1) $(p \wedge q) \vee (p \wedge r) \vee (r \wedge q) \rightarrow F$

2) a) $P(x) :=$ x has a blue shirt.
$q(x) :=$ x can swim.
$r(x) :=$ x was taught.
Domain - All people in the world.

$(\exists x)(P(x) \wedge (r(x) \rightarrow q(x)))$

b) $P(x) :=$ x has a scholar ship.
$q(x) :=$ x has a rich acuaintance.
$r(x) :=$ x had to pay for colledge.
Domain - All students.

$\forall x ((\sim P(x) \wedge \sim q(x)) \Rightarrow r(x))$

3) In step f The incorrectly assumed that the c in step e and d was the same.

4) Proof. We must show $(A-B) \cup (B-A) \cup (A \cap B) \subseteq A \cup B$ and $A \cup B \subseteq (A-B) \cup (B-A) \cup (A \cap B)$.

First choose any $x \in (A-B) \cup (B-A) \cup (A \cap B)$ Then either $x \in (A-B)$ or $x \in (B-A)$ or $x \in (A \cap B)$. ~~Clearly if~~

Case 1. $x \in A-B$ Then $x \in A$ and thus $x \in A \cup B$

Case 2 $x \in B-A$ then $x \in B$ and thus $x \in A \cup B$

Case 3 $x \in A \cap B$ Then $x \in A$ and thus $x \in A \cup B$

In every case $x \in A \cup B$ thus $(A-B) \cup (B-A) \cup (A \cap B) \subseteq A \cup B$.

Now choose any $x \in A \cup B$ Then either $x \in A$ and $x \notin B$ or $x \in B$ and $x \notin A$ or $x \in A$ and $x \in B$.

Case 1. $x \in A$ and $x \notin B$ then $x \in A-B$ and therefore $x \in (A-B) \cup (B-A) \cup (A \cap B)$.

Case 2 $x \in B$ and $x \notin B$ then $x \in B-A$ and therefore $x \in (A-B) \cup (B-A) \cup (A \cap B)$.

Case 3 $x \in B$ and $x \in B$ then $x \in A \cap B$ and therefore $x \in (A-B) \cup (B-A) \cup (A \cap B)$.

In every case $x \in (A-B) \cup (B-A) \cup (A \cap B)$ so $A \cup B \subseteq (A-B) \cup (B-A) \cup (A \cap B)$.

Combining this with our work above we have shown $(A-B) \cup (B-A) \cup (A \cap B) = A \cup B$. $\square$

4) Also can be done with a truth table

A	B	$A-B$	$B-A$	$A\cap B$	$(A-B)\cup(B-A)\cup(A\cap B)$	$A\cup B$
T	T	F	F	T	T	T
T	F	T	F	F	T	T
F	T	F	T	F	T	T
F	F	F	F	F	F	F

Same
So They are $=$.

5) a) $f(x)=x^3$ is onto sine for any $y\in R$ $\sqrt[3]{y}$ is defined and $(\sqrt[3]{y})^3=y$.

It is also 1-1. Choose any $z\in R$ suppos $x^3=z$ and $y^3=z$ for $x,y\in R$. Then $x=\sqrt[3]{z}$ and $y=\sqrt[3]{z}$ and scince $\sqrt[3]{x}$ is a function this implies $x=y$ so x^3 is 1-1.

b) $f(x)=e^x$ is not onto scion $e^x>0$ so $-1\in R$ and There is no x such that $e^x=-1$.

It is however 1-1. Choose any $z\in R$ and suppos for $x,y\in R$ $e^x=z$ and $e^y=z$ Then $x=\ln z$ and $y=\ln z$. Since $\ln$ is a function $x=y$ and $f(x)=e^x$ is 1-1.

5)

c) $f(x) = x^2$ with $f: \mathbb{Z} \to \mathbb{N}$ is not onto for example $5 \in \mathbb{N}$ but $\sqrt{5} \notin \mathbb{Z}$ so there is no element of $\mathbb{Z}$ such that $x^2 = 5$.

It is not 1-1 either since $-4 \in \mathbb{Z}$ and $4 \in \mathbb{Z}$ and $(-4)^2 = (4)^2 = 16$.

6)

$a_0 = 1000$

$a_1 = 1.02(a_0) = 1.02(1000)$

$a_2 = 1.02(a_1) = 1.02(1.02(1000)) = (1.02)^2(1000)$

$a_3 = 1.02(a_2) = 1.02(1.02^2(1000)) = 1.02^3(1000)$

$\vdots$

$a_n = 1.02^n(1000)$ - This can be proved with induction.

An application wold be interest at a bank with a rate of 2%.

7)

$$A*(-2B) = \begin{bmatrix} 1 & 3 \\ 2 & 0 \\ 3 & 0 \end{bmatrix} \cdot \begin{bmatrix} -2 & -14 & -20 \\ 0 & -2 & -4 \end{bmatrix} = \begin{bmatrix} -2 & -20 & -32 \\ -4 & -28 & -40 \\ -6 & -42 & -60 \end{bmatrix}$$

no, $A+B$ is not possible. $A^T + B$ is possible since they Both have the same dimisions.

8) answers vary.

9) Treat AE as 1 letter so there are 4! ways.

10) By The generilzed Pigeonhole principle you must have $3 \cdot 26 + 1$ students.

11) If There are x men then there must be $x+2$ women on the committee but this would give

$$x + x + 2 = 5 \Rightarrow x = \frac{3}{2}$$

which is imposible so there are 0 ways.

12) This is a stars and bars problem with $n=3$ and $r=5$ So there are $C(3+5-1, 5)$ ways.

13) We must show $\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$.

Basis step: If $n=1$ The $\sum_{i=1}^{1} \frac{1}{(2i-1)(2i+1)} = \frac{1}{3} = \frac{1}{2(1)+1}$ So it hold for $n=1$.

Induction step We must show $\sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)} = \frac{k+1}{2(k+1)+1}$ given $\sum_{i=1}^{k} \frac{1}{(2i-1)(2i+1)} = \frac{k}{2k+1}$.

Assume $\sum_{i=1}^{k} \frac{1}{(2i-1)(2i+1)} = \frac{k}{2k+1}$. then

$$\sum_{i=1}^{k+1} \frac{1}{(2i-1)(2i+1)} = \frac{1}{(2(k+1)-1)(2(k+1)+1)} + \sum_{i=1}^{k} \frac{1}{(2i-1)(2i+1)}$$

$$= \frac{1}{(2(k+1)-1)(2(k+1)+1)} + \frac{k}{2k+1}$$

$$= \frac{k+1}{2(k+1)+1}$$

which completes The induction step

13) continued - We now have by Induction that $\sum_{i=1}^{n} \frac{1}{(2i-1)(2i+1)} = \frac{n}{2n+1}$

14) a) We must show There is always an odd number of verticies in a rooted binary Tree;

Basis Step: If There is only an vertex then this result hold trivally.

Induction step: We must show that it it holds that all binary tree with of height k. has odd number of verticies then all ~~binary~~ binary trees of height k+1 also have an odd number of verticies.

Suppose al binary trees of height k have an odd number of verticies, Now take a binary tree of height k+1 if you delete enough leaves so the tree has height k the you must have deleted an even number by constuction of the tree. further more the remaining tree of height k has an odd number of vertices. Thus the original tree of height k+1 has a odd number of verticies. this conclude the indution step

We have now shown by induction that all binary rooted trees have and odd number of verticies.

b)

15) Use The binomial theorem.

16) First algebraicly

$$\frac{n}{k}\binom{n-1}{k-1} = \frac{n(n-1)!}{k(k-1)!\,(n-1-k+1)!}$$

$$= \frac{n!}{k!\,(n-k)!}$$

$$= \binom{n}{k} \quad \square.$$

16) continued

$\binom{n}{k}$ counts the number of ways to choose K people to ride in a car with K seats when location does not matter from a group of n people;

n is the number of ways to choose 1 person from a group of n

$\binom{n-1}{k-1}$ is the number of ways to choose K-1 people to ride in a car with K seats from n-1 people were location does not matter.

Thus $\frac{n}{k}$ is the number of ways to choose one person divided by the number of seats he could sit in and multiplies this by $\binom{n-1}{k-1}$ gives us the total numbre of ways to fill the rest of the seats thus

$\binom{n}{k}$ conts the same set as $\frac{n}{k}\binom{n-1}{k-1}$.

17) This is a stars and bars problem with $n = 5$ $r = 20$ so the solution is $C(20+5-1, 20)$.

18) $$\frac{52!}{13!\,13!\,13!\,13!}.$$

22) see hw 4 solutions

23) no There cannot be an odd numbee of vertices of odd degree.

24) no problem to do

25) 2^4

26) $3 \cdot 2^4$

27) use Theorem 4 from 11.1 with $i = 10{,}000$. Then Ther are

$5(10{,}000)$ people who recive it and

$(5-1)10{,}000 + 1$ people who do not send it out.