Lecture #5

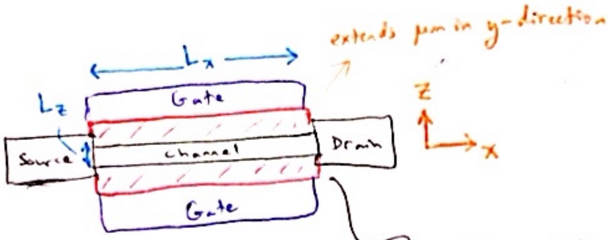
Quantization Effects

- Subbands

In a homogeneous solid (3D), electrons are free to move in all three directions and energy levels are classified: $E_b(k_x, k_y, k_z)$

bands, such as valence or conduction

What happens in this device?



Channel is a quantum well with e^-'s free to move only in x-y plane

z-confinement modeled as a ring of circumference L_z, with periodic B.C. that restrict to allowed values of k_z given by: $K_z = \frac{p2\pi}{L_z}$

For a ring-shaped wire or nanotube this works.

$$E_{b,p}(k_x, k_y) \simeq E_b\left(k_x, k_y, k_z = \frac{p2\pi}{L_z}\right)$$

$p \equiv$ subband index, 1, 2, 3, ...

The energy levels are now classified as:

$$E_{b,p}(k_x, k_y) \simeq E_b\left(k_x, k_y, k_z = \frac{p\pi}{L_z}\right)$$

for the structure shown above, where the insulator layers act like ∞ potential wells

This is an approximation, but shows the following critical points

★ Quantum wells have 1-D subbands, each having a 2-D dispersion relation, $E(k_x, k_y)$.

★ How small does L_z need to be for something to be a 2D (quantum well)?

A: When it has experimentally observable effects.

→ Typically, the discrete energy levels of $K_z = \frac{p\pi}{L_z}$ need to be $> k_B T$ (thermal energy)

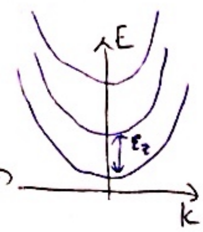
Due to smoothing out of Fermi function

Example with parabolic E-K:

$$E(\vec{K}) \simeq E_c + \frac{\hbar^2(k_x^2 + k_y^2 + k_z^2)}{2m_c}$$

E_c: conduction band Edge

m_c: effective mass in conduction band



confinement in z gives rise to subbands:

$$E_p(k_x, k_y) \simeq E_c + p^2 \varepsilon_z + \frac{\hbar^2(k_x^2 + k_y^2)}{2m_c}$$

where: $\boxed{\varepsilon_z = \frac{\hbar^2 \pi^2}{2m_c L_z^2} = \frac{m_0}{m_c}\left(\frac{10\text{ nm}}{L_z}\right)^2 \times 3.8\text{ meV}}$

m_0: mass of an e^-

Lecture # 5 (CONT)

Quantization Effects (CONT)

- Subbands (CONT)

Example: Si, ETSOI : when is it 2D? at R.T. [IN Class exercize]

~~m_c~~ $m_c = 0.2 m_0$, $k_B T \approx 26\, meV$ @ R.T.

$$\therefore \frac{1}{0.2}\left(\frac{10\,nm}{L_z}\right)^2 (3.8\, \cancel{meV}) > 26\, \cancel{meV}$$

$$\frac{1900\,nm}{L_z^2} > 26 \Rightarrow \boxed{L_z < 8.5\,nm}$$

Ok, now what happens if we confine our channel in the y-direction as well to create a 1-D nanowire structure? Also called "quantum wires"

$$E_{n,p}(k_x) \approx E_c + n^2 \varepsilon_y + p^2 \varepsilon_z + \frac{\hbar^2 k_x^2}{2 m_c}$$

$$\varepsilon_y = \frac{\hbar^2 \pi^2}{2 m_c L_y^2}$$

✱ Quantum wires (1D) have 2D subbands, each having a 1-D dispersion relation

- Density of States

From Lecture #1 we found the 3D DOS : $D_{3D}(E) = \frac{m\sqrt{2mE}}{\pi^2 \hbar^3} \cdot V$ (includes spin) ↑ volume

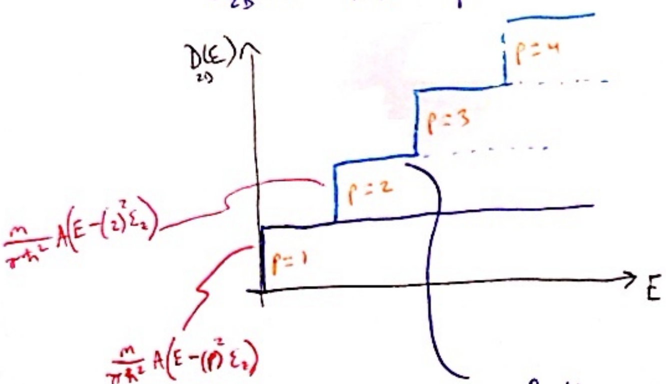
Following the same approach for 2D gives: $D_{2D}(E) = \frac{m}{\pi \hbar^2} \cdot A$ ↑ Area

for 1D: $D_{1D}(E) = \frac{m}{\pi \hbar}(2mE)^{-1/2} \cdot L$ ↑ length

HOWEVER, the D_{2D} and D_{1D} are for only one subband! Must sum over all subbands:

$$D_{2D}(E) = \frac{m}{\pi \hbar^2} A \sum_p \vartheta(E - p^2 \varepsilon_z)$$

unit step function



each of these subbands is called a "mode" for carrier transport

Lecture # 5 (CONT)

Back to G_Q

What is conductance of a material that is 1D, with perfect contacts?

Before, we said: $G_Q = \frac{2q^2}{h}$

Now, to consider the entire electronic structure:

$$G = \underbrace{M(E)}_{\text{\# of modes at } E}\, G_Q$$