

PHYS410 Quantum Mechanics

Schrödinger equation: $i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) = H\Psi(x,t)$

Expectation value of operator O : $\langle O \rangle = \int_{-\infty}^{+\infty} dx \Psi^* O \Psi$

Momentum operator: $p = -i\hbar \frac{\partial}{\partial x}$

Standard deviation: $\sigma_O^2 = \langle O^2 \rangle - \langle O \rangle^2$

Stationary states: $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$

$\psi(x)$ satisfies the time-independent Schrödinger equation (eigenvalue equation): $H\psi = E\psi$

General solution:

$$\Psi(x,t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar}$$

Orthonormality of states ψ_n : $\int dx \psi_m^*(x) \psi_n(x) = \delta_{mn} = \langle \psi_m | \psi_n \rangle$

$$c_n = \int dx \psi_n^*(x) \psi(x, 0)$$

Infintie square well:

$$V(x) = \begin{cases} 0, & 0 \leq x \leq a \\ \infty, & x < 0, x > a \end{cases}, \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}, \quad \psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

Harmonic Oscillator:

$$V(x) = \frac{1}{2} m \omega^2 x^2, \quad E_n = \left(n + \frac{1}{2}\right) \hbar \omega$$

$$\psi_n(x) = \frac{1}{\sqrt{n!}} (a_+)^n \psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n\left(\sqrt{\frac{m\omega}{\hbar}} x\right) e^{-\frac{m\omega}{2\hbar} x^2}$$

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar} x^2}$$

$$H_{2k}(x) = H_{2k}(-x), H_{2k+1}(x) = -H_{2k+1}(-x)$$

Ladder operators:

$$a_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}} (\mp i p + m\omega x), x = \sqrt{\frac{\hbar}{2m\omega}} (a_+ + a_-), p = i\sqrt{\frac{\hbar m\omega}{2}} (a_+ - a_-)$$

$$a_+\psi_n = \sqrt{n+1}\psi_{n+1}$$

$$a_-\psi_n = \sqrt{n}\psi_{n-1}$$

Dirac delta: $\int_{-\infty}^{+\infty} f(x)\delta(x-a)dx = f(a)$

Eigenfunctions/eigenvalues: $Of = qf$

Hermitian operators: $\langle f|Og\rangle = \langle Of|g\rangle$, $\int f(x)^*(Og(x))dx = \int (Of(x))^*g(x)dx$

Momentum space wave-function:

$$\tilde{\Psi}(p, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \Psi(x, t) e^{-ipx} dx$$

$$\Psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \tilde{\Psi}(p, t) e^{ipx} dp$$

Uncertainty principle:

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [A, B] \rangle \right)^2$$

$$\frac{d}{dt} \langle O \rangle = \frac{i}{\hbar} \langle [H, O] \rangle + \left\langle \frac{\partial O}{\partial t} \right\rangle$$