Problem set #4: AY 254C (Spring 2014)
Due Tuesday April 1, 2014 (5pm)

1. **Supernova classification**:

   Check the recent literature (Google scholar is a great resource...or you could start with Alex's 1997 review article and see who cites it) to find answers to the following: What are the distinguishing observational features of Type Ia, Ib, Ic, II-P, IIb, II-n, II-pec and II-L supernovae (e.g., spectra, light curves, velocities, location in galaxies, event rates)? What are the leading theoretical models for each? In each, during what epoch(s) is radioactivity an important contributor to the light curve? Keep your answers simple but clear.

2. **Type Ia Supernovae**: Consider a model for a Type Ia supernova to be a white dwarf with mass 1.39 $M_\odot$ composed of 50% each (by mass) ^{12}C and ^{16}O. The net binding energy (internal energy plus gravitational potential energy) is -5.0 $\times$ 10^{50} erg. How much energy per unit mass (in erg/g) is released when carbon and oxygen burn to ^{56}Ni? Suppose 0.8 $M_\odot$ of the star burns to ^{56}Ni (for simplicity, neglect the synthesis of intermediate- mass elements here). What will the net energy released in the supernova? If it expands to infinity, what would be the typical velocity? (Relevant binding energies: ^{12}C, 92.163 MeV; ^{16}O 127.621 MeV; ^{56}Ni, 484.003 MeV; although, you should be able to read these off from R&B Fig 4.6).

3. **SN gamma-rays**

   Suppose that SN 2011fe produced 0.5 $M_\odot$ of ^{56}Co ($\tau_{1/2} = 77.3$ day). Assuming that after 100 days the supernova became transparent to the γ-rays produced during the decay to ^{56}Fe (not true for SN II, but approximately true for SN I), what would be the flux of such γ-rays at the Earth? (The sensitivity limit of the Fermi γ-ray detector is about 10^6 photons cm^{-2} sec^{-1}.) Although it is not necessary for your result, each ^{56}Co decay produces gamma rays of 847 keV and 1236 keV.

4. **A new source of energy for supernovae? (via Cole Miller, UMD)**

   Dr. Sane has realized that there is another possible energy source for supernovae, which has been overlooked by lesser scientists. His calculations suggest that during the collapse of the core, various processes result in about 1% of the neutrons escaping from nuclei as free particles (i.e., basically in zero density environments). His proposal is that these neutrons then decay into protons, electrons, and electron antineutrinos, and that this energy release powers the supernova. Barry Barish, president of the American Physical Society, is investigating this model for a possible award of a major prize to Dr. Sane. He asks you to do a calculation to see if this model works. The input: the core itself starts at 1.4 M of mass, of which initially about half is neutrons. Supernovae produce $\sim 10^{51}$ ergs of energy in photons and kinetic energy. Give Dr. Sane the maximum benefit of the doubt by assuming that all of the energy in his neutrinos couples to matter and helps drive the supernova.

5. **SN 1987a predicted a loose cable in a 2011 Italian Muon Nuetrino Experiment...take that "Einstein was wrong" folks**

   (a) If 10^{53} erg were released in neutrinos (of mean energy 20 MeV) from SN 1987A in the Large Magellanic Cloud (LMC), how many passed through each person on Earth?

   (b) Of the 20 total neutrinos detected, twelve electron antineutrinos were detected in Japan via Cherenkov radiation from positron recoil in 3 kilotonnes of water. Estimate how many people on Earth could have perceived a flash of light due to Cherenkov radiation produced by the same process, when an antineutrino travelled through their eyeball. Assume eyeballs are composed primarily of water, each about 10 gm in mass. There were about 5 billion people alive in 1987. If you'd like to comment on retinal flashes (related to squiggly floaters...)[1] I cannot stop you.

   (c) Sciencey-types placed an upper limit on the mass of the (electron) neutrino based on the 20 neutrinos detected from SN1987A. They observed a $\sim$13 second spread of neutrino arrival times. If the neutrino has a mass, neutrinos of different energies will have different speeds. An instantaneous pulse of neutrinos with a variety of energies will spread out as it travels the large distance between the LMC and Earth, with the more energetic neutrinos arriving first. Show that the travel time of ultrarelativistic neutrinos of mass m_0 and energy E from a distance D to Earth is given approximately by

   $$t \approx (D/c)[1 + (1/2)(m_0 c^2/E)]$$

   Then find an expression relating the delay relative to a massless neutrino (Δt) to m_0 and E.

   Use this expression with $\Delta t \approx 13$ seconds and $E \approx 20$ MeV to constrain the neutrino mass. More detailed considerations give a mass upper limit of ≈ 15 eV (see The Astrophysical Journal, 318, L63 for details if you are interested). The reason this is only an upper limit rather than a direct mass measurement is that it takes $\sim$10 seconds for neutrinos to diffuse out of the hot, proto-neutron star. The next time a supernova goes off nearby, scientists hope to collect many more neutrinos so that they can study in detail the time structure of the neutrino emission and thereby look inside a collapsing star. What an awesome sight awaits them!

6. **Expanding shells in a supernova.**

   Consider an expanding sphere of initial radius 5×10^8 cm and mass $1.4\ M_\odot$ initially heated to 10^{10} K. Assuming constant density, total ionization, homologous expansion (radial velocity $v \propto r$, where r is the radius), and Thompson opacity (i.e., $\kappa = 0.2$ cm^2 g^{-1} throughout), calculate the radius the expanding sphere would have when it first becomes optically thin. (As an aside, although the gas in a SN I does not remain fully ionized, the Doppler broadened forest of iron lines provides an opacity

[1] http://www.retinamd.com/retinal_detachments_flashes.htm

comparable to electron scattering.) If the expansion is adiabatic ($\rho \propto T^3$) as well as homologous, what is the temperature inside the star when it becomes transparent? (Your result should be lower than the actual value because radioactive decays provide some heating.)