1. Write the following propositions in the form "if p then q" in English.

   (a) It is necessary to do well in the test to get an A in the course.

   (b) To pass quality control test, it is enough to get an error below 0.1mm.

   (c) You can access the web site only if you pay a subscription fee.

   (d) You will not get a passing grade unless you submit the project by tomorrow.

   (e) This rectangle is not a square.

2. When three strangers are seated in a restaurant, the hostess asks them: Does anyone want coffee? The first one says: "I don't know." The second one says: "Yes, someone wants it." The third one says: "Looks like exactly one person wants coffee." With no further questions, hostess serves coffee to those who wanted. Who did she serve coffee?

3. Use truth-table method to determine if the following is a tautology:

   (a) $((\neg q) \wedge (p \to q)) \to \neg q$

   (b) $\neg(p \to q) \to (p \wedge \neg q)$

   (c) $((p \to q) \wedge (q \to r)) \wedge (\neg r)) \to \neg p$

4. Write a propositional logic expression F involving variables p, q and r that is true if and only if exactly one of the three variables is true. Construct a circuit to implement F. (It takes as input p, q and r and outputs F.)

5. Show that the following predicates are NOT logically equivalent: (i) $\forall x(P(x) \vee Q(x))$ (ii) $\forall x\ P(x) \vee \forall x\ Q(x)$ (Choose a domain D with just two elements, and exhibit the truth table for P and Q with the property that expression 1 is true, but expression 2 is false.)

6. Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book."

7. Determine which of the following predicates are true when the domain is the set of integers. State the answer and provide a justification. Also assume that $even(x)$ is true if and only if x is an even integer.

   (a) $\exists x \forall y(x + y = y)$

   (b) $\exists x \exists y(x < y \wedge 2 * x > y)$

   (c) $\forall x \exists y(x < y \wedge 2 * x > y)$

   (d) $\forall x(even(x) \to \neg \exists y(x = 3 * y))$

8. Express each of these system specifications using predicates, quantifiers, and logical connectives.

   (a) At least one mail message, among the nonempty set of messages, can be saved if there is a disk with more than 10 kilobytes of free space.

   (b) Whenever there is an active alert, all queued messages are transmitted.

   The diagnostic monitor tracks the status of all systems except the main console.

   Each participant on the conference call whom the host of the call did not put on a special list was billed.

9. Give a proof by contradiction of the assertion: "If 3n + 2 is odd, then n is odd."

10. For each of these arguments, state which rules of inference are used for each step.

    (a) "Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. Therefore, someone in this class can get a high-paying job."

    (b) "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution."

11. If $m+n$ is an odd integer and $n+p$ is an even integer (where m, n and p are integers), can be always conclude that $m+p$ is an odd integer? Justify your answer. (If false, give a counterexample. If true, prove it.)

12. Prove that if n is an integer, these four statements are equivalent: (i) n is even, (ii) $n+1$ is odd, (iii) $3n+1$ is odd, (iv) $3n$ is even.