CS 357 / MATH 357
Spring 2014

Numerical Methods – Homework 9
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Due: 04/29/2014

Note: Homework is due **5pm** on the due date. Please submit your homework as a PDF via Compass2g.

Problem 1 [4pt] Write a Python program to estimate

$$\int_0^1 \int_2^4 x^2 + xy + y^2 \ dy \ dx$$

using Monte Carlo integration with 10000 points. What is the estimate you obtained? Include your source code.

Problem 2 [12pt] Consider the following set of points.

x	0	2	4
y	0	3	8

We wish to perform a least-squares best fit of a linear polynomial $a_0 + a_1 x = y$ to the above data.

(a) **[2pt]** Construct the overdetermined system of equations in matrix form.

(b) **[1pt]** What does least squares minimize?

(c) **[1pt]** If the condition number of A is some value α, what is the condition number of $A^T A$? Is this good or bad?

(d) **[4pt]** Find the linear polynomial of best fit (by hand) for the above points using the normal equations.

(e) **4pt** Use Python to find the singular value decomposition. Use the SVD to find the least squares solution (you may use Python for this as well).

Problem 3 [4pt] Use Python to implement Gram-Schmidt orthogonalization as described in the pseudocode in the lecture notes. Use your implementation to find the QR factorization of the following matrix. Include your source code.

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 0 & 4 \\ -3 & 2 & 1 \\ 8 & 4 & 12 \end{bmatrix}$$