Problem Set #1: AY/PH 254C (Spring 2014)
Due Friday Feb 7, 2013 (5pm)
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1. 60s Rocket Person

   The 1960s saw the genesis of modern X-ray astronomy, fostered by a series of remarkable high-altitude rocket experiments carrying little more than glorified Geiger counters. The Giacconi, Gursky, & Speybroeck (1968) review makes for a good read and a useful reference[1] on these early days. Here we'll try to reproduce the basic results of those experiments.

   (a) First, construct a quantum efficiency curve (from 1.0 to 100 keV) of a toy Geiger counter consisting of a tube carrying pure Argon gas enclosed by a Beryllium window. As in some of Giacconi's first experiments[2], assume an 0.002 inch Be window and 7 milligram cm^{-2} of Argon. You might consider checking out the NIST page for mass-attenuation tables[3].

   (b) Next, plot the optical depth to X-rays (from 1.0 to 100 keV) for a source emitting at $d = 1$ kpc from Earth and detected by a set of Geiger counters at an altitude 80 km. Assume an ISM particle density of pure hydrogen with $n = 1$ cm^{-3} and that the atmosphere to that depth has a mass 10 milligram cm^{-2} (as reported in Gursky et al. 1963). At what distance d would the X-ray emitting source need to be be for interstellar absorption to be comparable to the atmospheric absorption for X-rays with energy of 1 keV?

   (c) Assume that the X-ray emitting source is a neutron star radiating at Eddington luminosity with a blackbody spectrum at $kT = 1$ keV. Assume also that the effective collection area of the Geiger counters on your rocket is $A = 500$ cm^2. How many photons per second will you accumulate for that source at $d = 1$ kpc? (Assume you have no other noise or contaminating sources. Also assume that if a photon is stopped in the argon gas it will be recorded by the detector electronics). Compare your answer to typical count rate in the original Giaconni papers.

2. Dont Trap me, Bro

   Here we'll explore the importance of the "trapping radius" for spherical accretion. Consider a spherical flow of ionized hydrogen onto a BH with mass M that dissipates (and radiates) an energy per gram of $\approx GM/r$ as it falls from r to $r/2$ (this is called "maximally dissipative"). Even though some of the energy must clearly be escaping as radiation, presume that the matter is still falling inward at roughly the free-fall

[1] "Observational Techniques in X-Ray Astronomy" Annual Review of Astronomy and Astrophysics, Vol. 6: 373-416.

[2] "Two Source of Cosmic X-rays in Scorpius and Satittarius" Giacconi et al., Nature, 1964, 4962, 981.

[3] http://www.nist.gov/pml/data/xraycoef/index.cfm

speed (we'll ignore the transsonic radius and assume this is true for all r) all the way to the BH.

(a) Calculate the Thomson optical depth from an inner radius r to ∞ as a function of $\dot{M}$. Simplify your expression using the Schwartzchild radius $r_S = 2GM/c^2$ and $\dot{M}_{\rm Edd} = L_{\rm Edd}/c^2$.

(b) At what accretion rate (call this $\dot{M}_c$) does this optical depth become unity at $r = r_S$? How does $\dot{M}_c$ compare to $\dot{M}_{\rm Edd}$?

(c) For $\dot{M} \gg \dot{M}_c$, the inner parts of the flow become optically thick and the flow can drag back in the photons that are trying to diffuse outward through it. The black hole can then swallow these photons, so this may be a way to escape the Eddington limit on mass accretion. Show that the critical **trapping radius** (r_t) is equal to $(\dot{M}/\dot{M}_c) r_S$. Inside this radius, the accretion flow drags the diffusing photons back into the black hole.

(d) What is the approximate luminosity, $L_{\rm esc}$, that escapes to infinity in the regime $\dot{M} \gg \dot{M}_c$ How does the efficient, η, depend on the ratio $\dot{M}/\dot{M}_c$ in this regime? Here you may assume that all radiation created inside the trapping radius gets dragged back into the black hole, and you do not need to invoke a specific emission mechanism for the radiation.

3. Change in orbital period due to mass transfer.

(a) In a close binary system where orbital angular momentum is conserved, show that the change in the orbital period produced by mass transfer is given by

$$\frac{1}{P}\frac{dP}{dt} = 3\dot{M}_1 \frac{M_1 - M_2}{M_1 M_2}$$

(b) U Ceph (Algol type binary) has an orbital period of 2.49 days that has increased by about 20 s in the past 100 yr. The masses of the two stars are $M_1 = 2.9 M_\odot$ and $M_2 = 1.4 M_\odot$. Assuming that this change is due to the transfer of mass between the two stars, estimate the mass transfer rate. Which of the stars is gaining mass?

4. Adapted from problem 4.1 (FKR2) A degenerate star has a mass-radius relation of the form

$$R_2 = K m_2^{-1/3}$$

where K is a constant and m_2 is measured in solar masses.

(a) Show that if the star fills the Roche lobe in a close binary with $q < 1$ we have the period: $P \propto m_2^{-1}$ (JSB notes: recall that q in FKR is the inverse of what we did in class and in Melia...that is, q is $= m_2/m_1$).

(b) From the relation we showed in class $\bar{\rho} \approx 115 P_{\rm hr}^{-2}$ gm cm^{-3}, show that the approximate mass-radius relation of a low-mass main sequence star ($M_2 \approx R_2/R_\odot$) leads to a period-mass relation of $M_2 \approx 0.11 P_{\rm hr}$.

(c) If $K = 2 \times 10^9\,\mathrm{cm}$, show that this relation and the main-sequence relation above intersect at about $P = 0.6$ hr, $M_2 = 0.07$. This shows that there is a minimum orbital period for CVs, since the secondary cannot be smaller than its radius when fully degenerate.

5. Adapted from problem 4.2 (FKR2) For the degenerate secondary in the previous problem, show that

$$-\frac{\dot{M}_2}{M_2} = -\frac{\dot{J}/J}{2/3 - q}$$

6. Adapted from problem 4.5 (FKR2) Consider the Roche potential Φ_R in Cartesian coordinates with the orbital plane at $z = 0$ and the x-axis ($y = 0$) defined by the line connecting the two stars. Assume the Lagrange point is at $(x_1, 0, 0)$.

   (a) Show that, to order of magnitude, that the potential at a nearby point $(x_1, \Delta y, 0)$ is

   $$\Delta\Phi_R \approx \omega^2(\Delta y)^2$$

   where ω is the binary frequency $2\pi/P$.

   (b) Use an energy argument to show that matter escapes from a lobe-filling star in a patch of radius:

   $$H = \Delta y \approx c_s/\omega$$

   around L_1 and hence that the instantaneous mass transfer rate is:

   $$-\dot{M}_2 \sim \frac{1}{4\pi}\rho_{L_1} c_s^3 P^2$$

   where c_s and ρ_{L_1} are the stellar sound speed and density near L_1.

7. Adapted from problem 4.6 (FKR2) For the case of a lower main-sequence star filling the Roche lobe ($P =$ few hours, surface temperatures of 3000–4000 K) use the result from above to those that

$$-\dot{M}_2 \sim 10^{-8}(\rho_{L_1}/\rho_{\mathrm{ph}}) M_\odot \mathrm{yr}^{-1}$$

where $\rho_{\mathrm{ph}} \sim 10^{-6}$ gm cm^{-3} is the photospheric density.