CH 101 - Practice problems – 2/05/2013

Waves and quantum mechanics I.

1. The speed of sound in dry air at 20°C is 350 m/s and the frequency of the sound from the middle C note on a piano is 250 Hz. Calculate the wavelength of the sound and the time it will take to travel 70. m across a concert hall in seconds.

$$\lambda = \frac{v}{\nu} = \frac{350\ m/s}{250\ 1/s} = 1.4\ m$$

$$T = \frac{d}{v} = \frac{70.\ m}{350\ \frac{m}{s}} = 0.20\ s$$

2. How many times more energy is transferred when light with a wavelength of 5.0 nm is absorbed compared to absorption of light with a wavelength of 1.0×10^2 nm?

$$Ratio_\lambda = \frac{5.0\ nm}{1.0 \times 10^2\ nm} = 0.05$$

$$Ratio_E = Ratio_\nu = \frac{1}{Ratio_\lambda} = \frac{1}{0.05} = 20.$$

3. Which of the following beams of electromagnetic radiation transfers the greatest amount of energy per quantum to an electron cloud in a metal?

↳ It doesn't matter how many photons (power), as we are comparing the amount of energy of a single quantum/photon.

Hence, shortest wavelength, highest ν => highest E.

(a) 100 J/s (watt) with wavelength (λ) = 600 nm

(b) 1 J/s (watt) with wavelength (λ) = 400 nm

(c) 0.01 J/s (watt) with wavelength (λ) = 0.01 nm

(d) 0.01 J/s (watt) with wavelength (λ) = 11000 nm

4. Using the information in problem 3 calculate the following:

$E = \frac{hc}{\lambda}$

a. Energy of a photon in each beam of electromagnetic radiation.

(a) 3.3×10^{-19} J

(b) 5.0×10^{-19} J

(c) 2.0×10^{-14} J

(d) 1.8×10^{-20} J

b. Number of photons per second in each beam of electromagnetic radiation.

$\frac{\#photon}{s} = \frac{\cancel{E}/s}{\cancel{E}/photon} = \frac{photons}{s}$ power

(a) 3.0×10^{20}

(b) 2.0×10^{18}

(c) 5.0×10^{11}

(d) 5.6×10^{17}

c. Which beam of light transfers more energy per second.

Here it doesn't matter how many photons you need, simply look at the powers.

Answer : (a)

5. A beam of red light has a frequency of 4×10^{14} Hz.

a. Calculate the energy of a photon.

$E_{photons} = 2.7 \times 10^{-19}$ J

$E = h\nu$

b. What is the energy of 1.325 mol of these photons?

$E_{1.325\ mol} = E_{photons} \cdot \frac{\#\ photons}{1\ mol} \cdot 1.325\ mol$

$= 215\ kJ \quad \left(= 2.7 \times 10^{-19} \frac{J}{\cancel{photon}} \cdot 6.022 \times 10^{23} \frac{\cancel{photons}}{\cancel{mol}} \cdot 1.325\,\cancel{mol} \right)$

6. Fluorescent bulb emits light of several different wavelengths from each major region of the visible spectrum so that its light appears white to our eyes. Assume that a 45 watt fluorescent bulb emits equal amounts of red, green and blue light. Assume that the blue wavelength is 450 nm. How many energy units (photons) of blue light are emitted each second by the matter composing the fluorescent bulb? Recall that 1 watt =1 J/s, and assume the bulb operates at 70% efficiency.

45 W power of total light $\Rightarrow \frac{1}{3} 45 = 15$ W power blue light.

$$E_{photon} = h\nu = \frac{hc}{\lambda} = 4.42 \times 10^{-19} \text{ J}$$

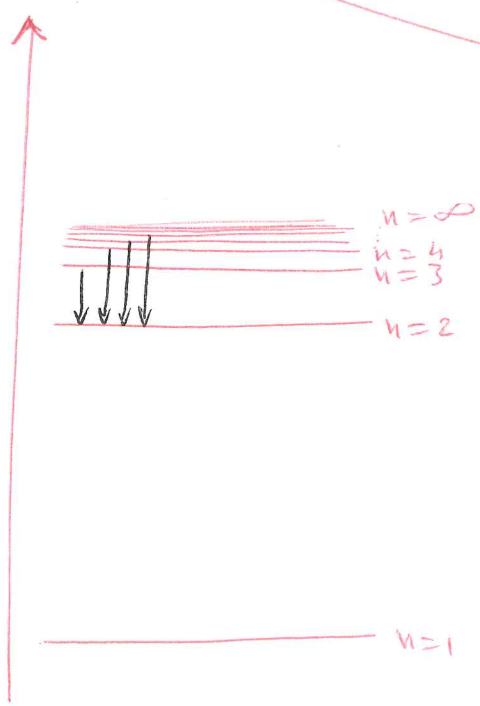
$$\frac{\#\text{photons}}{s} = \frac{E/s}{E/\text{photon}} = \frac{15 \text{ J/s}}{4.42 \times 10^{-19} \text{ J/s}} = 3.4 \times 10^{19} \text{ photons/s @ 70\%.}$$

$$100\% \text{ of photons} = 3.4 \times 10^{19} \times \frac{100}{70} = 4.9 \times 10^{19} \text{ photons/s}$$

7. Sketch an energy-level diagram for electronic transitions in the Bohr model for the hydrogen atom.
   a. Explain (providing mathematical support) why the energy levels get closer together as they increase.
   b. Draw the arrows for the transitions that generate the visible lines we observed in class during the demo. (If you forgot how it looked like, you can refer to picture 1.10 in the textbook.)

Balmer Series $\Rightarrow n_{FINAL} = 2$.

$$\Delta E = hR \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$



Since the "n" are at the denominator and squared, the differences will decrease when n increase.

8. Neutron diffraction is used in determining the structures of molecules.

a. Calculate the wavelength of a neutron moving at 1.00% of the speed of light.

$$\lambda = \frac{h}{m_n\, v} = \frac{6.63\times10^{-34}\ m^2\ \cancel{Kg}\ \cancel{s^{-1}}}{1.67\times10^{-27}\ \cancel{Kg}\cdot 3.0\times10^{6}\ \cancel{\tfrac{m}{s}}}$$

Known constant

$0.01\times 3.0\times10^{8} = 3.0\times10^{6}\ \frac{m}{s}$

De Broglie

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$= 1.3\times10^{-13}\ m$

b. Calculate the velocity of a neutron with a wavelength of 75pm.

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}}{1.67\times10^{-27}\times 7.5\times10^{-11}} = 5.2\times10^{3}\ \frac{1}{s}$$

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9. The work function for lithium is 279.7 KJ/mol.

$$\Phi_{photon} = \frac{279.7\ \frac{\cancel{KJ}}{\cancel{mol}}\cdot\frac{1000\ J}{1\ \cancel{KJ}}}{6.022\times10^{23}\ photons/\cancel{mol}} = 4.6\times10^{-19}\ J$$

a. What is the maximum wavelength of light that can remove an electron from an atom on the surface of lithium metal?

Max λ => min E => no leftover energy => KE = 0

$KE = h\nu - \Phi$, if KE = 0 => $h\nu = \Phi$ => $\nu = \frac{\Phi}{h}$

$$\lambda = \frac{c}{\nu} = \frac{c}{\Phi/h} = \frac{hc}{\Phi} = \frac{6.63\times10^{-34}\times 3.0\times10^{8}}{4.6\times10^{-19}} = 4.3\times10^{-7}\ m = 430\ nm$$

b. What is the maximum wavelength of light capable of removing an electron from an atom on the surface of lithium with a linear momentum of 13.94×10^{-24} kg m/s?

We can calculate v (velocity) $= p/m = \frac{13.94\times10^{-24}\ \cancel{Kg}\ m\ s^{-1}}{9.109\times10^{-31}\ \cancel{Kg}} = 1.5\times10^{7}\ \frac{m}{s}$

$$KE = \frac{1}{2}mv^2 = 0.5\times 9.109\times10^{-31}\times(1.5\times10^{7})^2 = 1.025\times10^{-16}\ J$$

$$KE = h\nu - \Phi = \frac{hc}{\lambda} - \Phi \Rightarrow \lambda = \left(\frac{KE+\Phi}{hc}\right)^{-1}$$

$$= \left(\frac{1.025\times10^{-16}\ \cancel{J} + 4.6\times10^{-19}\ \cancel{J}}{6.63\times10^{-34}\ \cancel{J}\ \cancel{s}\cdot 3.0\times10^{8}\ \frac{m}{\cancel{s}}}\right)^{-1} = (5.2\times10^{8})^{-1} = 1.92\times10^{-9}\ m$$

10. The Li^{2+} ion has only one electron, therefore can be approximately explained with the Bohr model for the hydrogen atom. Compare the energy of the 4th energy level (n=4) of the Li^{2+} ion with that of the 6th energy level (n=6) of the H atom. How many times bigger/smaller is it going to be? (Note: The problem is not asking for the absolute value, only the ratio).

H $n = 6$

$Z = 1$

$E = -hR \cdot \frac{1^2}{6^2}$

Li^{2+} $n = 4$

$Z = 3$

$E = -hR \cdot \frac{3^2}{4^2}$

Ratio Li^{2+} v. H

$$\frac{\cancel{-hR}\,\frac{9}{16}}{\cancel{-hR}\,\frac{1}{36}} = \frac{9}{\cancel{16}_{4}} \cdot \frac{\cancel{36}^{9}}{1} = \frac{81}{4}$$

11. Does a photon of visible light (l = 400 to 700) have enough energy to excite an electron in the hydrogen atom...

a. ...from n=2 to n=5?

$\nu = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$, $R = 3.29 \times 10^{15}$ Hz

$\nu = 3.29 \times 10^{15}\,\text{Hz}\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 0.69 \times 10^{15}$ Hz

Max ν of visible light $= 7.5 \times 10^{14}$ Hz

7.5×10^{14} Hz $>$ 6.9×10^{14} Hz $\Rightarrow$ Yes

b. ...from n=2 to n=6?

$\nu = 3.29 \times 10^{15}\left(\frac{1}{2^2} - \frac{1}{6^2}\right) = 7.3 \times 10^{14}$ Hz

7.5×10^{14} Hz $>$ 7.3×10^{14} Hz $\Rightarrow$ Yes

c. ...from n=1 to n=4?

$\nu = 3.29 \times 10^{15}\left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 3.08 \times 10^{15}$ Hz

7.5×10^{14} Hz $<$ 3.08×10^{15} Hz $\Rightarrow$ No

12. The Cosmic microwave background radiation fits the Planck equations for a blackbody at 2.76 K.

a. What is the wavelength at the maximum intensity of the spectrum of the background radiation ?

Wien's law $T\lambda_{max} = 2.9\ K.mm$

$$\lambda = \frac{2.9\ \cancel{K}.mm}{2.76\ \cancel{K}} = 1.1\ mm$$

b. What is the frequency of the radiation at the maximum?

$$\nu = \frac{c}{\lambda} = 2.7 \times 10^{8}\ \frac{1}{s}$$

c. What is the total power incident on Earth from the background radiation? (Hint: The total power = Intensity of the radiation X area of incidence)

$$\text{Total Intensity} = 5.67 \times 10^{-8}\ \frac{W}{m^2 . \cancel{K^4}} . (2.9\ \cancel{K})^4 = 4.0 \times 10^{-6}\ \frac{W}{m^2}$$

$$\text{Power} = \text{Intensity} \times \text{Area} = 4.0 \times 10^{-6}\ \frac{W}{\cancel{m^2}} \times 5.1 \times 10^{14}\ \cancel{m^2}$$

$$= 20. \times 10^{8}\ W$$

Note: Area = Earth surface = $5.1 \times 10^{14}\ m^2$