CS 242 Discrete Structures for Computer Science
Spring 2014

Home Work # 1
Due: February 5, 2014

Problems on Section 1.1

[1- 5] Chapter 1, Section 1.1: Problems 10, 15, 20, 30, 32

10. **a)** The election is not decided.
b) The election is decided, or the votes have been counted.
c) The election is not decided, and the votes have been counted.
d) If the votes have been counted, then the election is decided.
e) If the votes have not been counted, then the election is not decided.
f) If the election is not decided, then the votes have not been counted.
g) The election is decided if and only if the votes have been counted.
h) Either the votes have not been counted, or else the election is not decided and the votes have been counted. Note that we were able to incorporate the parentheses by using the words *either* and *else*.

15. (a) r ^ ~p
(b) p ^q ^ r
(c) r → (q ↔ ~p)
(d) ~q ^ ~p ^ r
(e) (q → (~p ^ ~r)) ^ (~(~p ^ ~r) → q))
(f) (p ^ r) → q

20. **a)** The employer making this request would be happy if the applicant knew both of these languages, so this is clearly an inclusive *or*.
b) The restaurant would probably charge extra if the diner wanted both of these items, so this is an exclusive *or*.
c) If a person happened to have both forms of identification, so much the better, so this is clearly an inclusive *or*.
d) This could be argued either way, but the inclusive interpretation seems more appropriate. This phrase means that faculty members who do not publish papers in research journals are likely to be fired from their jobs during the probationary period. On the other hand, it may happen that they will be fired even if they do publish (for example, if their teaching is poor).

30. A truth table will need 2^n rows if there are n variables.
a) $2^2 = 4$ **b)** $2^3 = 8$ **c)** $2^6 = 64$ **d)** $2^5 = 32$

32. To construct the truth table for a compound proposition, we work from the inside out. In each case, we will show the intermediate steps. In part **(d)**, for example, we first construct the truth tables for $p \wedge q$ and for $p \vee q$ and combine them to get the truth table for $(p \wedge q) \rightarrow (p \vee q)$. For parts **(a)** and **(b)** we have the following table (column three for part **(a)**, column four for part **(b)**).

p	$\neg p$	$p \rightarrow \neg p$	$p \leftrightarrow \neg p$
T	F	F	F
F	T	T	F

For parts **(c)** and **(d)** we have the following table.

p	q	$p \vee q$	$p \wedge q$	$p \oplus (p \vee q)$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	F	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	F	T

For part **(e)** we have the following table.

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

For part **(f)** we have the following table.

p	q	$\neg q$	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Problems on Section 1.2

A king posed the following challenge to a prisoner. He set up three rooms one of which contained a lady and the other two contained tigers. The rooms had signs that read as follows:

I	II	III
A TIGER IS IN THIS ROOM	A LADY IS IN THIS ROOM	A TIGER IS IN ROOM II

The king explained that at most one of the three signs was true. The challenge was to open the room that contained the lady. Which room should the prisoner open? Justify your answer.

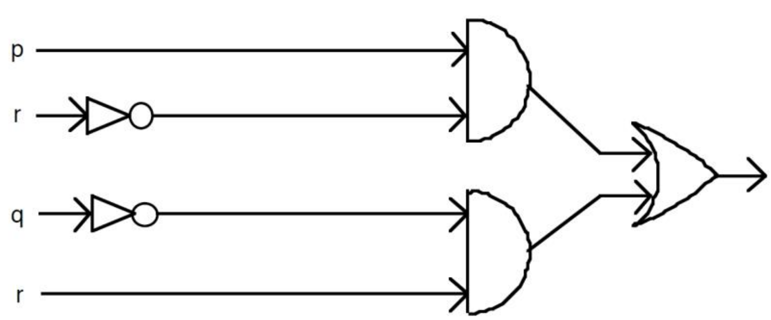
Answer: Suppose sign 1 was true. Then, signs 2 and 3 are both false. The negation of sign 2 (which is true) states that a lady in not in room 2 which means a tiger is in room 2. The negation of sign 3 (which must also be true) states that a tiger is not in room 2. This results in a contradiction. Thus, our opening assumption (namely sign 1 is true) must be incorrect. Thus sign 1 is false. Since it states that a tiger is in room 1, we conclude that a tiger is not in room 1 which implies a lady is in room 1. ***So the prisoner should open room 1.***

[7 – 9] Chapter 1, Section 1.2: Problem 12, 28, 42

12. This system is consistent. We use L, Q, N, and B to stand for the basic propositions here, "The file system is locked," "New messages will be queued," "The system is functioning normally," and "New messages will be sent to the message buffer," respectively. Then the given specifications are $\neg L \to Q$, $\neg L \leftrightarrow N$, $\neg Q \to B$, $\neg L \to B$, and $\neg B$. If we want consistency, then we had better have B false in order that $\neg B$ be true. This requires that both L and Q be true, by the two conditional statements that have B as their consequence. The first conditional statement therefore is of the form $\mathrm{F} \to \mathrm{T}$, which is true. Finally, the biconditional $\neg L \leftrightarrow N$ can be satisfied by taking N to be false. Thus this set of specifications is consistent. Note that there is just this one satisfying truth assignment.

28. Suppose that A is the knight. Then B's statement is true, so he must be the spy, which means that C's statement is also true, but that is impossible because C would have to be the knave. Therefore A is not the knight. Next suppose that B is the knight. His true statement forces A to be the spy, which in turn forces C to be the knave; once more that is impossible because C said something true. The only other possibility is that C is the knight, which then forces B to be the spy and A the knave. This works out fine, because A is lying and B is telling the truth.

42. We have the inputs come in from the left, in some cases passing through an inverter to form their negations. Certain pairs of them enter AND gates, and the outputs of these enter the final OR gate.



Problems on Section 1.3

[10 – 13] Section 1.3: Problems 8, 20, 41, 58

8. We need to negate each part and swap "and" with "or."

a) Kwame will not take a job in industry and will not go to graduate school.

b) Yoshiko does not know Java or does not know calculus.

c) James is not young, or he is not strong.

d) Rita will not move to Oregon and will not move to Washington.

20. It is easy to see from the definitions of the logical operations involved here that each of these propositions is true in the cases in which p and q have the same truth value, and false in the cases in which p and q have opposite truth values. Therefore the two propositions are logically equivalent.

41 (p ^ q ^ ~r) V (p ^ ~q ^ r) V (~p ^ q ^ r)

58. If we want the first two of these to be true, then p and q must have the same truth value. If q is true, then the third and fourth expressions will be true, and if r is false, the last expression will be true. So all five of these disjunctions will be true if we set p and q to be true, and r to be false.

Problems on Section 1.4

[14 – 16] Section 1.4: Problems 8, 16, 28

8. Note that part **(b)** and part **(c)** are not the sorts of things one would normally say.

a) If an animal is a rabbit, then that animal hops. (Alternatively, every rabbit hops.)

b) Every animal is a rabbit and hops.

c) There exists an animal such that if it is a rabbit, then it hops. (Note that this is trivially true, satisfied, for example, by lions, so it is not the sort of thing one would say.)

d) There exists an animal that is a rabbit and hops. (Alternatively, some rabbits hop. Alternatively, some hopping animals are rabbits.)

16. **a)** true ($x = \sqrt{2}$) **b)** false ($\sqrt{-1}$ is not a real number)

c) true (the left-hand side is always at least 2) **d)** false (not true for $x = 1$ or $x = 0$)

28. Let $R(x)$ be "x is in the correct place"; let $E(x)$ be "x is in excellent condition"; let $T(x)$ be "x is a [or your] tool"; and let the domain of discourse be all things.

a) There exists something not in the correct place: $\exists x \neg R(x)$.

b) If something is a tool, then it is in the correct place place and in excellent condition: $\forall x\, (T(x) \rightarrow (R(x) \wedge E(x)))$.

c) $\forall x\, (R(x) \wedge E(x))$

d) This is saying that everything fails to satisfy the condition: $\forall x \neg (R(x) \wedge E(x))$.

e) There exists a tool with this property: $\exists x\, (T(x) \wedge \neg R(x) \wedge E(x))$.