PHYS410 Quantum Mechanics

Schrödinger equation: $i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = H\Psi(x,t)$

Expectation value of operator O: $\langle O\rangle = \int_{-\infty}^{+\infty} dx\Psi^* O\Psi$

Momentum operator: $p = -i\hbar\frac{\partial}{\partial x}$

Standard deviation: $\sigma_O^2 = \langle O^2\rangle - \langle O\rangle^2$

Stationary states: $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$
$\psi(x)$ satisfies the time-independent Schrödinger equation (eigenvalue equation): $H\psi = E\psi$

General solution:

$$\Psi(x,t) = \sum_n c_n\psi_n(x)e^{-iE_n t/\hbar}$$

Orthonormality of states ψ_n: $\int dx\psi_m^*(x)\psi_n(x) = \delta_{mn}$

$$c_n = \int dx\psi_n^*(x)\psi(x,0)$$

Infintie square well:

$$V(x) = \begin{cases} 0, & 0 \le x \le a \\ \infty, & x < 0, x > a \end{cases}$$

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

Harmonic Oscillator:

$$V(x) = \frac{1}{2}m\omega^2 x^2$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\psi_n(x) = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{m\omega}{2\hbar}x^2}$$

$$H_{2k}(x) = H_{2k}(-x), H_{2k+1}(x) = -H_{2k+1}(-x)$$

Ladder operators:

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x), x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-), p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$

$$a_+\psi_n = \sqrt{n+1}\psi_{n+1}$$

$$a_-\psi_n = \sqrt{n}\psi_{n-1}$$

Free particle: $V = 0$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\phi(k)e^{i(kx-\frac{\hbar k^2}{2m}t)}dk$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\Psi(x,0)e^{-ikx}dx$$

Gaussian integral:

$$\int_{-\infty}^{+\infty} e^{-\alpha x^2} = \sqrt{\frac{\pi}{\alpha}}$$