Physics 236b, Week 1: Overview and Basics of Gravitational Waves

(Dated: Jan 9, 2014, Due Jan 16, 2014)

I. READING

This week, we discussed the basics of gravitational waves.

- See Misner, Thorne and Wheeler (MTW), *Gravitation*, Chapter 35 for basics of GW.
- See MTW, Secs. 36.1 – 36.4 for generation of gravitational waves. (Note that some estimates on how much GW energy is emitted from astrophysical sources were out-dated.)
- Sec. 26.3 – 26.5 of Blandford and Thorne, *Applications of Classical Physics*.

II. PROBLEMS

The maximum number of points you can get for this assignment is 50, although you could choose to do problems that worth more than 50 points.

1. **Order of magnitude estimates.** [15 Points]
   Use the quadrupole formula to estimate GW amplitude originating from the following earth-based events (very rough estimates will suffice), then estimate their detectability:

   (a) A meteorite (2 km in diameter) hitting the ground at a speed of 25 km/s.

   (b) A big chunk of (1-ton) Piezoelectric material driven to oscillate at 100 MHz.

   Here we have two important cautions: (i) gravitational waves only exist in the wave zone, and (ii) when an elastic object is driven to oscillate at very high frequencies, the resulting motion usually forms a standing-wave pattern within the object, and gravitational waves generated by different portions of this pattern may not superimpose coherently.

2. **Resonant mass detector** [15 Points]

   Analyze the spring-resonator-based resonant mass detector discussed in Lecture 1. In the frequency domain, calculate the transfer function from the gravitational-wave strain $h(\Omega)$ to the change in the gap $x(\Omega)$ between the middle test mass and the free test mass.

   Assume that the spring not only has a spring constant, but also velocity damping (i.e., damping force proportional to velocity). Show that energy is indeed dissipated.

   Since energy is extracted, the GW leaving the detector must have less energy. How does this happen?

3. **Geodesic motion in TT gauge** [15 Points]

   (a) Exercise 26.13 of BT, Geodesic motion in TT coordinate frame.

   (b) As part (a) shows, objects remaining "at rest" in the TT coordinate system are following geodesics. How can you measure gravitational waves if the test masses are not moving?

4. **Sensitivity of GW Detectors** [20 Points] To get a feel for what the possible space of GW astronomy may look like, plot the spectral density of strain noise for the following detectors on a single log-log plot:

   (a) Initial LIGO (aka LIGO-I)

   (b) Advanced LIGO (aka LIGO-II)

   (c) ET (Einstein Telescope)

(d) LISA
(e) BBO
(f) DECIGO
(g) Doppler tracking of satellite signals (including the Cassini mission)
(h) Pulsar timing variations
(i) Cosmic Microwave Background (WMAP)
(j) Cosmic Microwave Background (CMBPOL)

For each trace, list what reference(s) the data comes from. Be sure to avoid the trap of confusing units! All traces should be in the units of (strain/$\sqrt{\mathrm{Hz}}$) (i.e. the square root of the power spectral density of the strain.) Finding all of these will require the wise use of Google Scholar or something similar.