Computational Physics

Partial Differential Equations

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Forward-Time Backward-Space (FTBS) Discretization

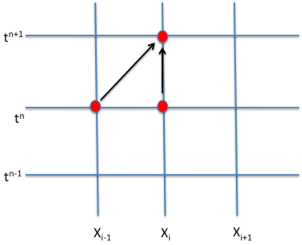
We will approximate the advection equation as

$$\frac{(\bar{\phi}_i^{n+1} - \bar{\phi}_i^n)}{\Delta t} + v\,\frac{(\bar{\phi}_i^n - \bar{\phi}_{i-1}^n)}{\Delta x} = 0$$

thus

$$\bar{\phi}_i^{n+1} = \bar{\phi}_i^n - C\,\left(\bar{\phi}_i^n - \bar{\phi}_{i-1}^n\right)$$

where $C \equiv \Delta t\, v/\Delta x$



Working with PDEs

- We will solve a simple advection equation: Equation

$$\frac{\partial \phi}{\partial t} + v\frac{\partial \phi}{\partial x} = 0$$

- Given the following initial data

$$\phi(t=0) = cos(kx)e^{-(x-x_0)^2/2\sigma^2}$$

$k = \frac{2\pi nn}{L}$, $nn = 10$, $L = xmax - xmin$, x_0 and σ are user inputs.

- Boundary conditions: at $i = 1$, set ϕ_{i-1} to ϕ_{N-1}; at $i = N$, set ϕ_{i+1} to ϕ_2

- Specify dx
- Specify dt: from von Neumann stability analysis we found that

$$dt \leq \frac{1}{v} dx$$

  In my code you will see

$$dt = \lambda \frac{1}{v} dx$$

  where λ is the Courant-Friedrichs-Lewy condition (CFL condition) and I have λ specified.
- Decide how your code will exit, I end after a specified number of cycles.

To Do

Once your code is up and running, perform the following tasks.

- Explore λ to get a sense of the stability situation.
- When the code is stable, what happens as you evolve ϕ? Why?
- Explore other choices for the FT stencil.