## CSE 150. Assignment 1

**Out:** *Tue Jan 14*
**Due:** *Tue Jan 21* (in class)
**Reading**: Russell & Norvig, Chapter 13; Korb & Nicholson, Chapter 1.

### 1.1 Kullback-Leibler distance

Often it is useful to measure the difference between two probability distributions over the same random variable. For example, as shorthand let

$$\begin{aligned} p_i &= P(X\!=\!x_i|E), \\ q_i &= P(X\!=\!x_i|E') \end{aligned}$$

denote the conditional distributions over the random variable $X$ for different pieces of evidence $E \neq E'$. The Kullback-Leibler (KL) distance between these distributions is defined as:

$$\mathrm{KL}(p,q) = \sum_i p_i \log(p_i/q_i).$$

(a) By sketching graphs of $\log x$ and $x-1$, verify the inequality

$$\log x \leq x-1,$$

with equality if and only if $x\!=\!1$. Confirm this result by differentiation of $\log x - (x\!-\!1)$. (Note: all logarithms in this problem are *natural* logarithms.)

(b) Use the previous result to prove that $\mathrm{KL}(p,q) \geq 0$, with equality if and only if the two distributions $p_i$ and $q_i$ are equal.

(c) Provide a counterexample to show that the KL distance is not a symmetric function of its arguments:

$$\mathrm{KL}(p,q) \;\neq\; \mathrm{KL}(q,p).$$

Despite this asymmetry, it is still common to refer to $\mathrm{KL}(p,q)$ as a measure of distance. Many algorithms in machine learning are based on minimizing KL distances between probability distributions.

### 1.2 Conditional independence [RN 13.10]

Show that the following three statements about random variables $X$, $Y$, and $Z$ are equivalent:

$$\begin{aligned} P(X,Y|Z) &= P(X|Z)P(Y|Z) \\ P(X|Y,Z) &= P(X|Z) \\ P(Y|X,Z) &= P(Y|Z) \end{aligned}$$

You should become fluent with all these ways of expressing that $X$ is conditionally independent of $Y$ given $Z$.

## 1.3 Creative writing

Attach events to the binary random variables $X, Y$, and $Z$ that are consistent with the following patterns of commonsense reasoning. You may use different events for the different parts of the problem.

(a) Explaining away:

$$\begin{aligned} P(X=1|Y=1) &> P(X=1), \\ P(X=1|Y=1,Z=1) &< P(X=1|Y=1) \end{aligned}$$

(b) Accumulating evidence:

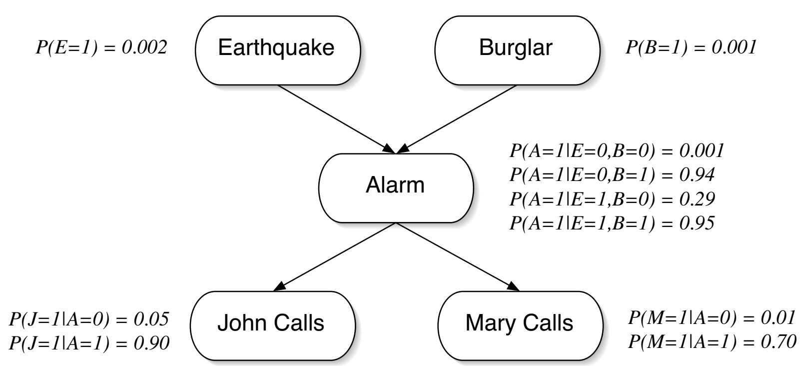
$$P(X=1) < P(X=1|Y=1) < P(X=1|Y=1,Z=1)$$

(c) Conditional independence:

$$\begin{aligned} P(X=1,Y=1) &\neq P(X=1)P(Y=1) \\ P(X=1,Y=1|Z=1) &= P(X=1|Z=1)P(Y=1|Z=1) \end{aligned}$$

## 1.4 Probabilistic inference

Recall the probabilistic model that we described in class for the binary random variables $\{E = \text{Earthquake}, B = \text{Burglary}, A = \text{Alarm}, J = \text{JohnCalls}, M = \text{MaryCalls}\}$. We also expressed this model as a belief network, with the directed acyclic graph (DAG) and conditional probability tables (CPTs) shown below:




Compute numeric values for the following probabilities, exploiting relations of conditional independence as much as possible to simplify your calculations. You may re-use numerical results from lecture, but otherwise show your work. Be careful not to drop significant digits in your answer.

(a) $P(E=1|A=1)$ (c) $P(A=1|M=1)$ (e) $P(A=1|M=0)$
(b) $P(E=1|A=1,B=0)$ (d) $P(A=1|M=1,J=0)$ (f) $P(A=1|M=0,B=1)$

Consider your results in (b) versus (a), (d) versus (c), and (f) versus (e). Do they seem consistent with commonsense patterns of reasoning?