Examples

Generate a Gaussian distribution of width σ

$$P(x)dx = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

also generate y with same distribution

$$P(x)\,P(y)\,dx\,dy = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} - \frac{(y-\mu)^2}{2\sigma^2}\right) dx\,dy$$

change to polar coords

$$x = \rho\cos\theta + \mu$$
$$y = \rho\sin\theta + \mu$$
$$u = \rho^2/2\sigma^2$$
$$a = \theta/2\pi$$

$$P(u)\,P(a)\,du\,da = e^{-u}\,du\,da$$ Box-Muller

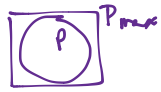
We generate exponentially distributed random # u

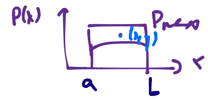
uniform deviate a

Use our coords above + we get Gaussian x + y

Try $\mu = 0$, $\sigma = 1$

Example 2 Generate π using acceptance - rejection





Want $P(x)$ for random variable b/t a + b

Select $P_{max} \geq P(x)$ for all x

Pick trial value $x_t = a + (b-a) R_1$

R_1 is a uniform deviate

Calculate $P(x_t)$

accept x_t if $P(x_t)$ $R_2 P_{max}$

$(x, y) = (a + (b-a)R_1,\ R_2 P_{max})$ accept if on or below $P(x)$

$A_O = \pi \left(\frac{d}{2}\right)^2$ $\qquad A_\square = d^2$

$A_O / A_\square = \frac{\pi d^2}{4 d^2}$

$\pi = \frac{4 A_O}{A_\square}$