Review Mid-term # 2 Spring 2014

Material

The Exam will have material from the following sections:

- 3.1 sorting algorithms
- 9.1 relations
- 6.1-6.5 counting principals

Study tips

- Step through various algorithms we discussed in Chapter 3 with examples.
- Learn all the counting formulas first and when to use them. (Table 1 in page 427 and Theorem 3 in page 428 cover most of the formulas needed for the test.) Then learn their proofs.
- Work on the problems listed below. Then, go over examples from class, old quizzes, and in class work.
- Know what a combinatorial proof is and how to do one. There are lots of problems involving combinatorial proofs in section 6.4.
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Practice Problems

1. Suppose the insertion sort algorithm is run on the input (7, 6, 10, 12, 9, 8, 1).

   (a) What is output after the first iteration of the outer loop?

   (b) What is the total number of data swaps performed?

   (c) What is the total number of comparisons performed?

2. Recall the greedy algorithm to convert a given amount using the standard coin denominations. For the currency set consisting of $\{1, 4, 9, 21\}$, describe how the greedy algorithm will make the target amount of 47 units. Exhibit a target amount that will not be converted otpimally (i.e., with the fewest number of coins) using the greedy algorithm.

3. Shown below is the pseudo-code for binary search. Consider the input to this algorithm: $a_i = 3i + 1$, i = 1, 2, ..., 101 and x = 124. What are the successive values taken by i and what is the value of *location* output by the algorithm?

4. A person deposits $1000 in an account that yields 9% interest compounded annually.

   (a) Set up a recurrence relation for the amount in the account at the end of n years.

   (b) Find an explicit formula for the amount in the account at the end of n years.

5. Let R be a relation defined on the set Z of integers as follows: xRy if $x \leq y$.

(a) Is R a function? Explain.

(b) Is R transitive? Explain.

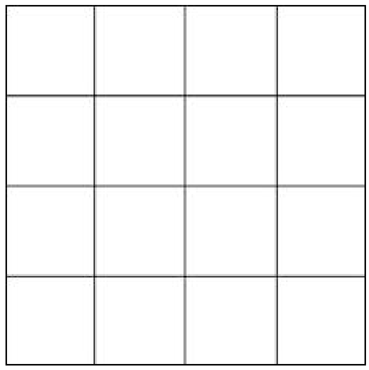
(c) Is R symmetric? Explain.

(d) What is $|A|$ if $A = \{x \mid x \; R \; 6\}$?

6. Describe why the expression $|A \cup B| = |A| + |B| - |A \cap B|$ is true.

7. How many 10 digit integers are there which have exactly two distinct digits? (Make sure to exclude numbers starting with 0.)

8. How many rectangles are in the figure shown below?



9. How many ways are there to arrange the letters A, B, C, D and E such that A never comes immediately after E or D and C always comes immediately before D?

10. There are 12 blue socks, 4 red socks, and 11 green socks in a box. What is the minimum number of socks that need to be drawn from the box to ensure each of the following?

    (a) There is at least one sock of each color.

    (b) There is at least one pair of socks of the same color.

    (c) There is at least one blue sock, and at least two red socks.

11. Suppose that a department contains 13 men and 9 women. How many ways are there to form a committee with 7 members if it must have more women than men?

12. Show that $k\binom{n}{k} = n\binom{n-1}{k-1}$ using a combinatorial argument.

13. Prove there are $\frac{n!}{n_1!n_2!...n_k!}$ number of ways to distribute n distinguishable objects into k distinguishable boxes so that n_i objects are placed into box i, for $i = 1, 2, \ldots k$.

14. How many strings with five or more characters can be formed from the letters in MISSISSIPPI?