PHYS410 Quantum Mechanics

Schrödinger equation: $i\hbar\frac{\partial\Psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) + V(\vec{r})\Psi(\vec{r},t) = H\Psi(\vec{r},t)$

Expectation value of operator $O$: $\langle O\rangle = \int_{-\infty}^{+\infty} dx\Psi^* O\Psi$

Momentum operator: $p = -i\hbar\frac{\partial}{\partial x}$

Standard deviation: $\sigma_O^2 = \langle O^2\rangle - \langle O\rangle^2$

Stationary states: $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$
$\psi(x)$ satisfies the time-independent Schrödinger equation (eigenvalue equation): $H\psi = E\psi$

General solution:

$$\Psi(x,t) = \sum_n c_n\psi_n(x)e^{-iE_nt/\hbar}$$

Orthonormality of states $\psi_n$: $\int dx\psi_m^*(x)\psi_n(x) = \delta_{mn} = \langle\psi_m|\psi_n\rangle$

$$c_n = \int dx\psi_n^*(x)\psi(x,0)$$

Infintie square well:

$$V(x) = \begin{cases} 0, & 0 \le x \le a \\ \infty, & x < 0, x > a \end{cases}, \quad E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}, \quad \psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

Harmonic Oscillator:

$$V(x) = \frac{1}{2}m\omega^2x^2, \quad E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

$$\psi_n(x) = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^nn!}}H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{m\omega}{2\hbar}x^2}$$

$$H_{2k}(x) = H_{2k}(-x), H_{2k+1}(x) = -H_{2k+1}(-x)$$

Ladder operators:

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x), x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-), p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$

$$a_+\psi_n = \sqrt{n+1}\psi_{n+1}$$
$$a_-\psi_n = \sqrt{n}\psi_{n-1}$$

Dirac delta: $\int_{-\infty}^{+\infty} f(x)\delta(x-a)dx = f(a)$

Eigenfunctions/eigenvalues: $Of = qf$

Hermitian operators:$\langle f|Og\rangle = \langle Of|g\rangle$, $\int f(x)^*(Og(x))dx = \int (Of(x))^*g(x)dx$

Orthogonal functions: $\langle f|g\rangle = \int f^*(x)g(x)dx = 0$
Momentum space wave-function:

$$\tilde{\Psi}(p,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \Psi(x,t)e^{-ipx/\hbar}dx$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} \tilde{\Psi}(p,t)e^{ipx/\hbar}dp$$

Uncertainty principle:

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[A,B]\rangle\right)^2$$

$$\frac{d}{dt}\langle O\rangle = \frac{i}{\hbar}\langle[H,O]\rangle + \left\langle\frac{\partial O}{\partial t}\right\rangle$$

$$I(k,p) = \int_0^\infty r^k e^{-pr/a}dr = k!\left(\frac{a}{p}\right)^{k+1}, k, p \text{ integers}, k \geq 0, p \geq -2$$

$$S_i = \frac{\hbar}{2}\sigma_i$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad .$$

Radial equation for a central potential:

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + \frac{\hbar^2 l(l+1)}{2mr^2}u(r) + V(r)u(r) = Eu(r) \ , \quad u(r) = rR(r)$$

Periodic table:
$1s$
$2s2p$
$3s3p$
$4s3d4p$
$5s4d5p$
$6s4f5d6p$
$7s5f6d7p$