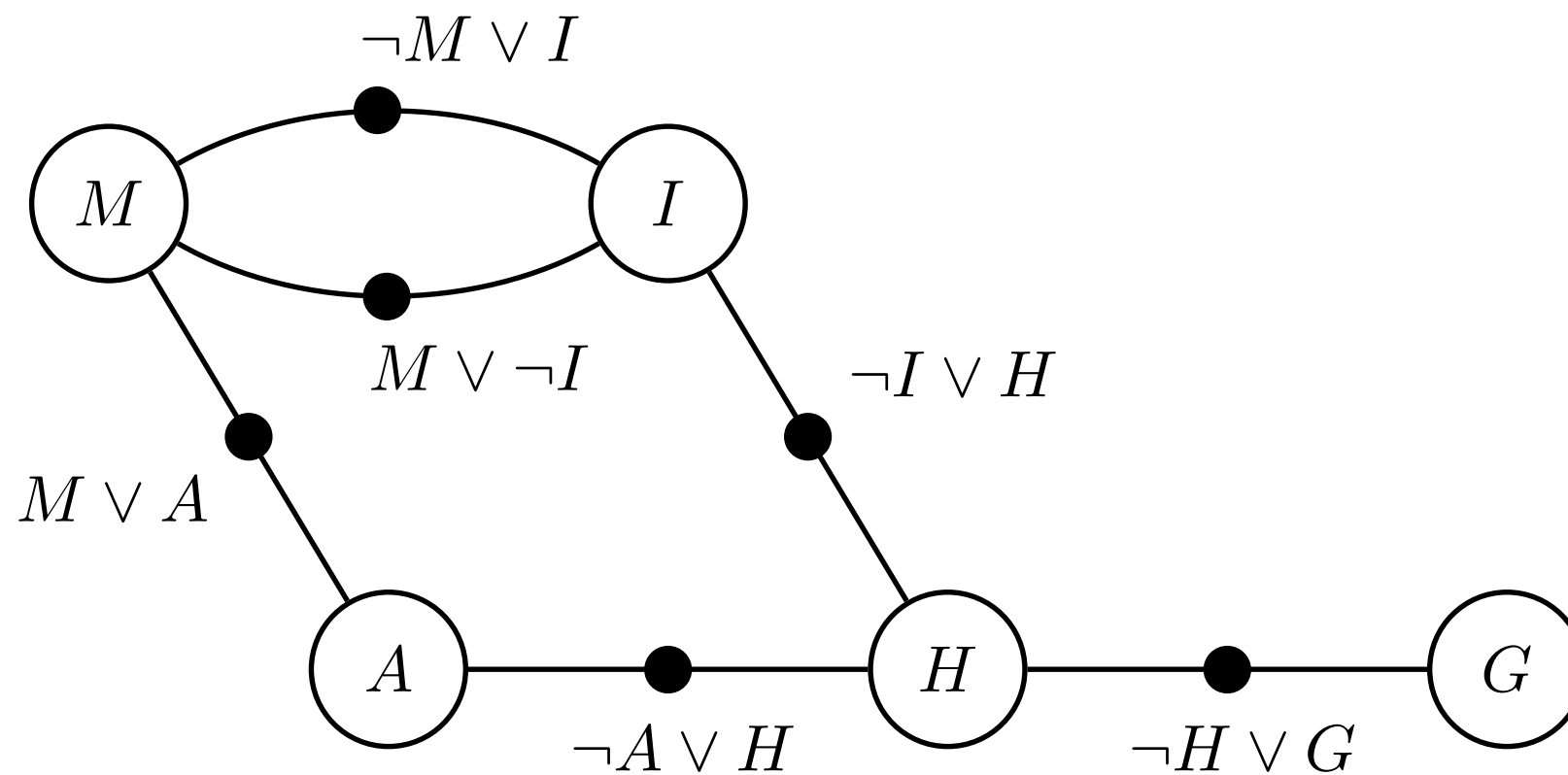
End

Boolean Satisfiability

$$(\neg M \vee I) \wedge (M \vee \neg I) \wedge (M \vee A) \wedge (\neg I \vee H) \wedge (\neg A \vee H) \wedge (\neg H \vee G)$$

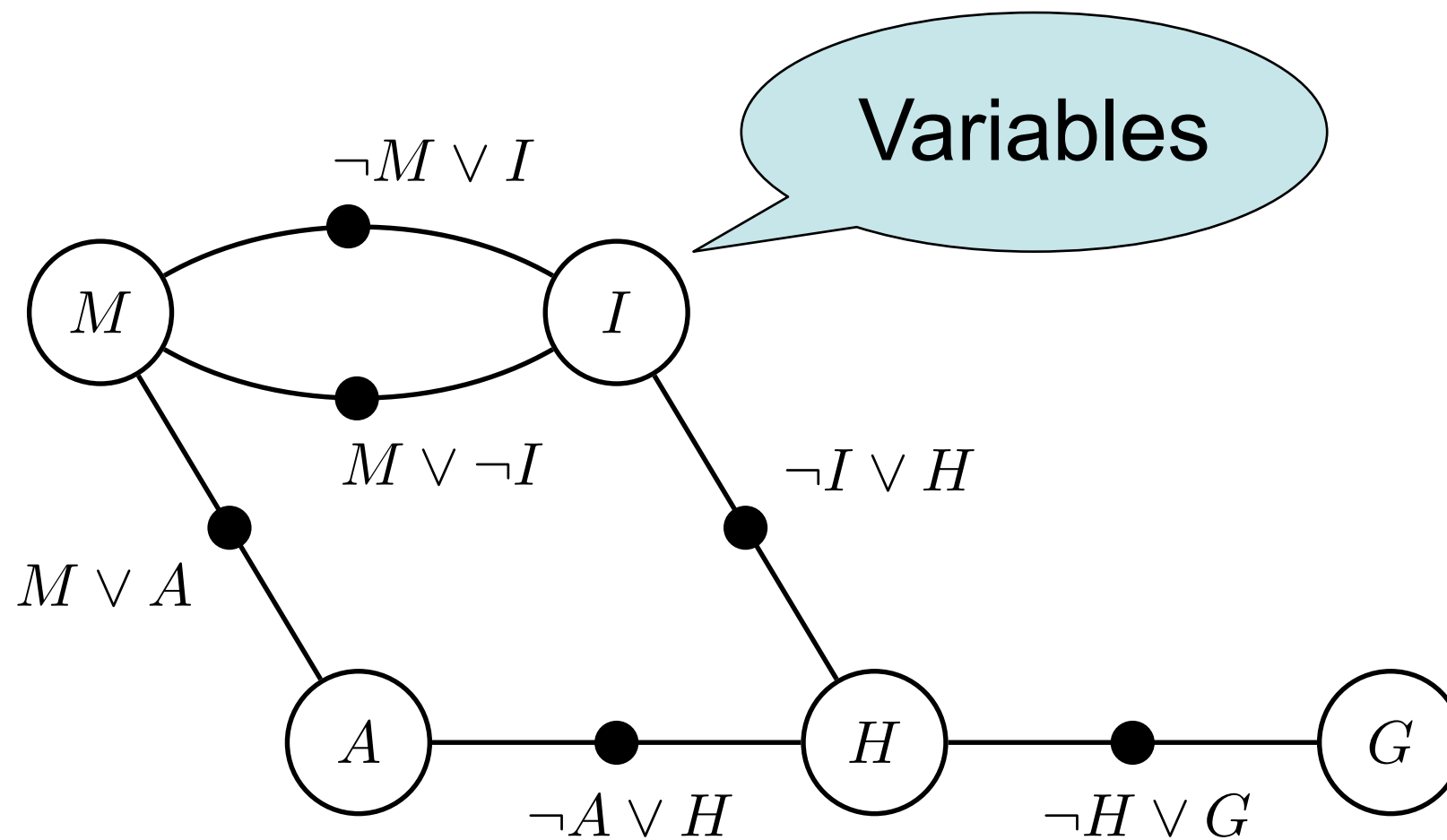
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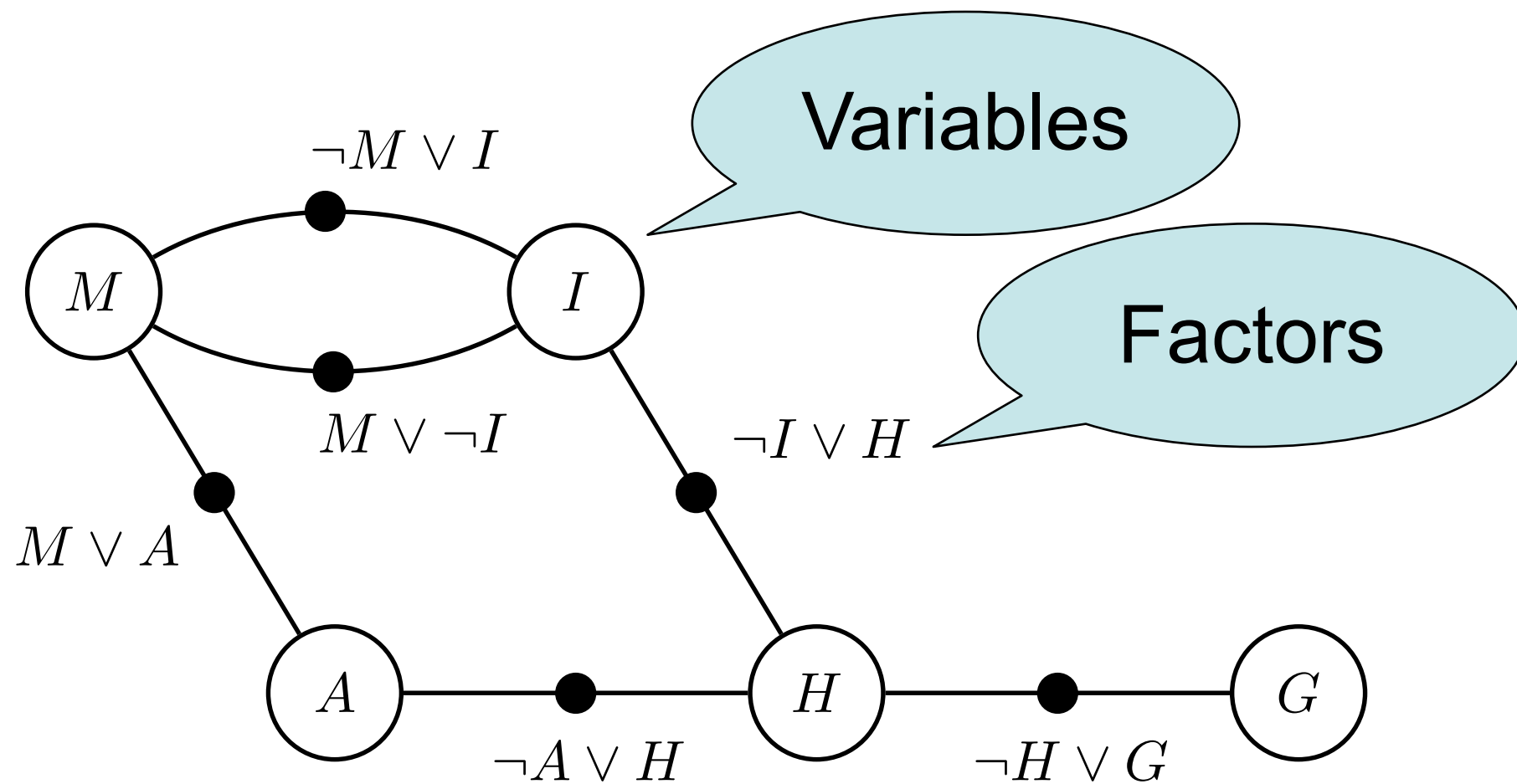
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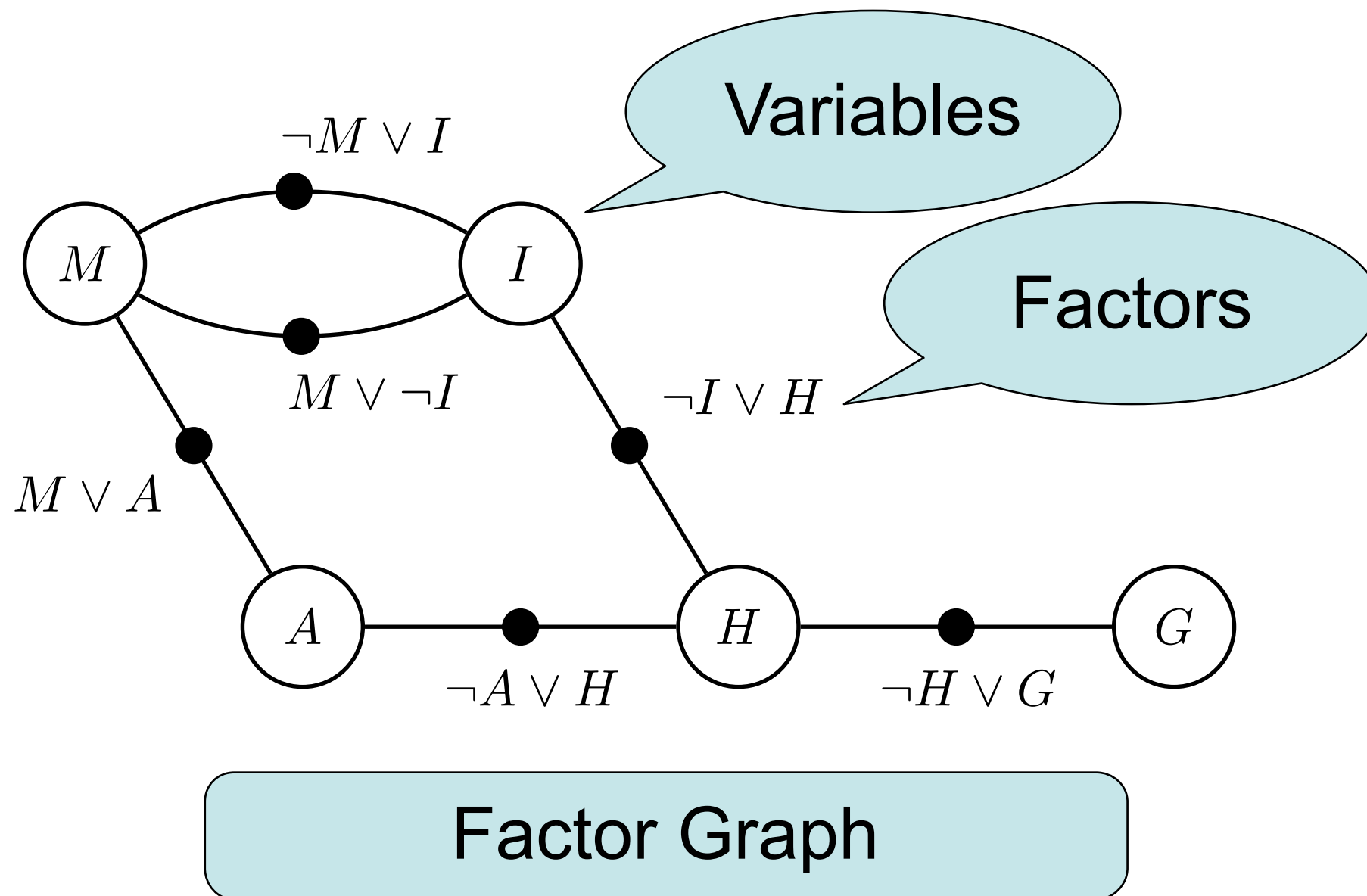
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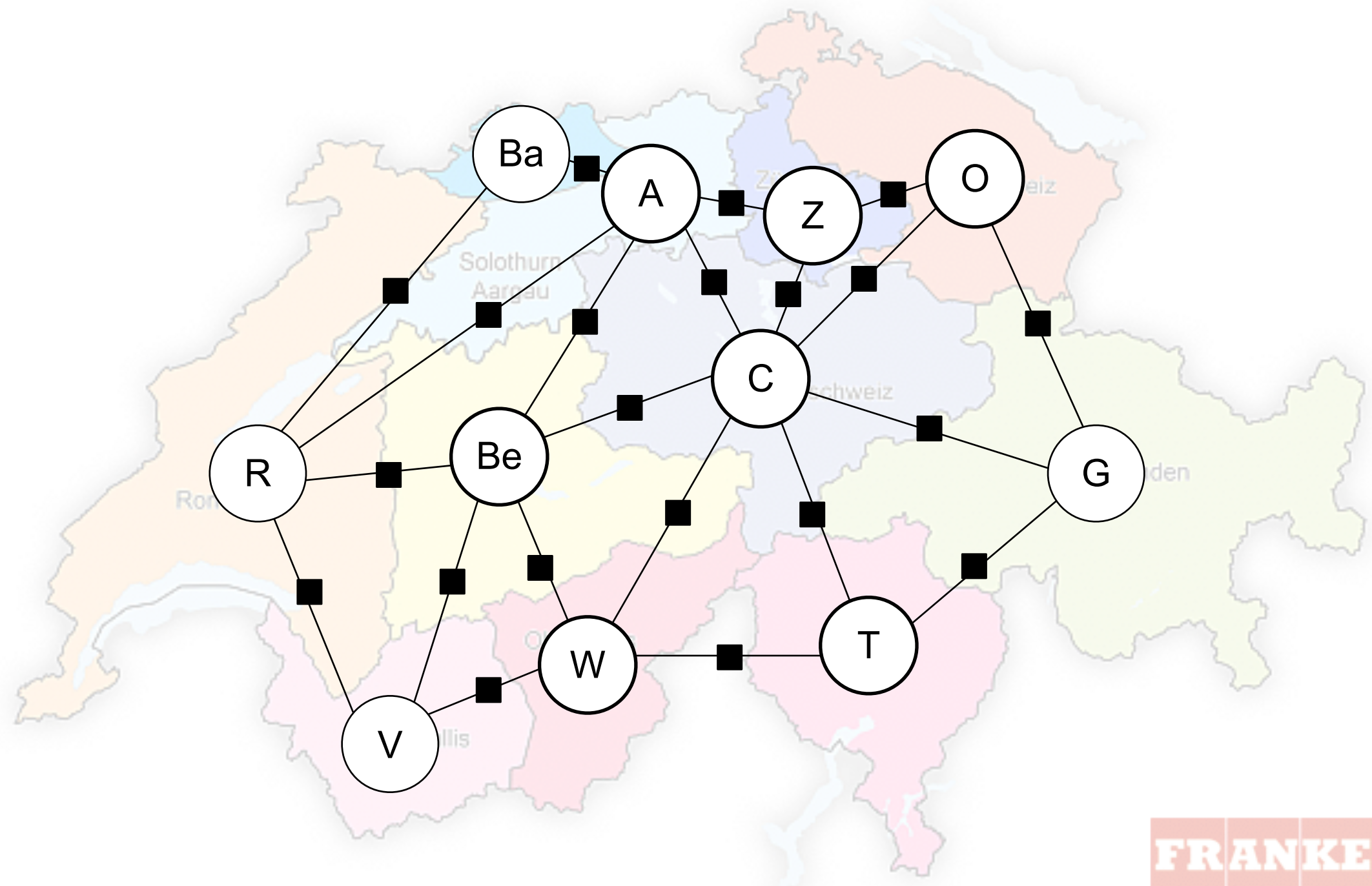
Constraint Satisfaction Problems



Constraint Satisfaction Problems

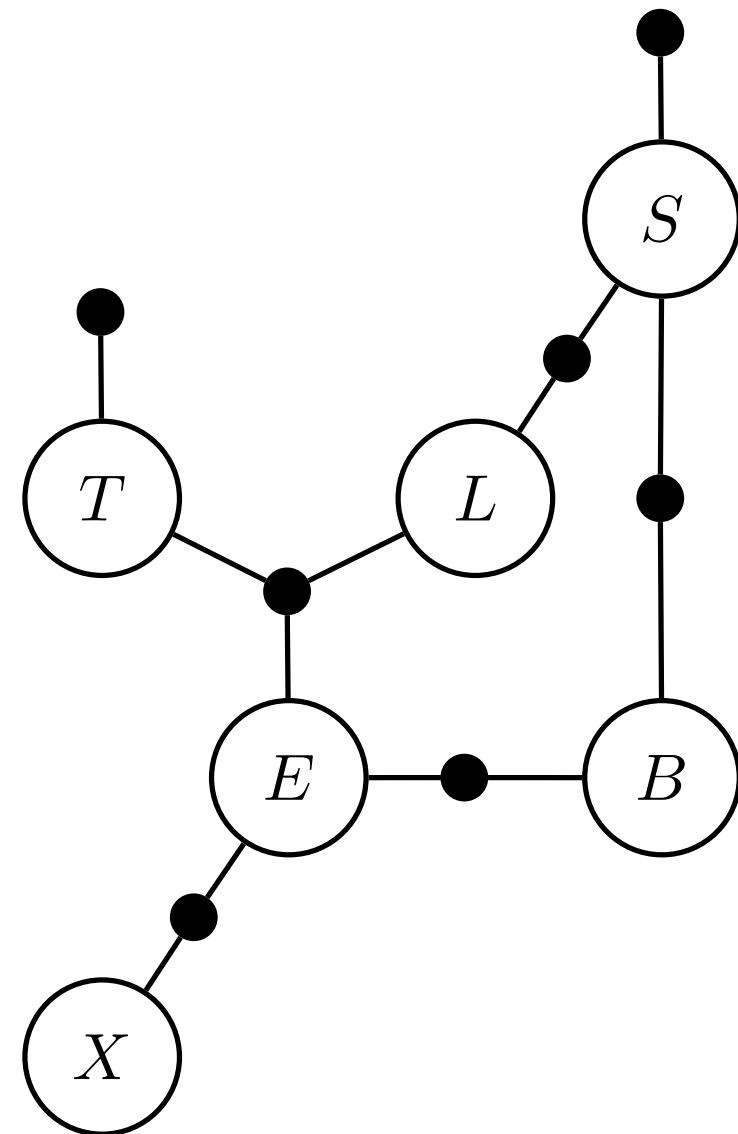
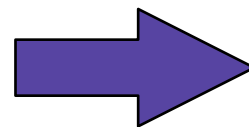
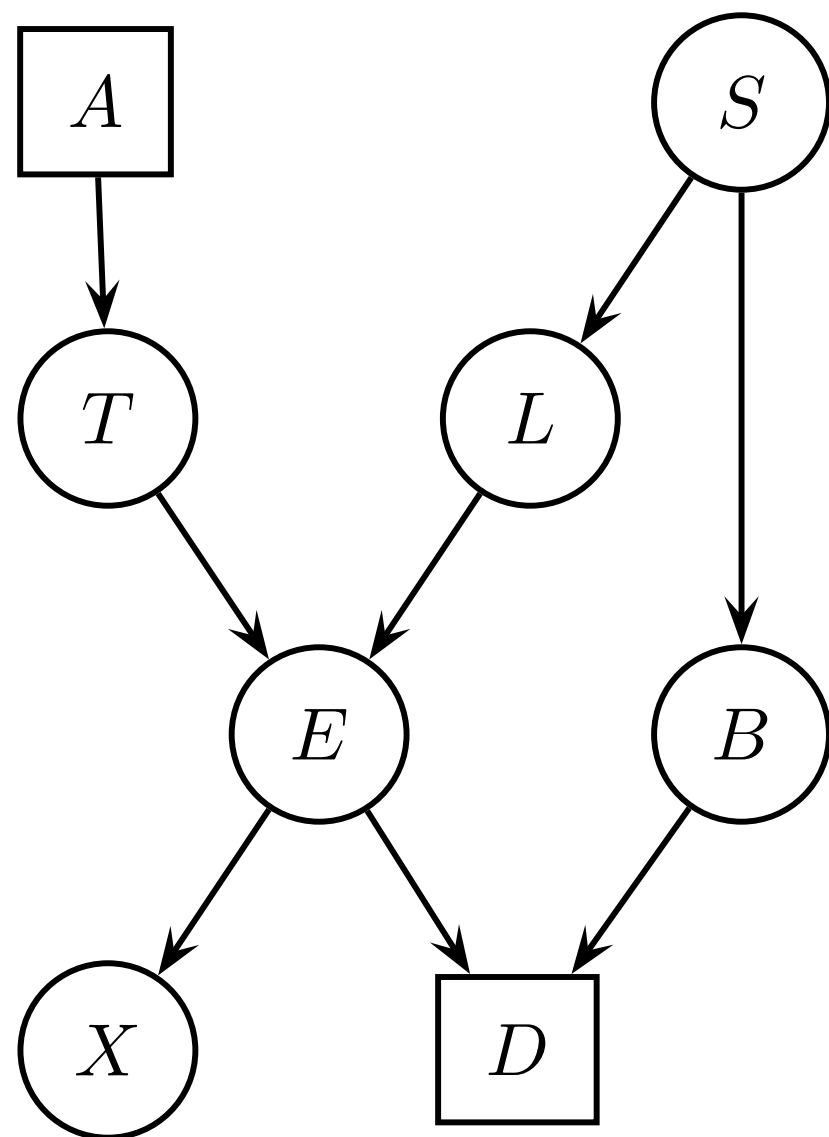


Constraint Satisfaction Problems



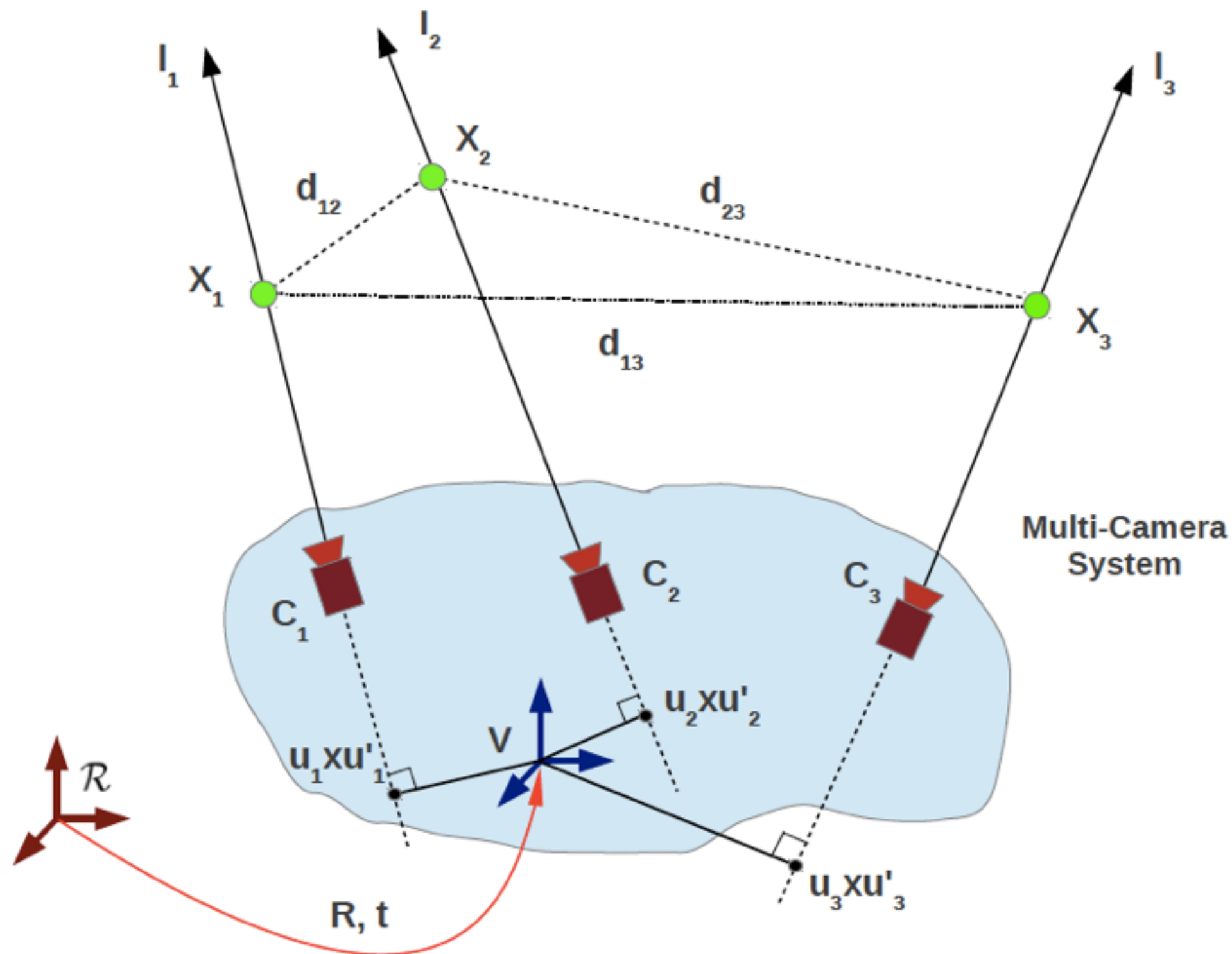
FRANKE

Graphical Models



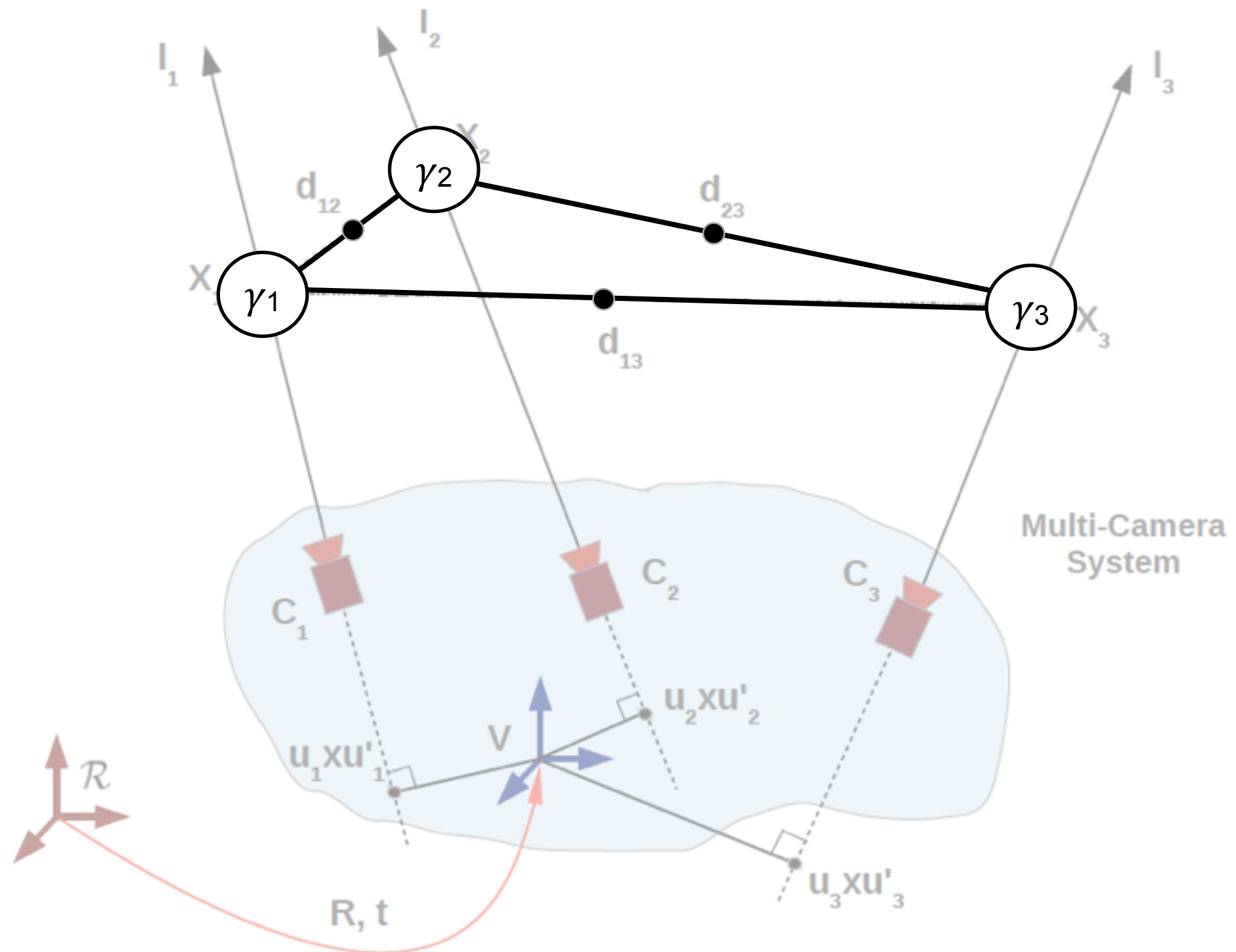
Polynomial Equations

Gim Hee Lee's thesis: multi-camera pose estimation

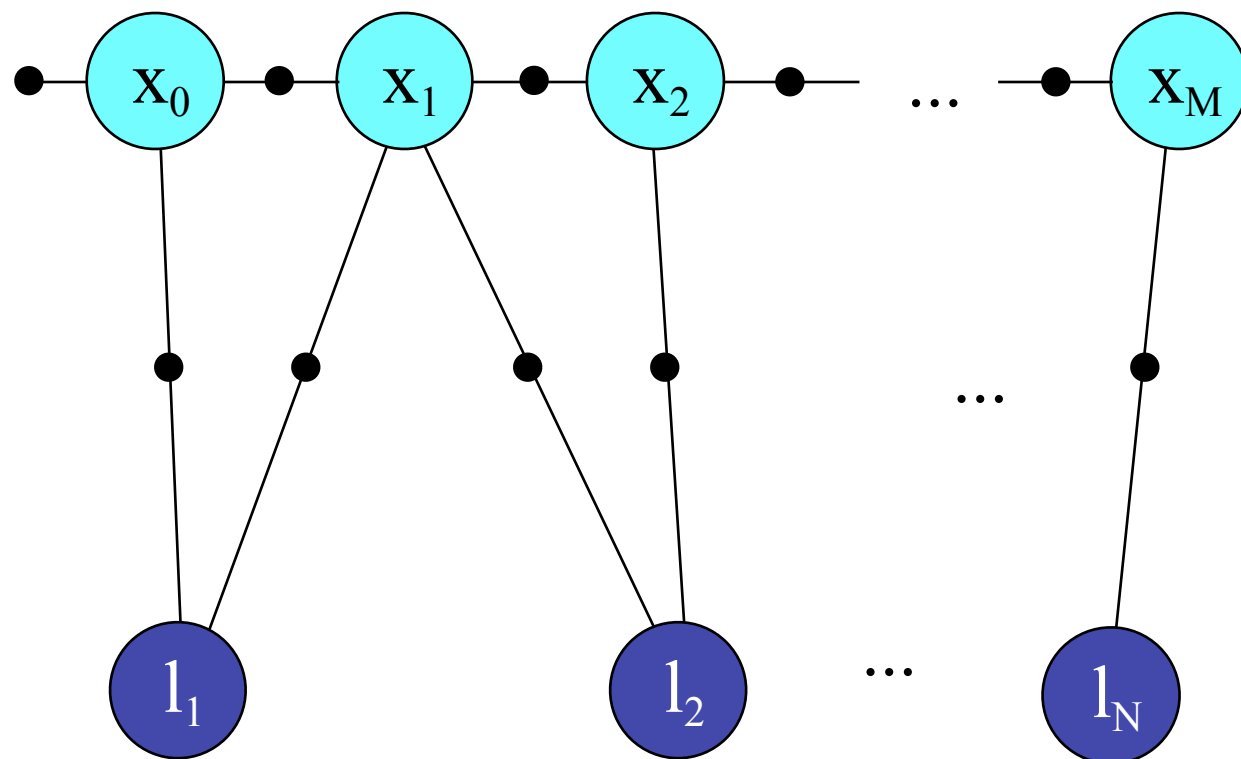


Polynomial Equations

Gim Hee Lee's thesis: multi-camera pose estimation



Continuous Probability Densities



- Trajectory of Robot

- Measurements

- Landmarks

$$P(X, M) = k^* P(x_0) \prod_{i=1}^M P(x_i | x_{i-1}, u_i) \times \prod_{k=1}^K P(z_k | x_{i_k}, l_{j_k})$$

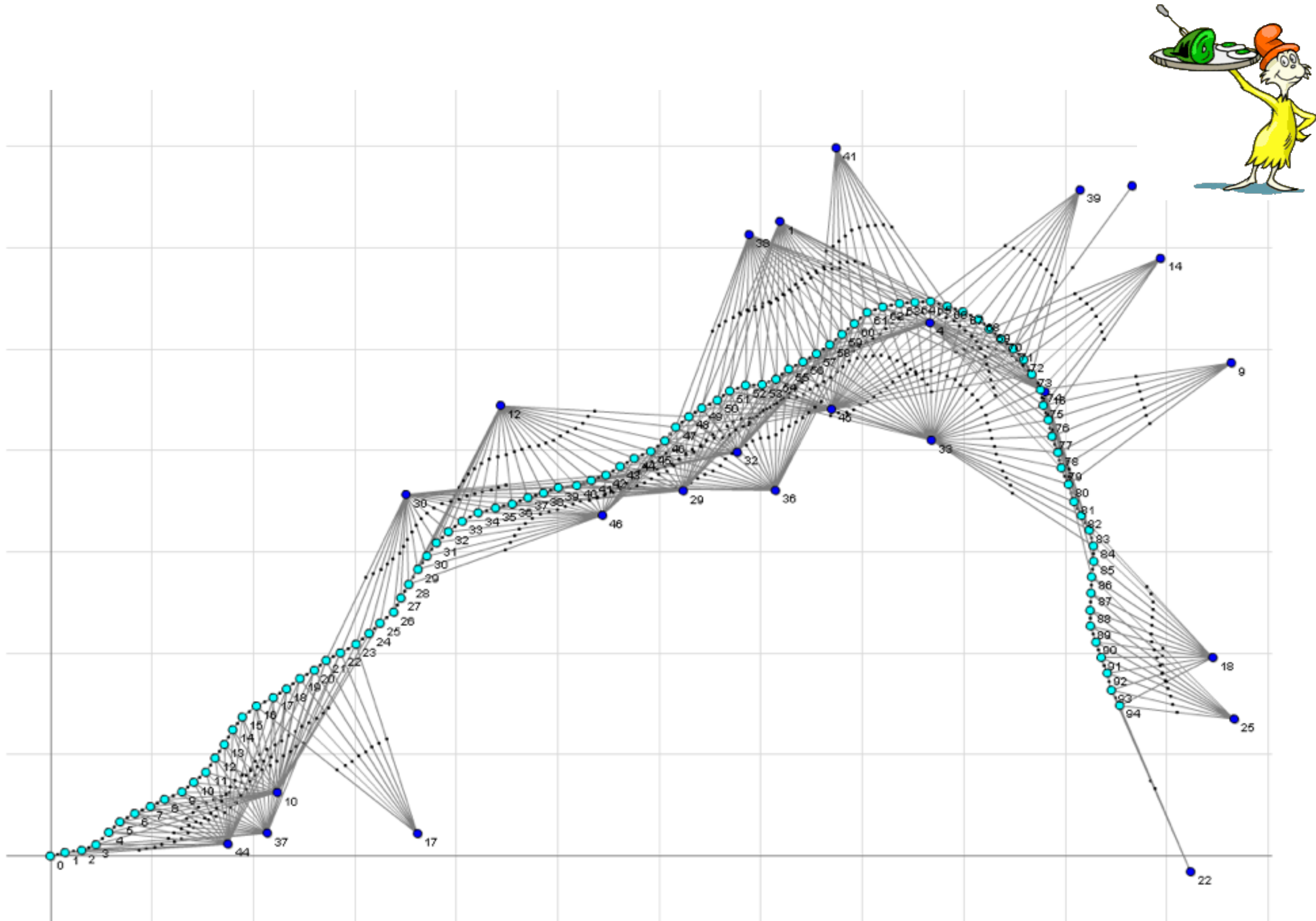
Simultaneous Localization and Mapping (SLAM)



Simultaneous Localization and Mapping (SLAM)

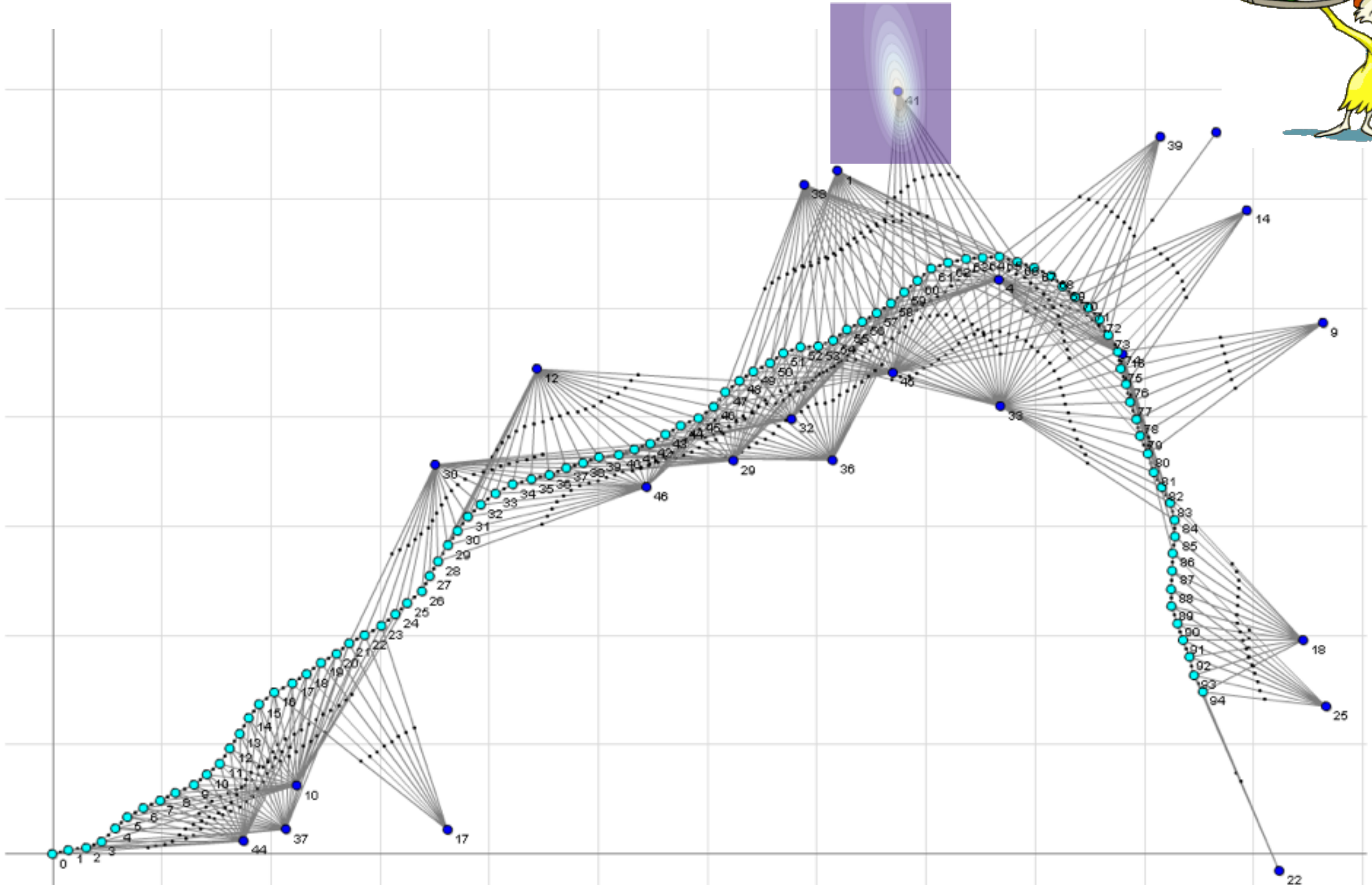


Factor Graph -> Smoothing and Mapping (SAM) !



Factor Graph -> Smoothing and Mapping (SAM) !

Variables are multivariate! Factors are non-linear!

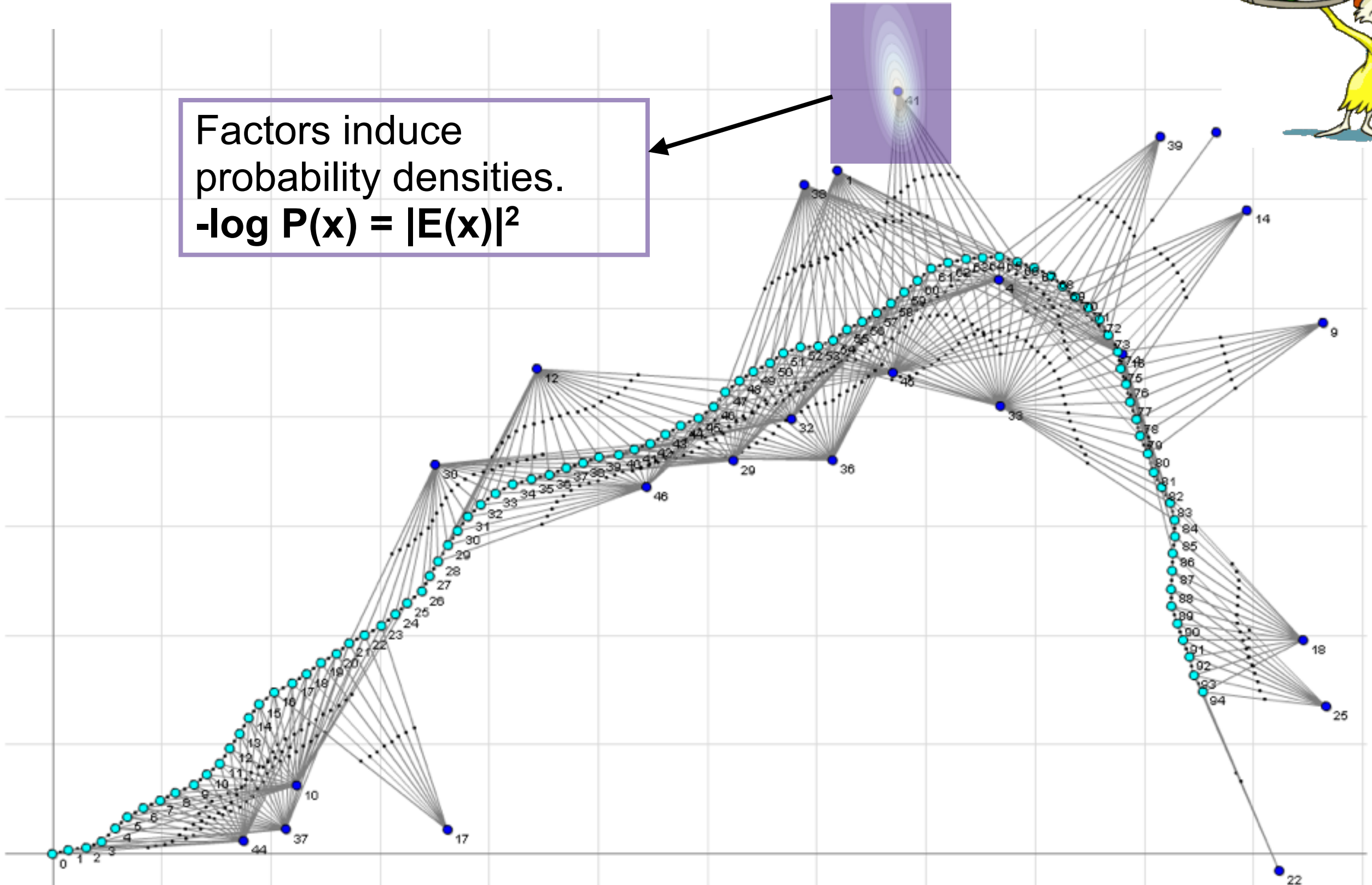


Factor Graph -> Smoothing and Mapping (SAM) !

Variables are multivariate! Factors are non-linear!



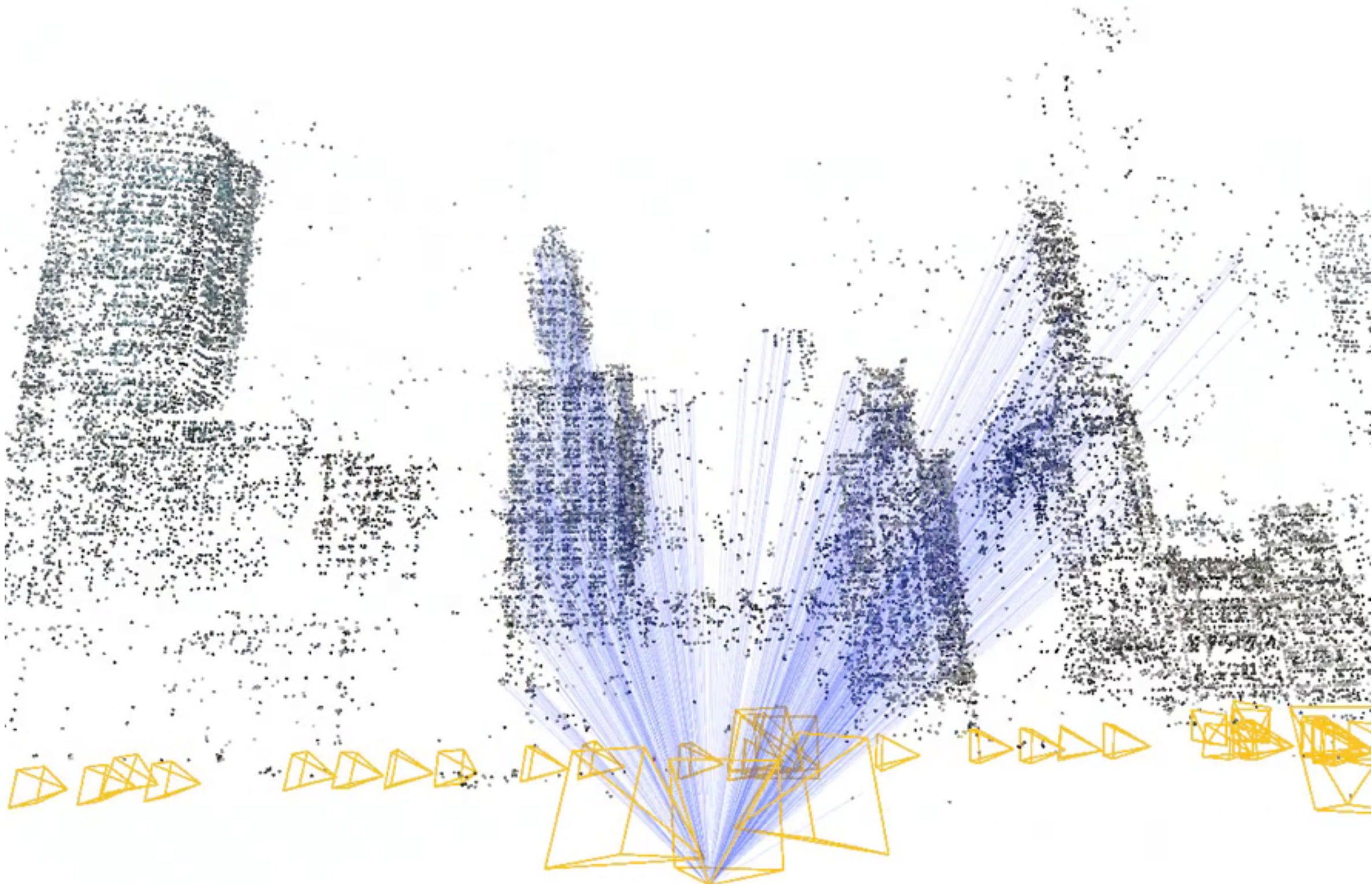
Factors induce probability densities.
 $-\log P(\mathbf{x}) = |\mathbf{E}(\mathbf{x})|^2$



Structure from Motion (Chicago, movie by Yong Dian Jian)

180 cameras, 88723 points
458642 projections
active camera: 4

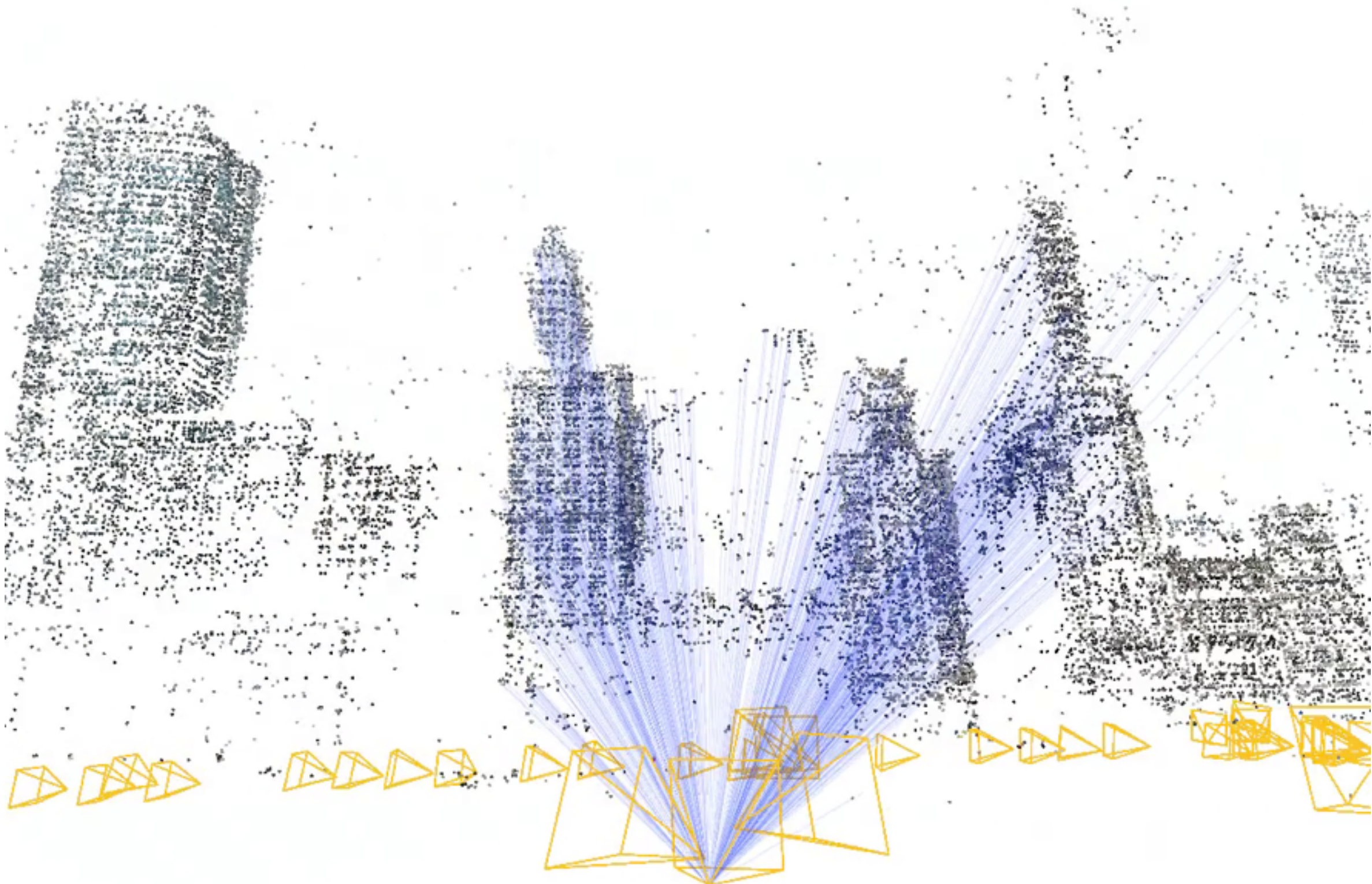
Original graph

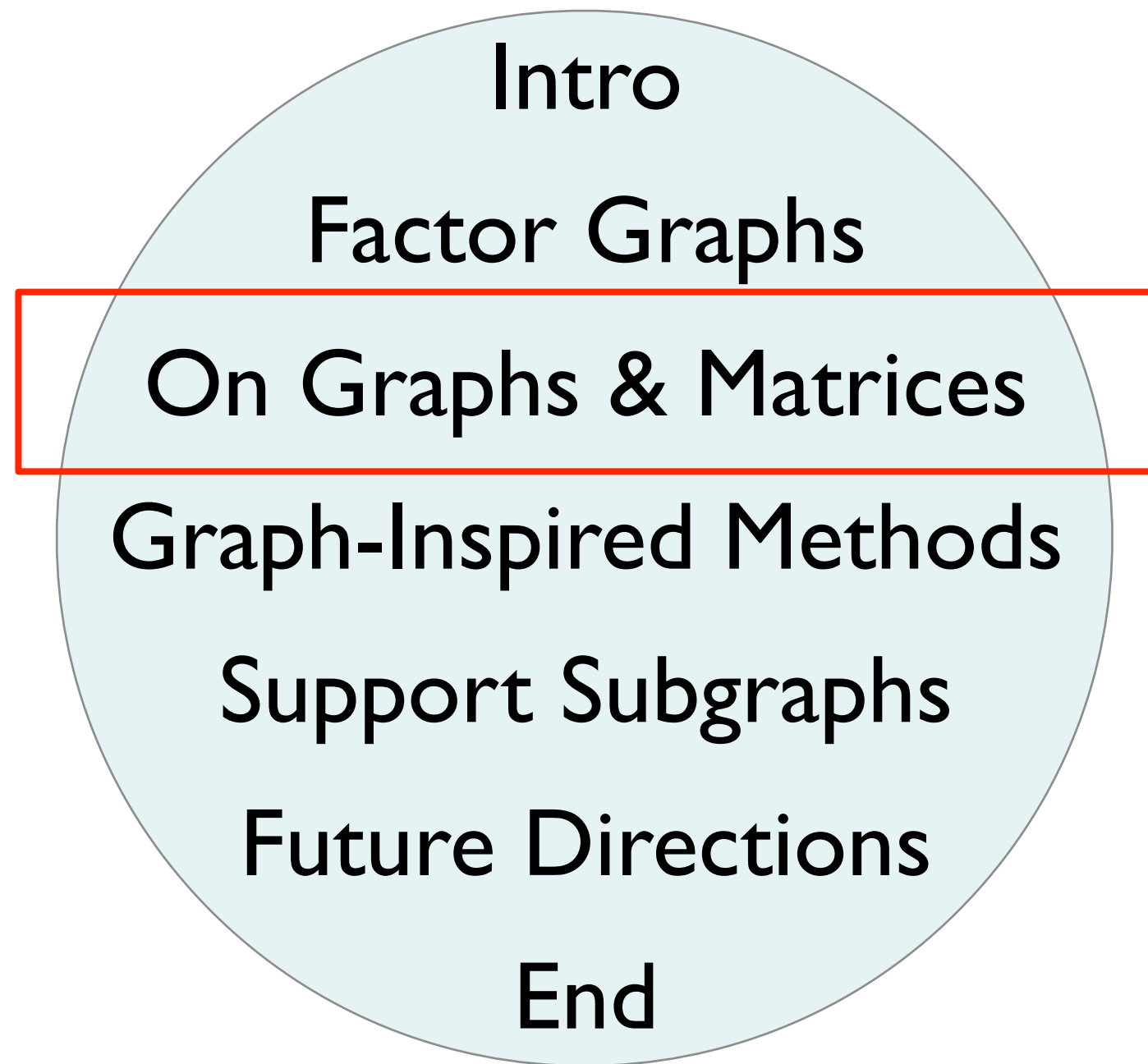


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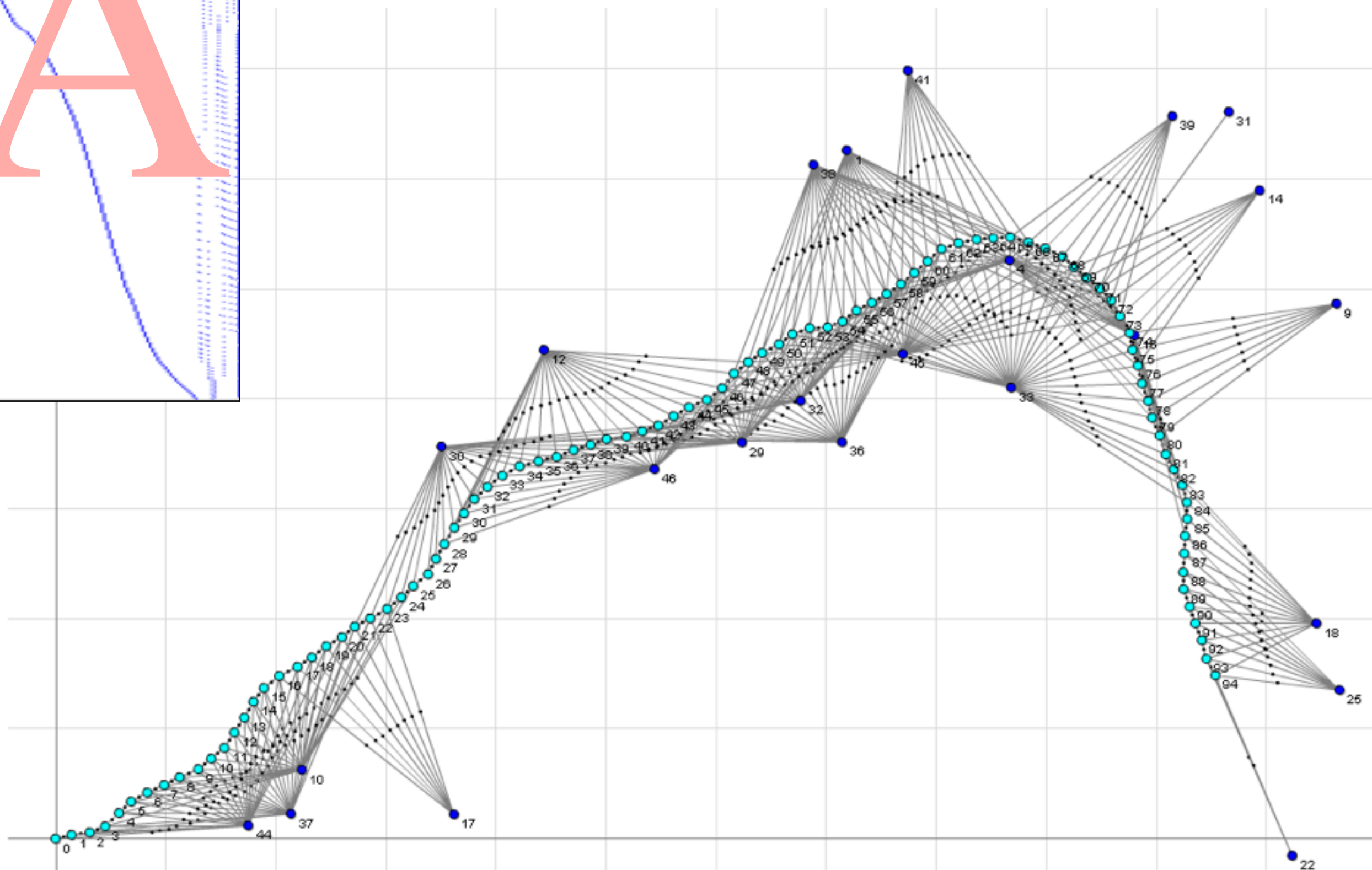
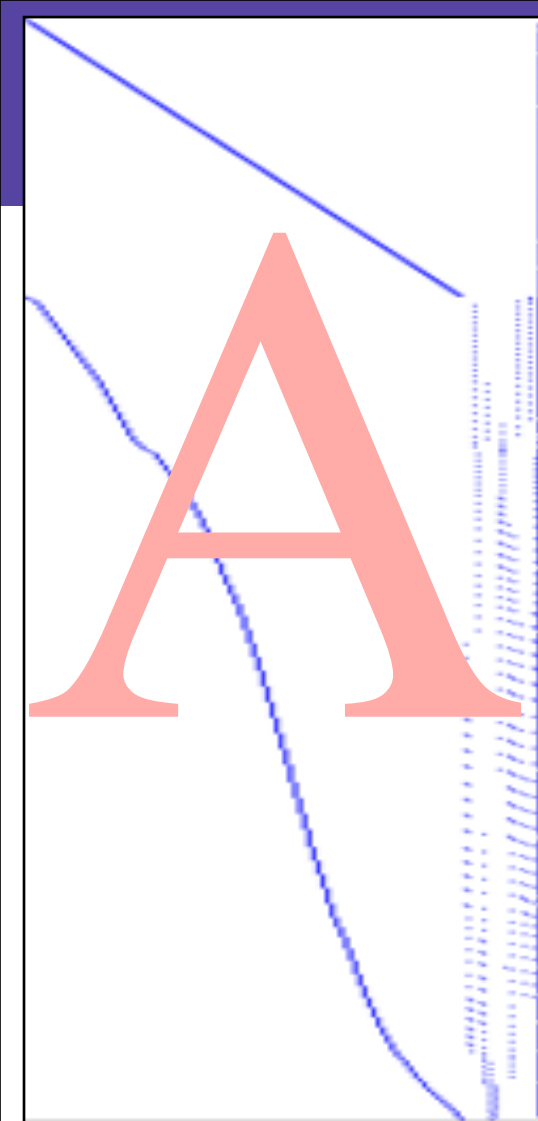
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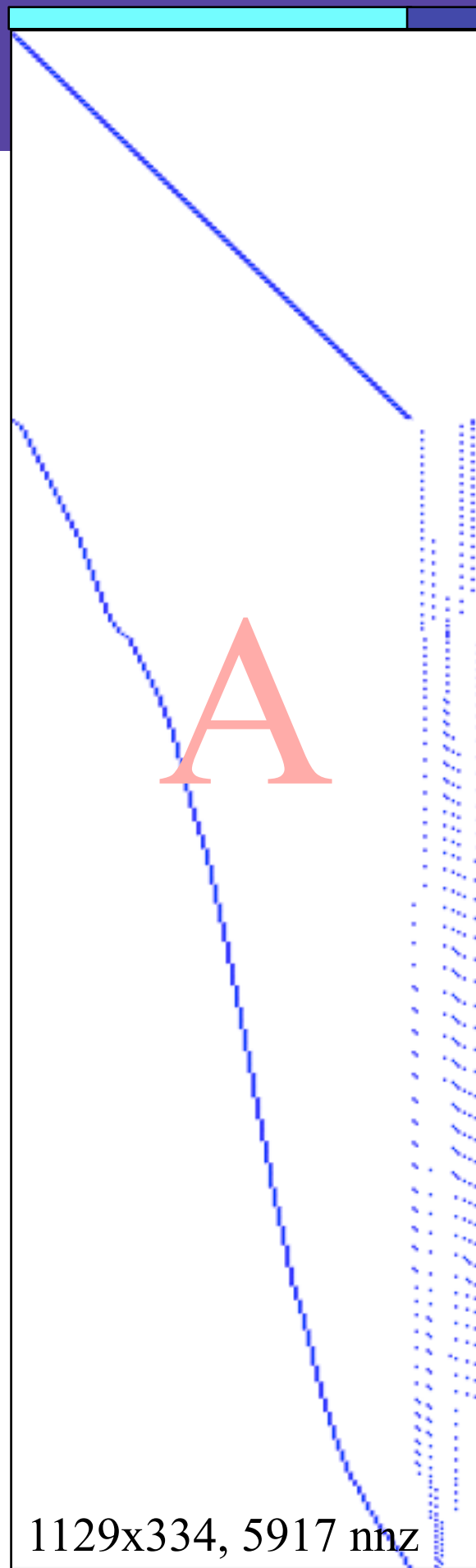




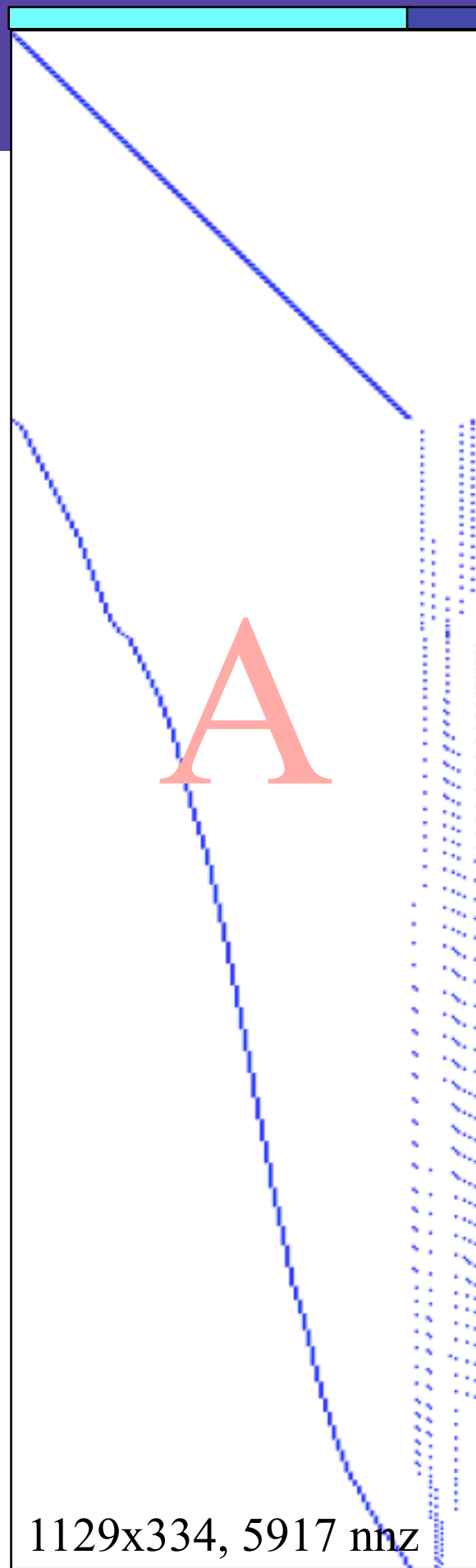
Gaussian Factor Graph == $m \times n$ Matrix



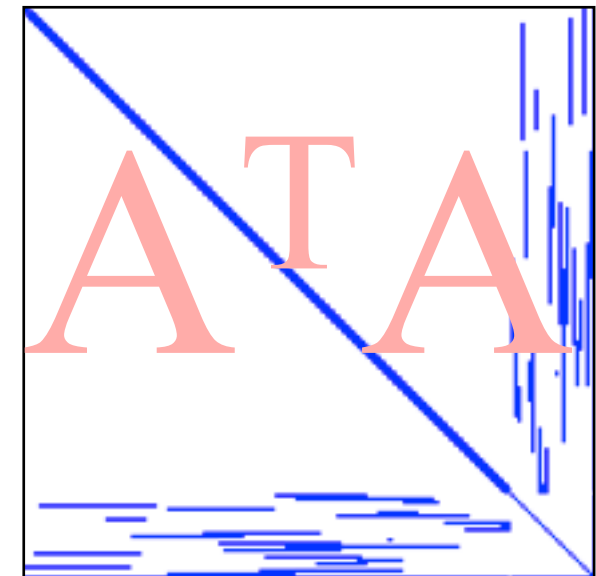
Linear Least Squares



Linear Least Squares

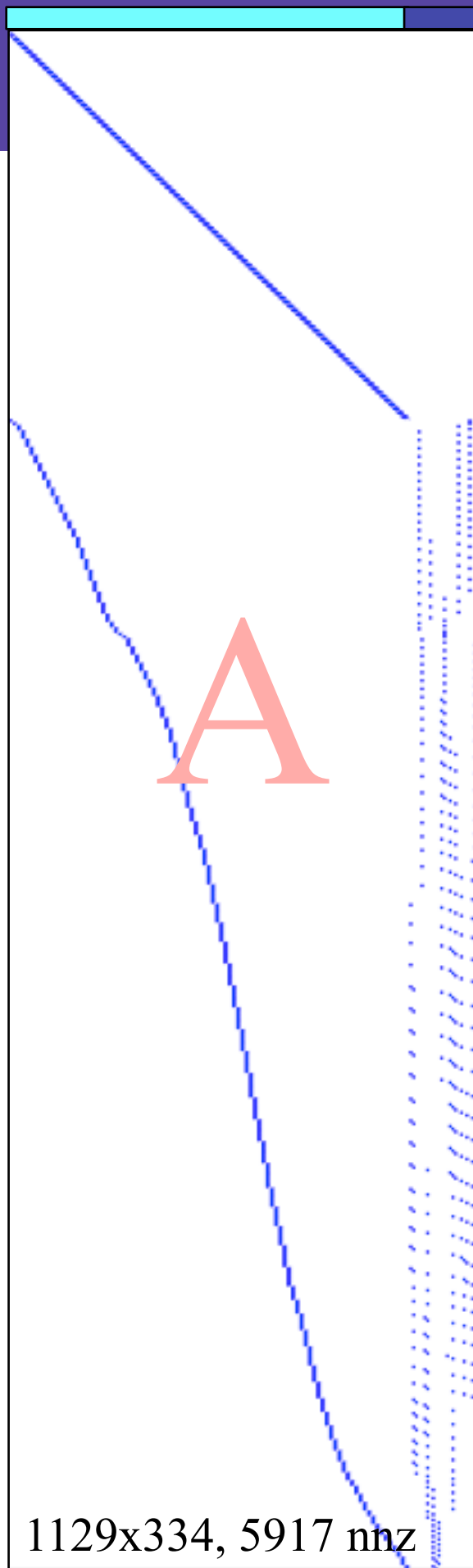


$$A^T A \theta^* = A^T b$$

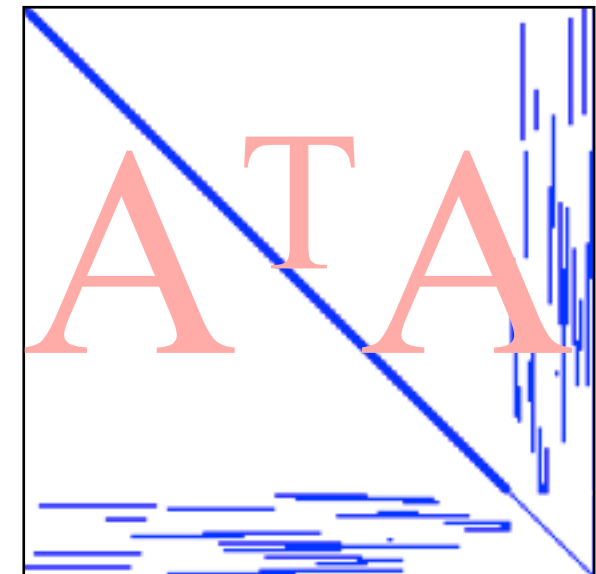


334x334, 7992 nnz

Linear Least Squares

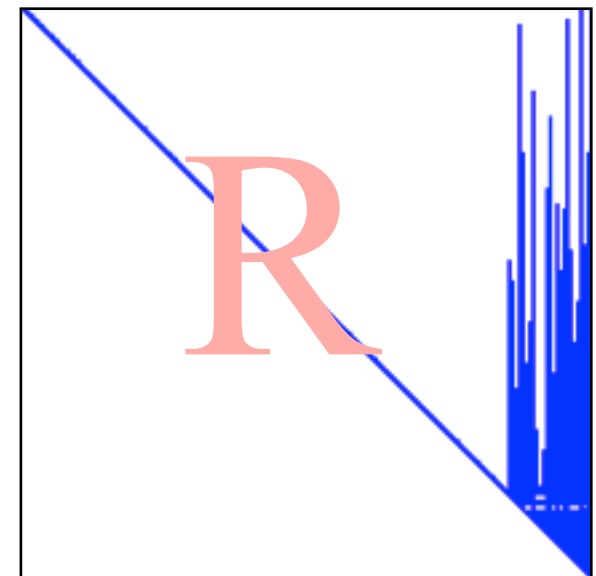


$$A^T A \theta^* = A^T b$$



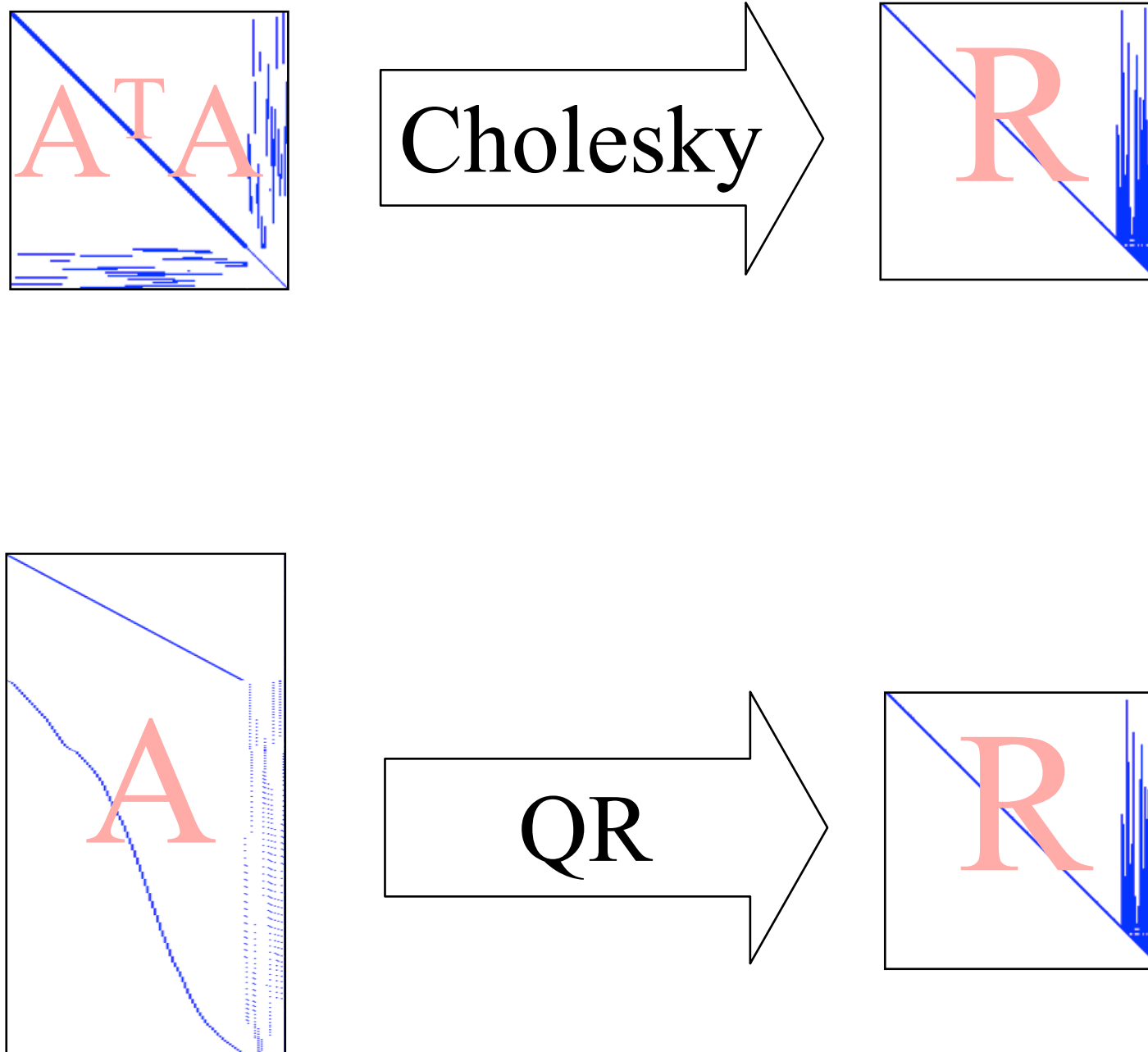
334x334, 7992 nnz

$$\mathcal{I} \triangleq A^T A = R^T R$$



334x334, 9606 nnz

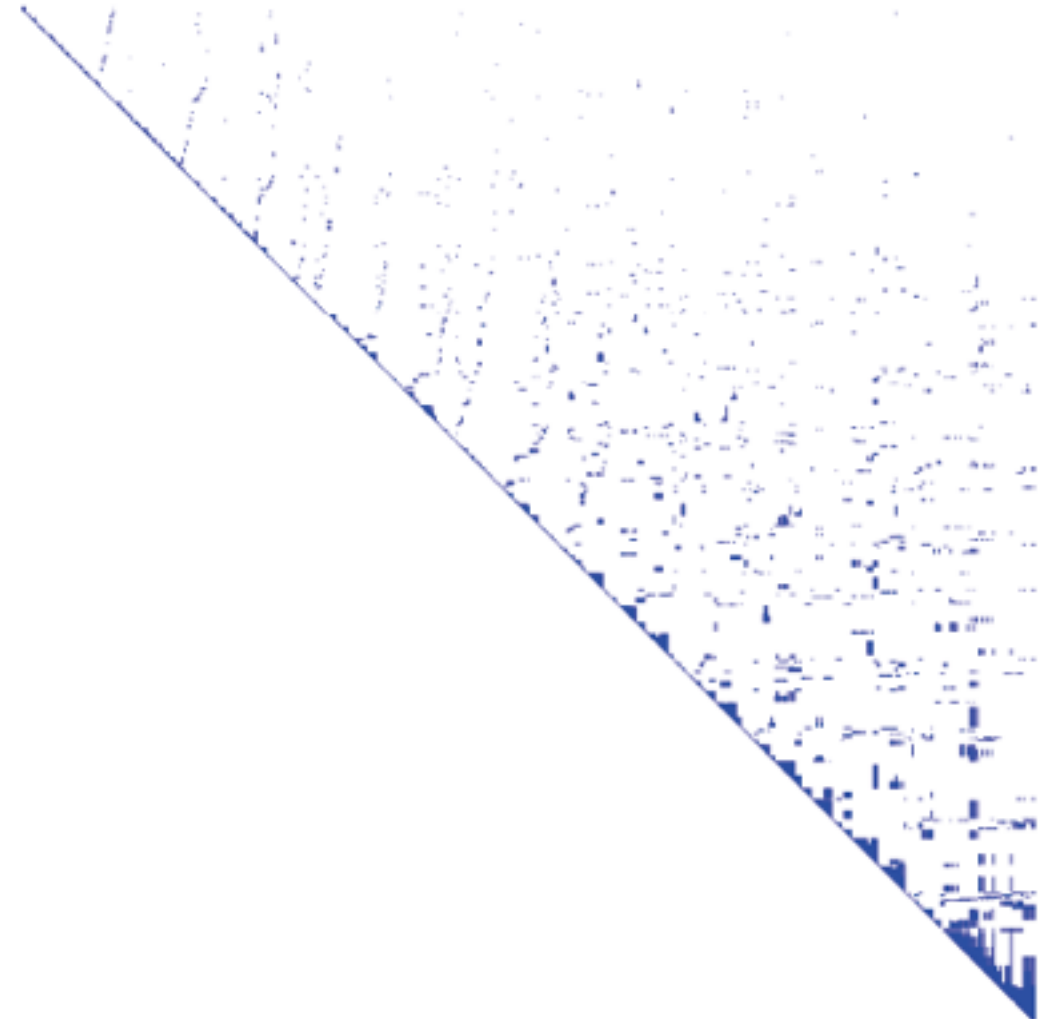
Square Root Factorization



Intel Dataset



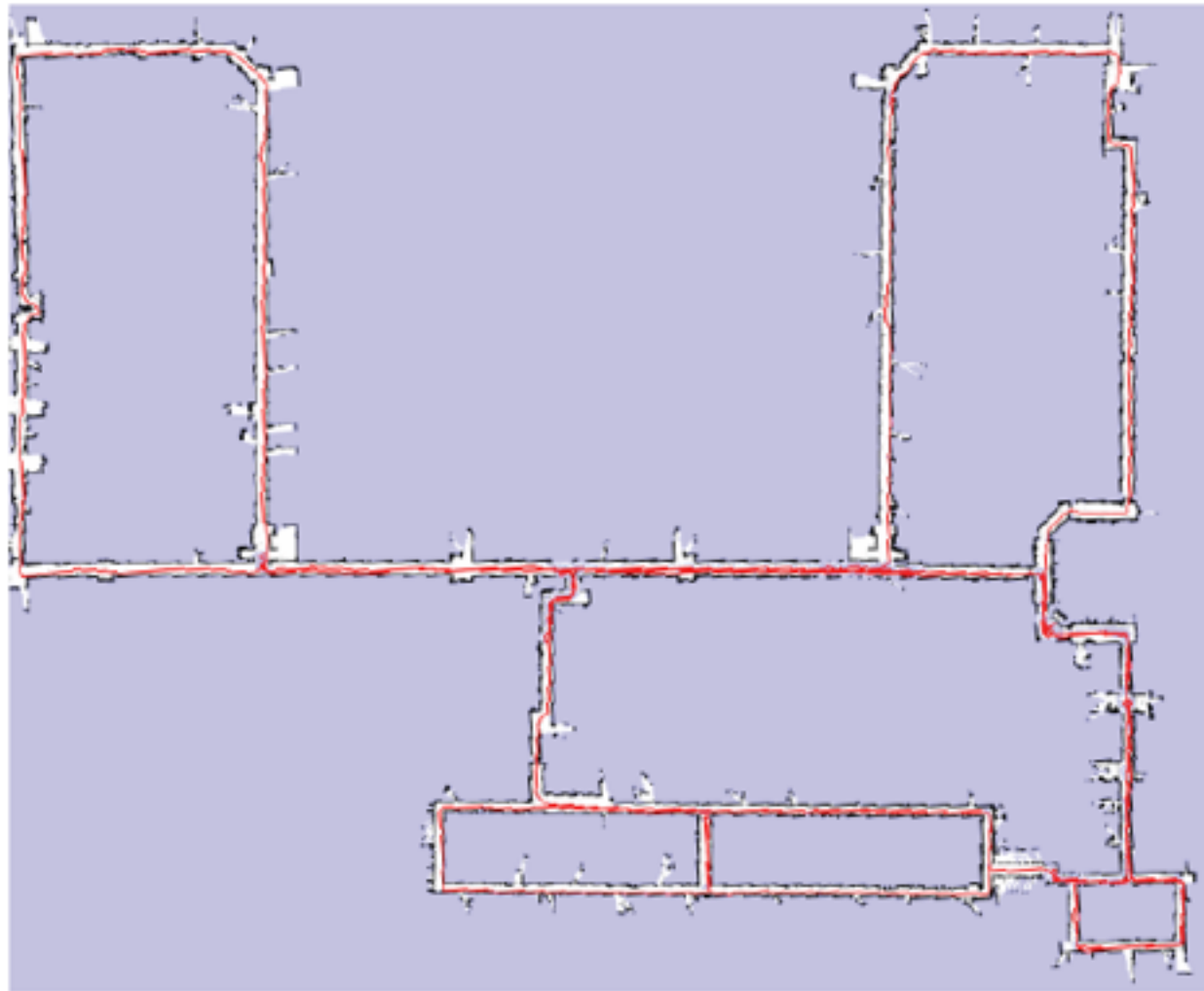
(b) Final trajectory and evidence grid map.



(c) Final R factor with side length 2730.

910 poses, 4453 constraints, 45s or 49ms/step
(Olson, RSS 07: iterative equation solver, same ballpark)

MIT Killian Court Dataset



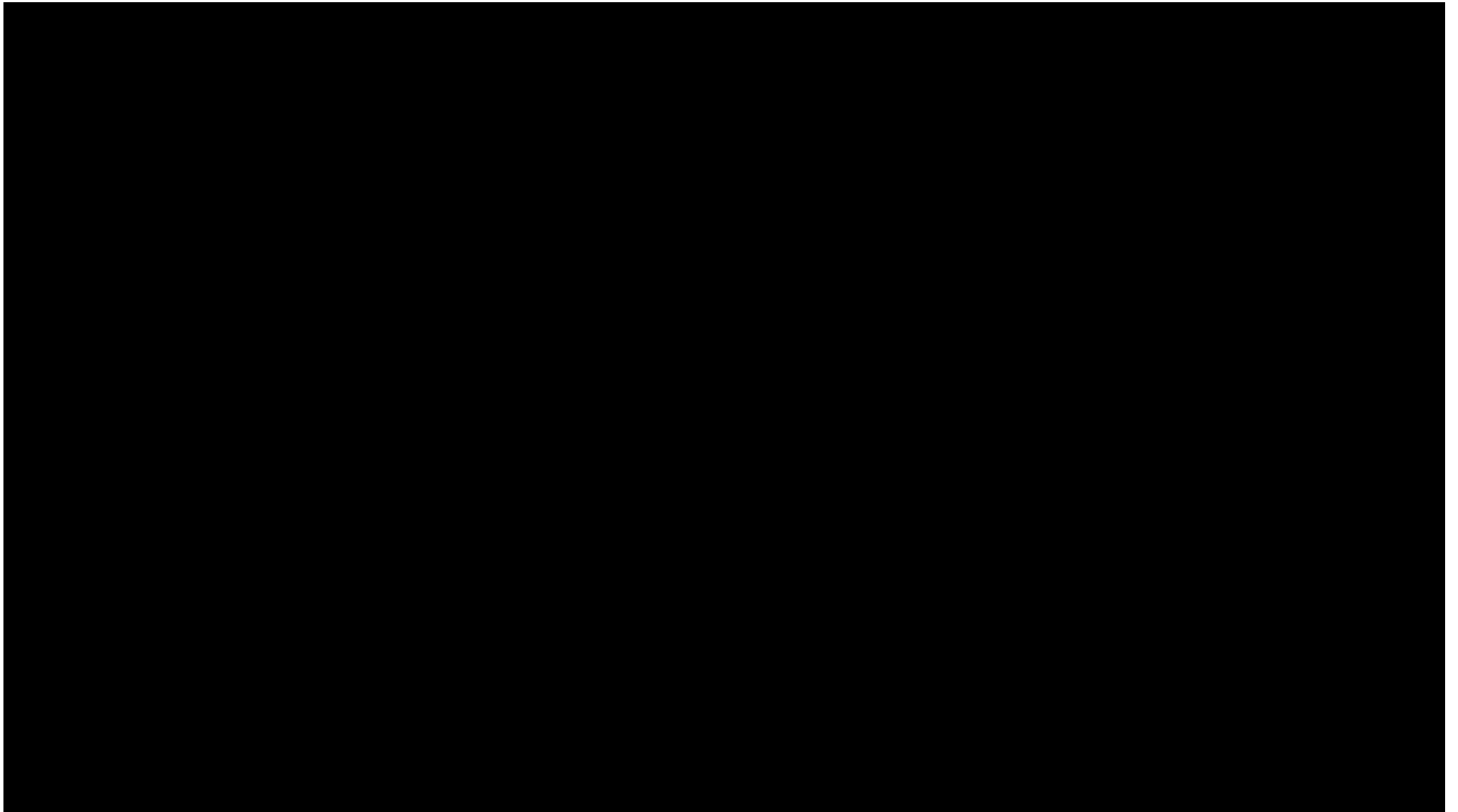
(b) Final trajectory and evidence grid map.



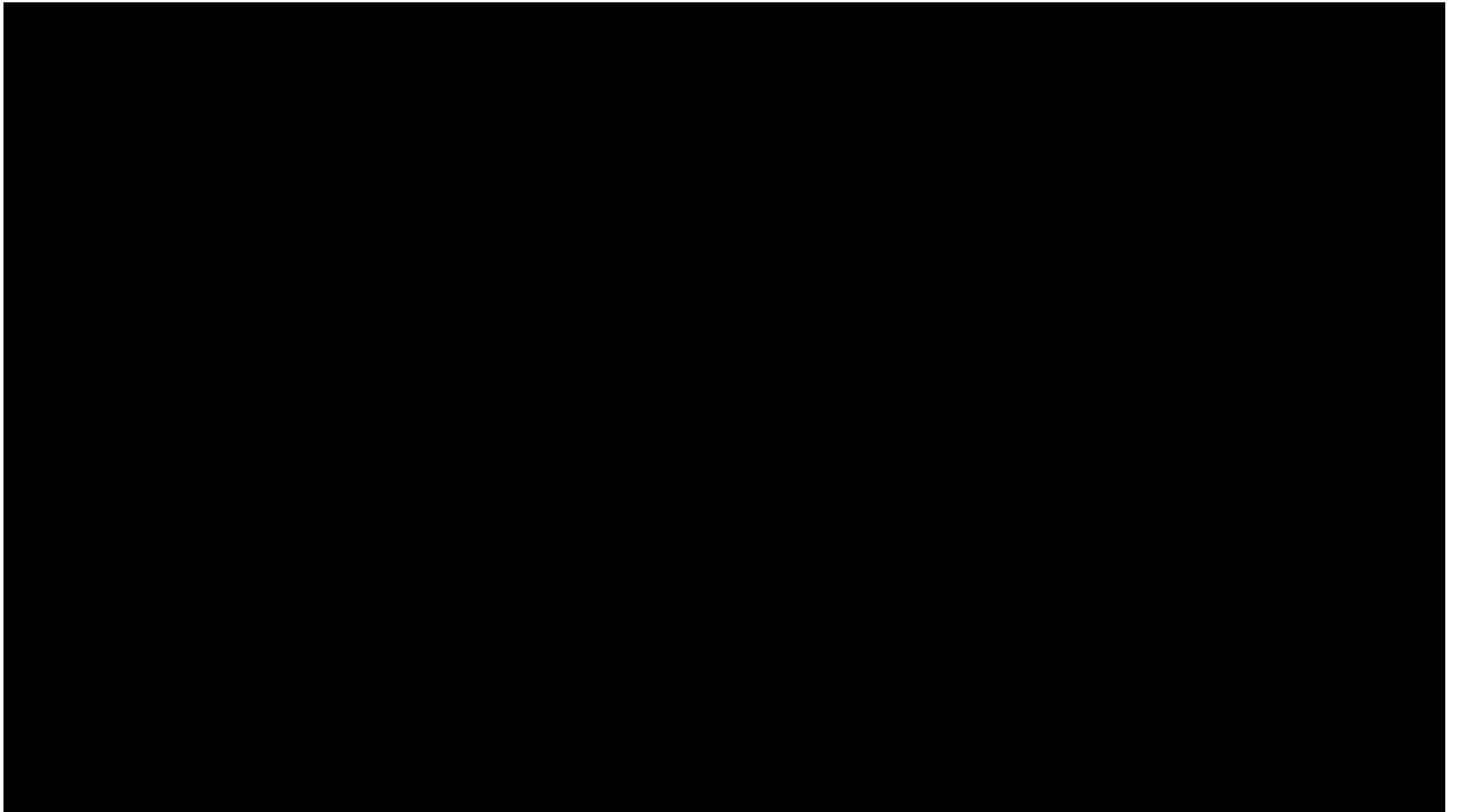
(c) Final R factor with side length 5823.

1941 poses, 2190 constraints, 14.2s or 7.3ms/step

- On Vesta and Ceres

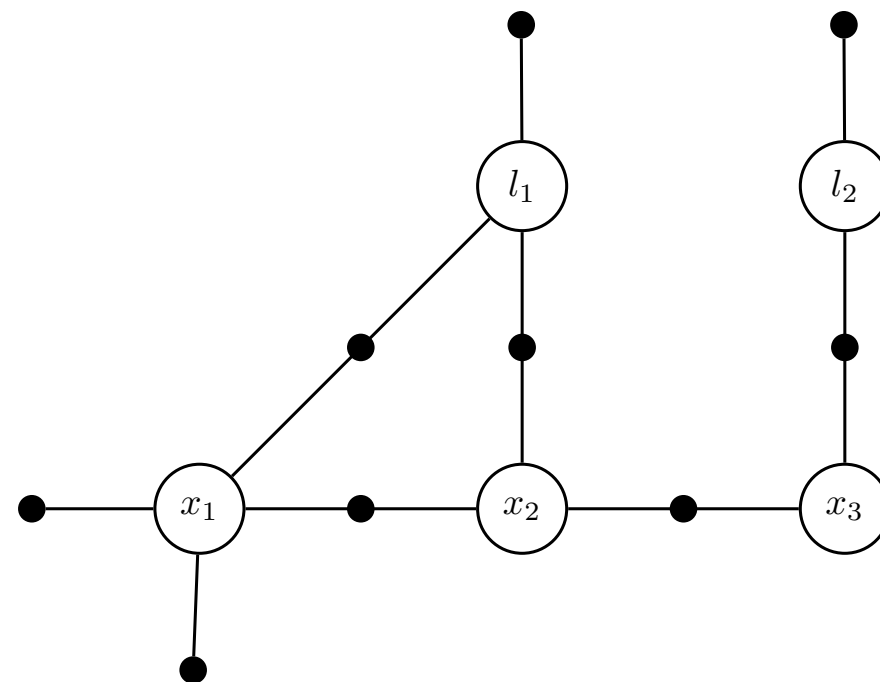


- On Vesta and Ceres



Variable Elimination

- Choose Ordering
- Eliminate one node at a time

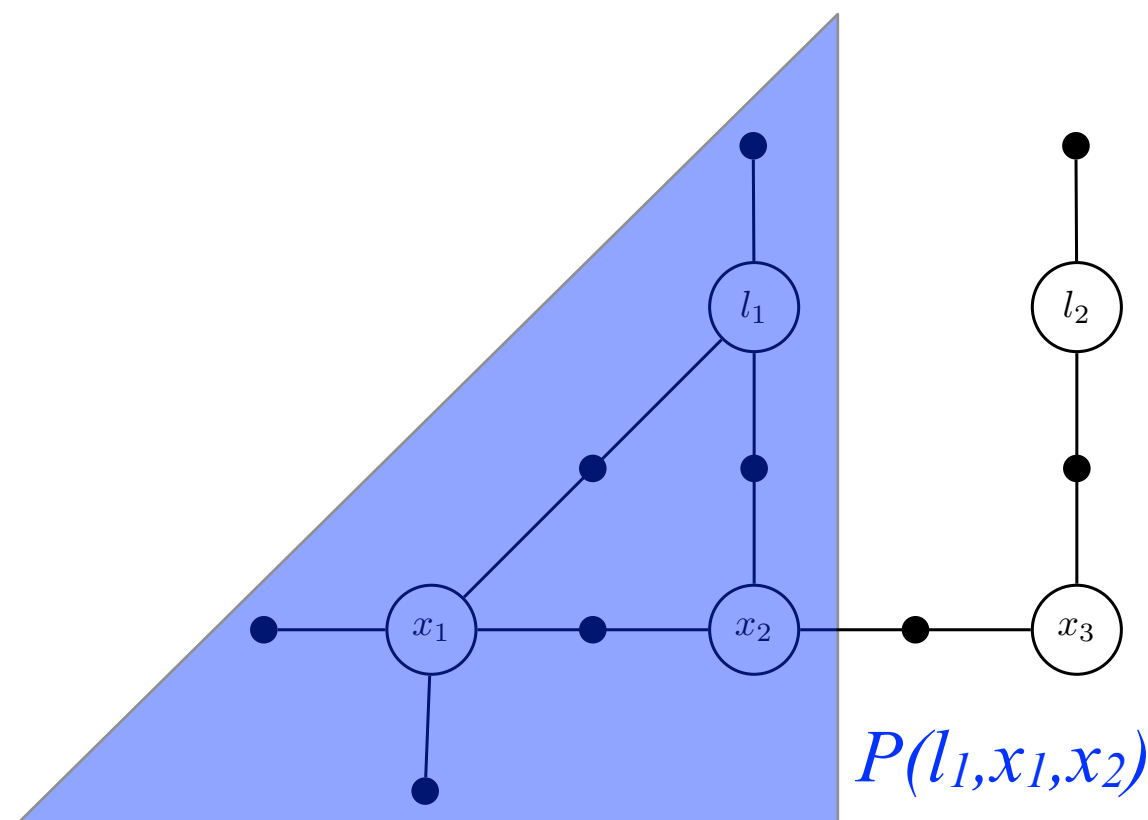


l_1	l_2	x_1	x_2	x_3
x				
x		x		
x			x	
	x			
	x			x
		x		
		x		
		x	x	
			x	x

- Express node as function of adjacent nodes

Variable Elimination

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- Eliminate one node at a time



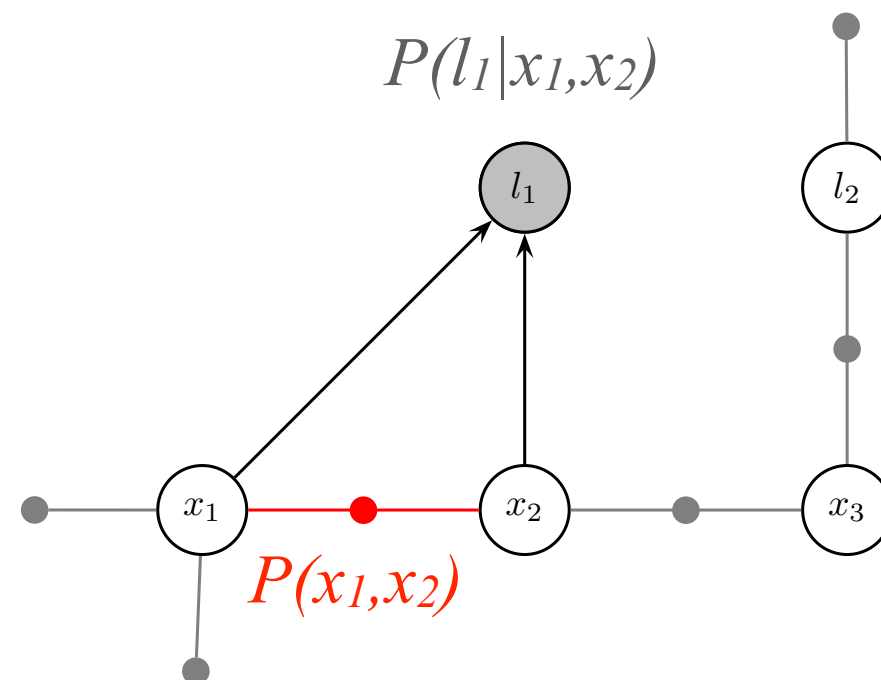
l_1	l_2	x_1	x_2	x_3
x				
x		x		
x			x	
	x			
	x			x
		x		
		x		
		x	x	
			x	x

- Express node as function of adjacent nodes

Basis = Chain Rule ! e.g. $P(l_1, x_1, x_2) = P(l_1 | x_1, x_2) P(x_1, x_2)$

Small Example

- Choose Ordering
- Eliminate one node at a time



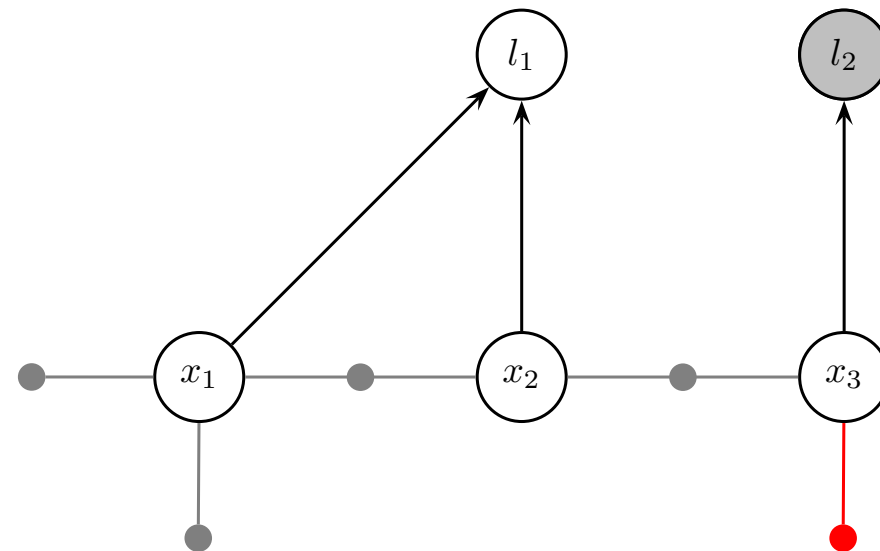
l_1	l_2	x_1	x_2	x_3
$\times$		$\times$	$\times$	
	$\times$			
	$\times$			$\times$
		$\times$	$\times$	
		$\times$		
		$\times$	$\times$	
			$\times$	$\times$

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Basis = Chain Rule ! e.g. $P(l_1, x_1, x_2) = P(l_1|x_1, x_2)P(x_1, x_2)$

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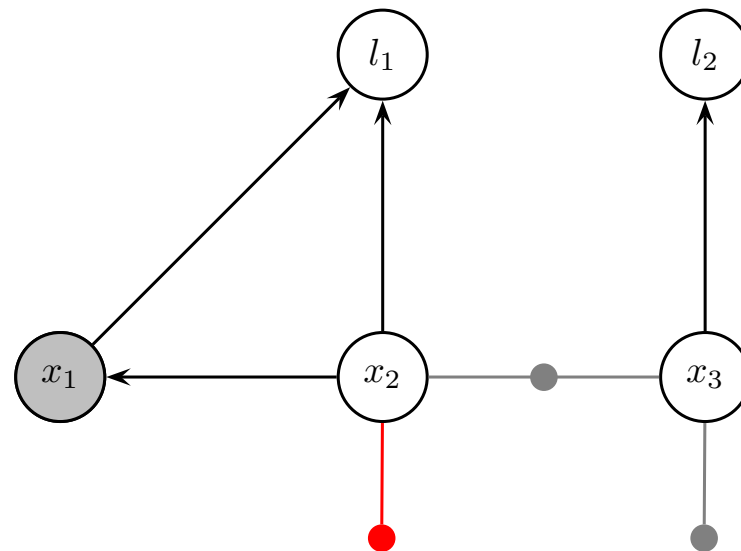


l_1	l_2	x_1	x_2	x_3
x		x	x	
	x			x
		x	x	
		x		
		x	x	
			x	x
				x

- Express node as function of adjacent nodes

Small Example

- Choose Ordering
- Eliminate one node at a time

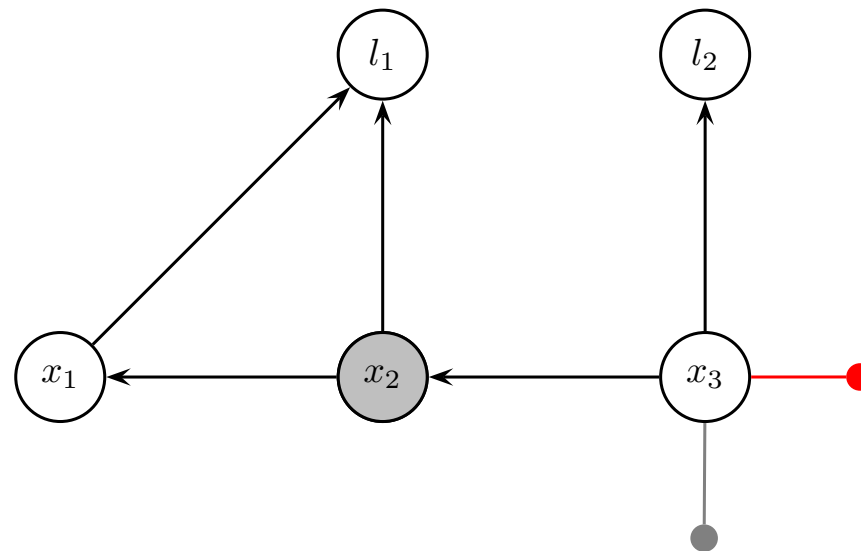


l_1	l_2	x_1	x_2	x_3
x		x	x	
	x			x
		x	x	
			x	
			x	x
				x

- Express node as function of adjacent nodes

Small Example

- Choose Ordering
- Eliminate one node at a time

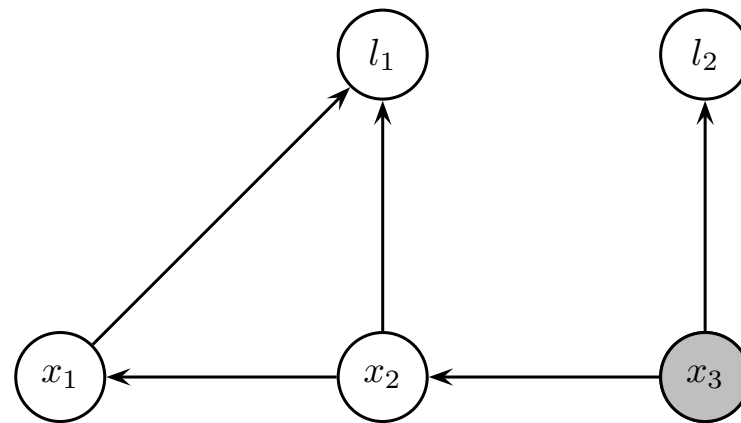


l_1	l_2	x_1	x_2	x_3
x		x	x	
	x			x
		x	x	
			x	x
				x
				x

- Express node as function of adjacent nodes

Small Example

- Choose Ordering
- Eliminate one node at a time

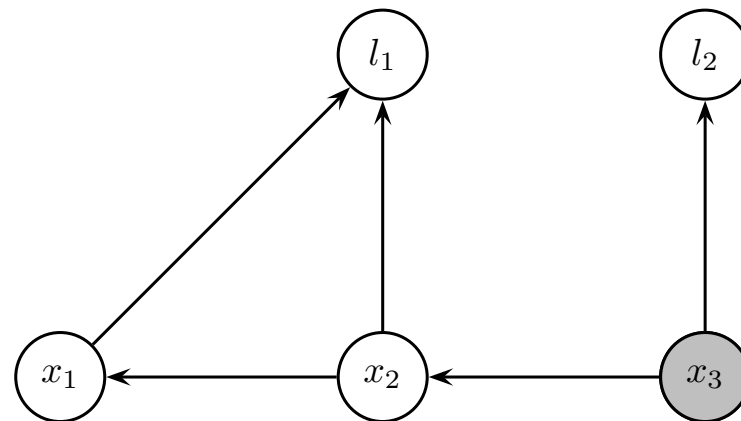


l_1	l_2	x_1	x_2	x_3
x		x	x	
	x			x
		x	x	
			x	x
				x

- Express node as function of adjacent nodes

Small Example

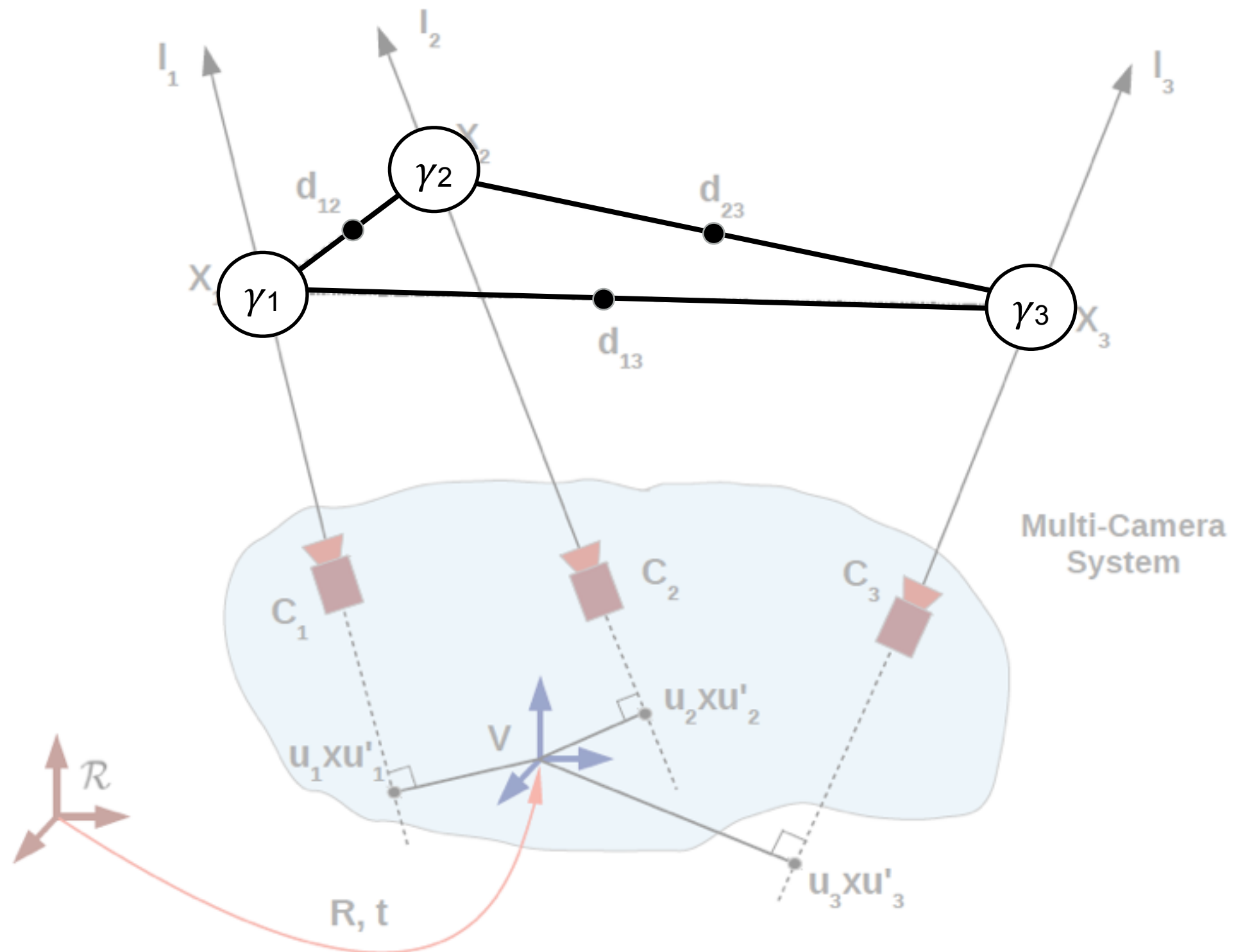
- End-Result = **Bayes Net** !



l_1	l_2	x_1	x_2	x_3
x		x	x	
	x			x
		x	x	
			x	x
				x

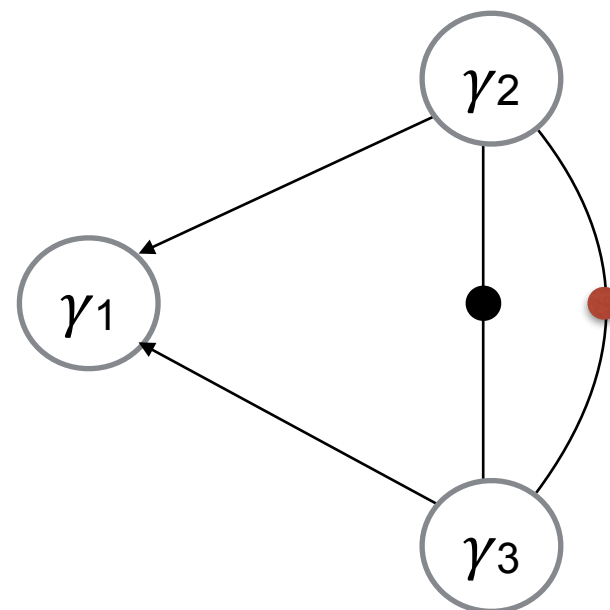
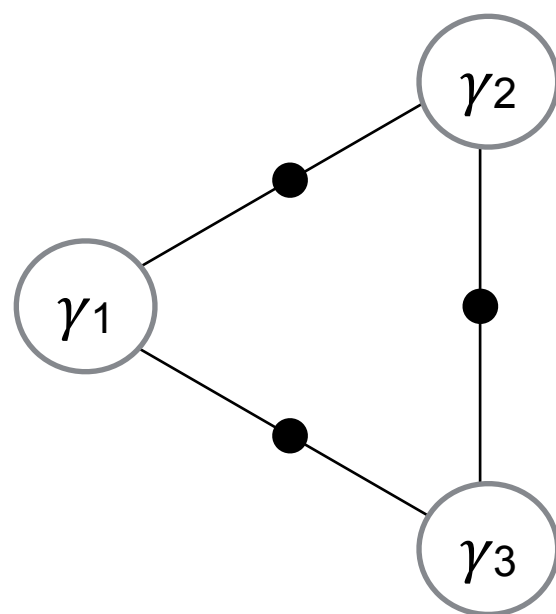
Polynomial Equations

Gim Hee Lee's thesis: multi-camera pose estimation

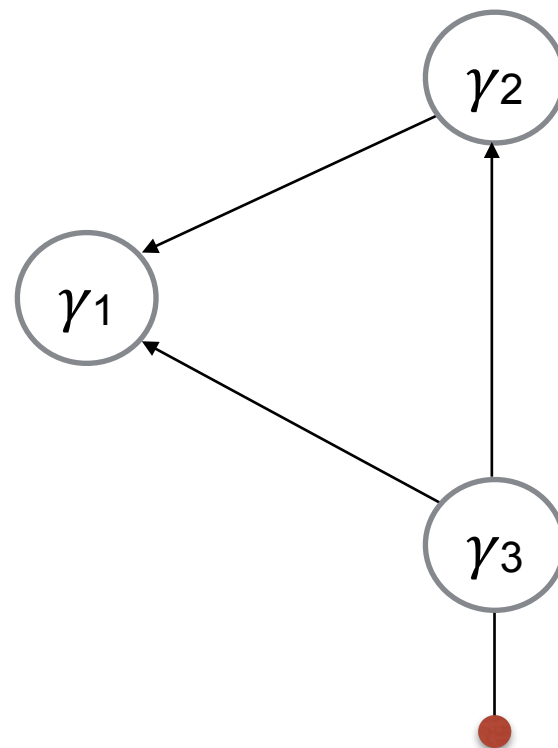


Polynomial Equations

Three degree 2 polynomials, each in only 2 variables

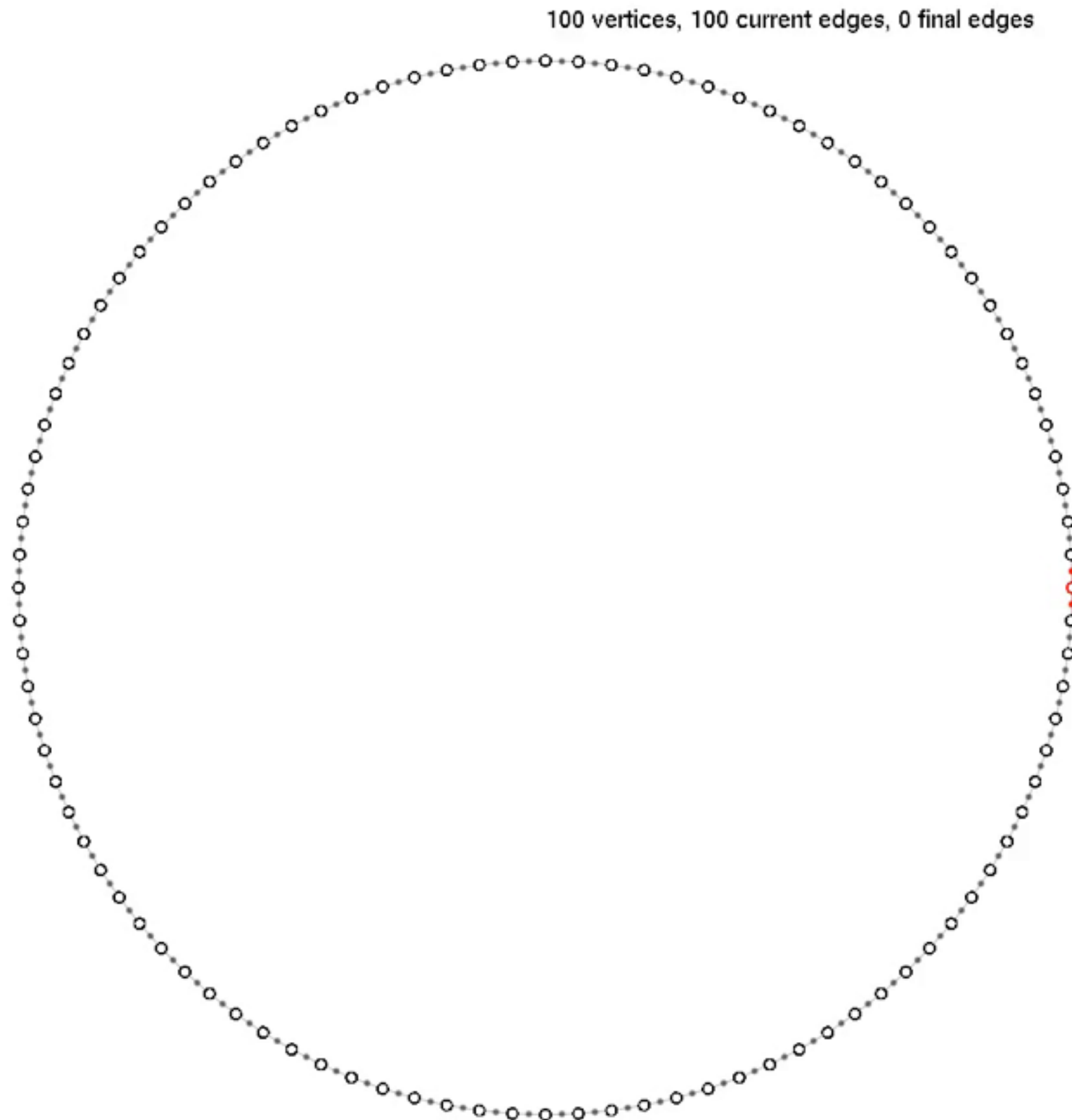


Degree 4 polynomial

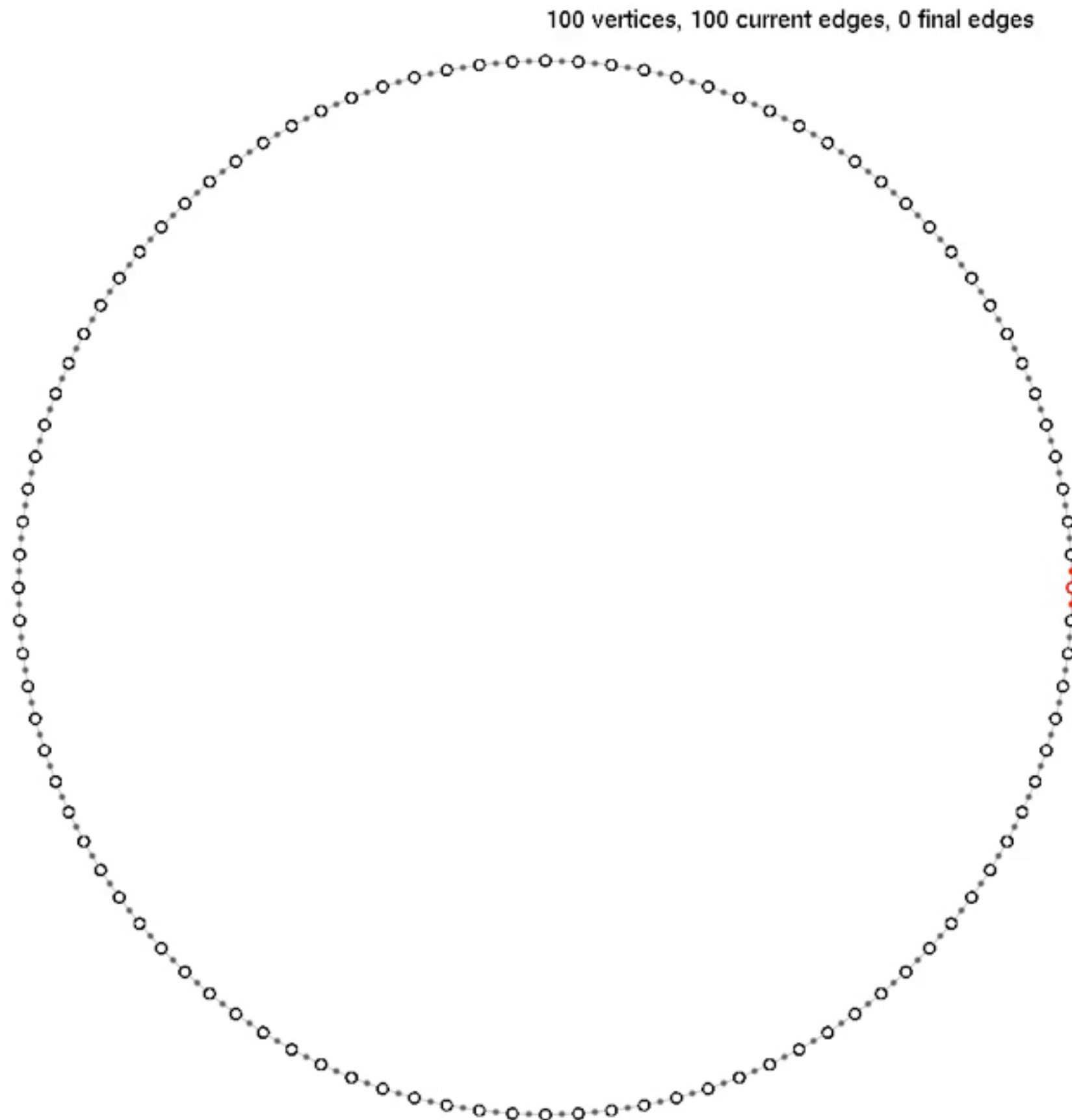


Degree 8 polynomial

Elimination at work

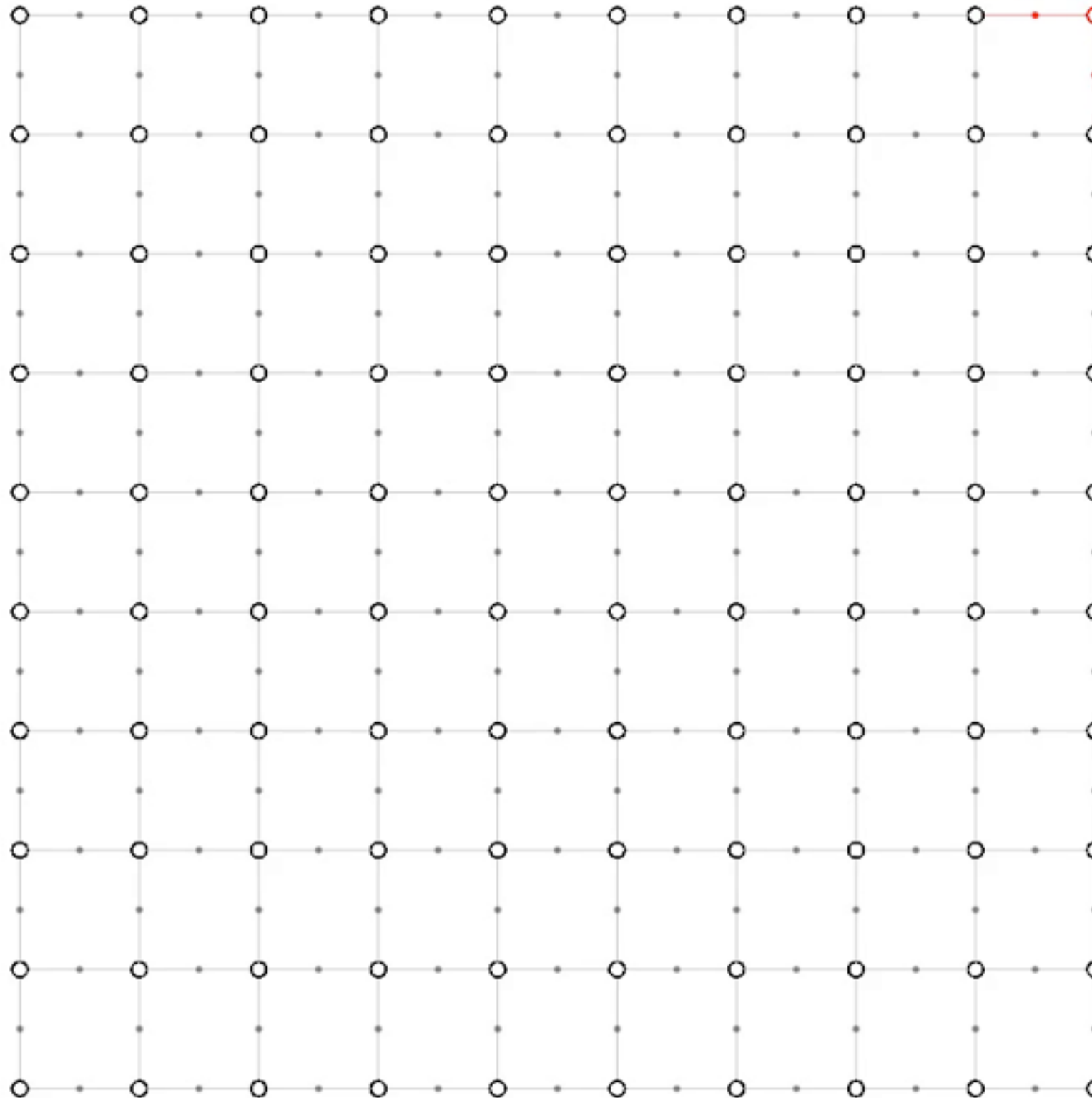


Elimination at work



Elimination at work

100 vertices, 180 edges



Elimination at work

100 vertices, 180 edges

