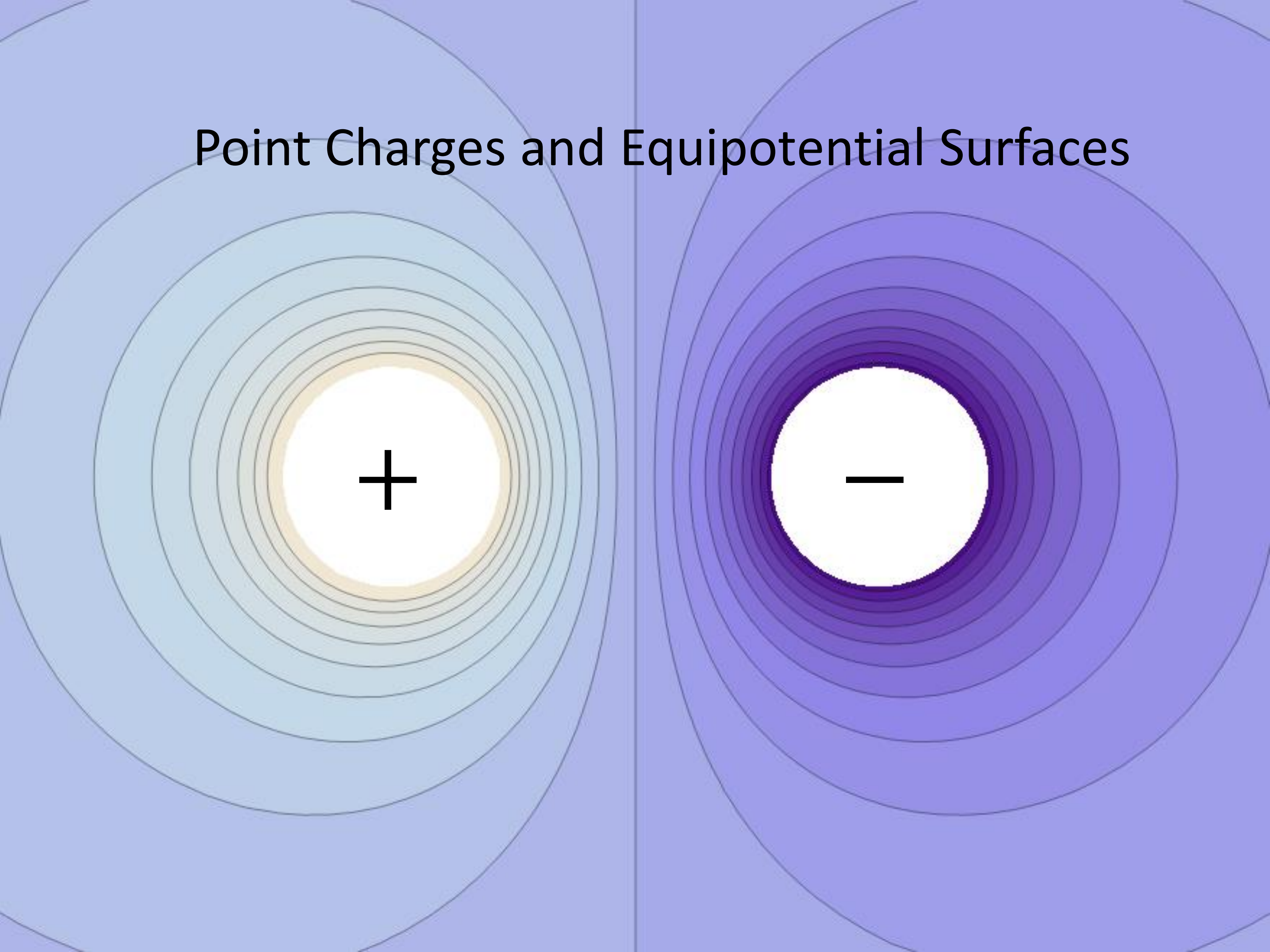
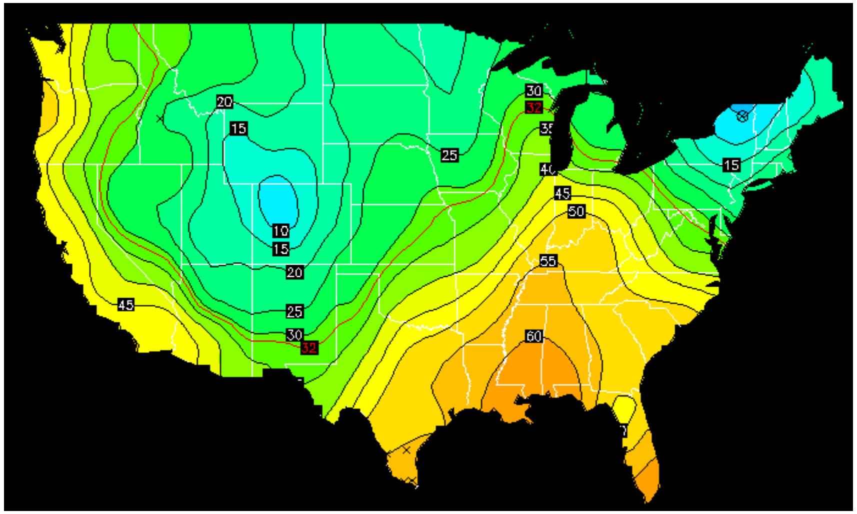
# Point Charges and Equipotential Surfaces

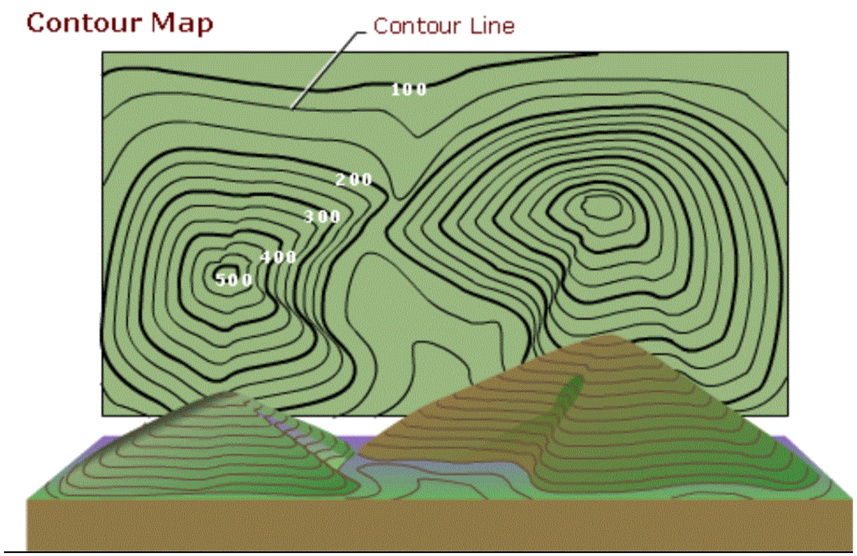


# Equipotentials in Other Fields

On the map below, the low temperatures for the day have been plotted over the United States. The people who made the map connected all of the weather stations that reported the same temperature, hence the lines on the map represent paths of equal temperature.




Topography adds a third dimension to the flat-map picture of the world. Sometimes elevations are indicated with different colors or by number labels



# Equipotentials in Electrostatics

(Optional Derivation)

$$W = \left|\vec{F}_e\right| |\Delta \vec{r}| \cos\theta = q\left|\vec{E}\right| |\Delta \vec{r}| \cos\theta$$

$$\Delta EPE = -W = -q\left|\vec{E}\right| |\Delta \vec{r}| \cos\theta = q\Delta V$$

$$\boxed{\Delta V = -\left|\vec{E}\right| |\Delta \vec{r}| \cos\theta}$$

$$\Rightarrow \frac{|\Delta V|}{|\Delta \vec{r}|} = \left|\vec{E}\right| \cos\theta$$

$$\theta = 0 \Rightarrow \boxed{\left(\frac{|\Delta V|}{|\Delta \vec{r}|}\right)_{\max} = \left|\vec{E}\right|}$$

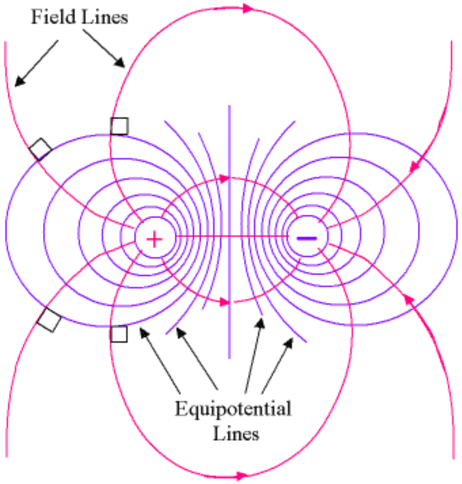
Main poin: Electric field lines can be obtained by finding the direction in which change of potential is maximum

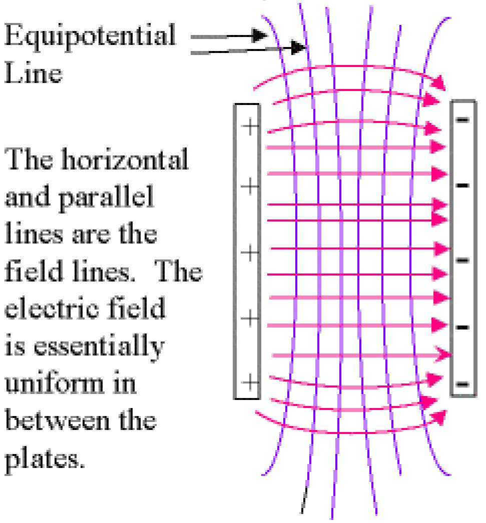
# Equipotentials in Electrostatics

$$\left(\frac{|\Delta V|}{|\Delta \vec{r}|}\right)_{\max} = |\vec{E}|$$

- Equipotentials are virtual lines (or surfaces in 3D) along which the potential **does not change**.
- Electric field lines point perpendicular to equipotentials
- Electric field points from higher $V$ toward lower $V$.
- If equipotentials are drawn so that $|\Delta V|$ is const. then $|\vec{E}|$ is higher in places where equipotentials are closer together.

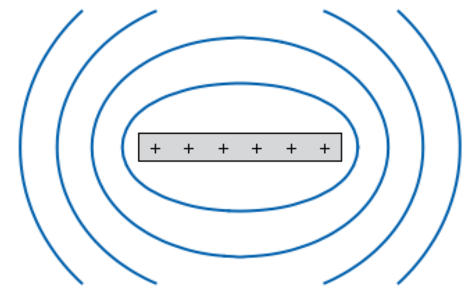
Simulation:
http://www.falstad.com/vector2de/



# Equipotentials of Objects

- For a point charge the equipotentials are concentric circles in 2D and spheres in 3D.
- Equipotentials of a sphere are also spheres
- Does it mean the equipotentials of a rod are boxes? Why?



# Energy and Equipotentials

- How much work is needed to move a charge along an equipotential line between points $A$ and $B$?

$$W = -\Delta EPE = -q\Delta V$$

- Between any points on an equipotential line the potential is the same by definition:

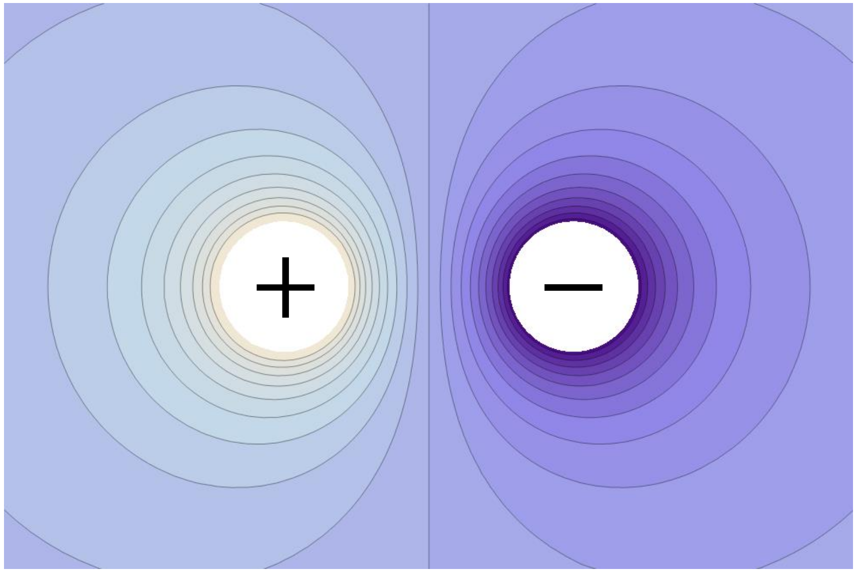
$$V_A = V_B \Rightarrow \Delta V_{AB} = V_A - V_B = 0$$
$$\Rightarrow \Delta EPE = 0 \Rightarrow W = 0$$

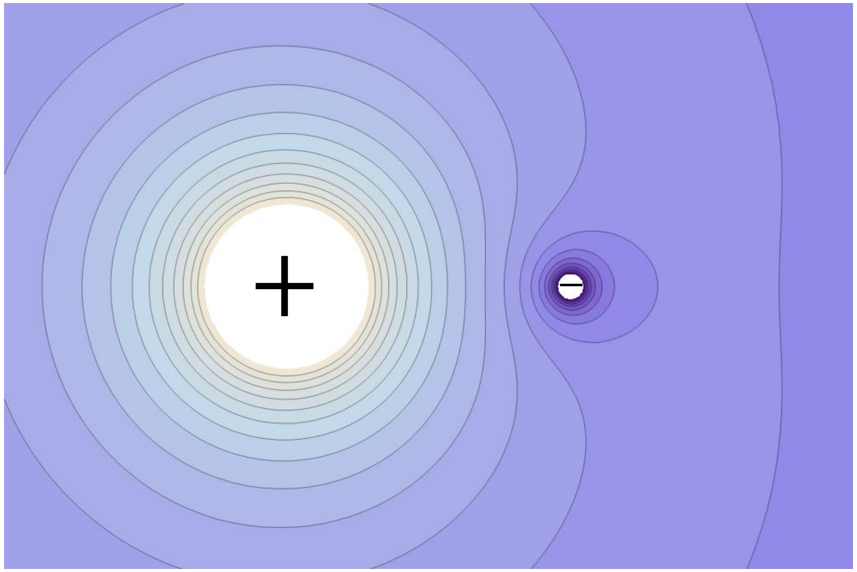
Simulation:
http://phet.colorado.edu/sims/charges-and-fields/charges-and-fields_en.html

# Equipotentials in Electrostatics

**Equipotential lines become farther apart away from a charge**

**If one charge is much larger than the other**



# Quickuiz

Consider a charged conductor at equilibrium. What is the electric field and electric potential inside it?

$A.\ |\vec{E}| = const \neq 0, V = 0$

$B.\ |\vec{E}| = 0, V = 0$

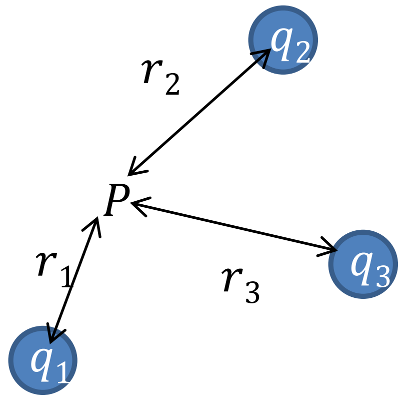
$C.\ |\vec{E}| = 0, V = const \neq 0$

$D.\ |\vec{E}| = const \neq 0, V = const \neq 0$

# Electric Potential due to Multiple Point Charges

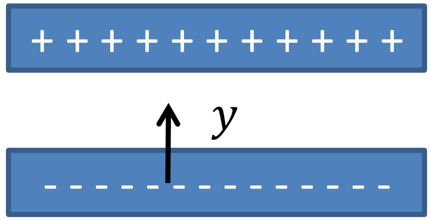
- To find the potential due to several charges at a point $P$, just add the potential due to all charges (superposition):

$$V_{tot,P} = V_{1,P} + V_{2,P} + V_{3,P}$$
$$= \frac{kq_1}{r_{1P}} + \frac{kq_2}{r_{2P}} + \frac{kq_3}{r_{3P}}$$



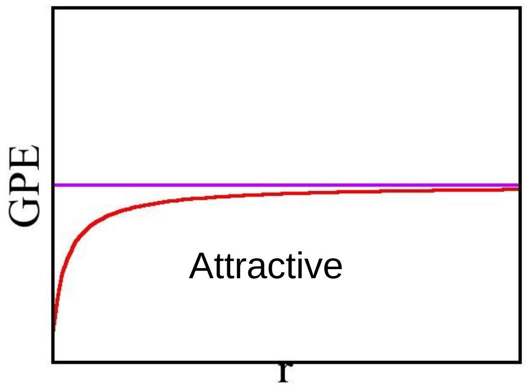

- If you have other configurations, for example parallel plates, you add that. For example, a uniform field:

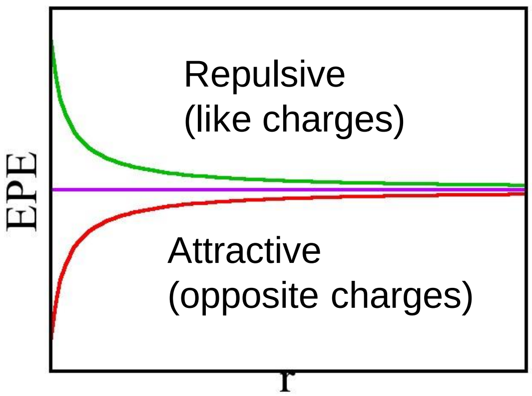
$$V = |\vec{E}|y$$



# Electrical Potential Energy of a Point Charge

| | Force | Potential Energy |
|---|---|---|
| Gravity | $F_g = \frac{Gm_1m_2}{r^2}$ | $GPE = -\frac{Gm_1m_2}{r}$ |
| Electricity | $F_e = \frac{kq_1q_2}{r^2}$ | $EPE = \frac{kq_1q_2}{r}$ |



# Quickuiz

A charge $q$ is at the origin. Consider two points $P_1 = (3\text{ m}, 4\text{ m})$ and $P_2(-5\text{m}, 0\text{m})$. Let $V_1$ be the potential at $P_1$ and $V_2$ at $P_2$. Recall for a point charge $V = kq/r$. The ratio $V_1/V_2$ is

*A.* 2

*B.* $q$

*C.* 1

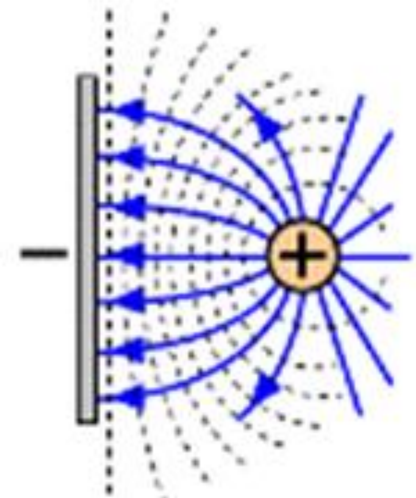
*D.* $-1$

$$V_1 = \frac{kq}{\sqrt{3^2 + 4^2}} = \frac{kq}{5\text{m}}$$

$$V_2 = \frac{kq}{\sqrt{5^2 + 0}} = \frac{kq}{5\text{m}}$$

# Demo Problem

- A very long negatively charged rod extends along the $y$-axis with a field intensity $\left|\vec{E}_r\right| = 2.0 \times 10^3\ \mathrm{N/C}$. At point $P = (10.0\ \mathrm{cm}, 0)$ there is a positive charge $q = 1.0\ \mathrm{nC}$.
- What is the total electric field at $M = (5.0\ \mathrm{cm}, 0)$?



$$\vec{E}_{net} = \vec{E}_r + \vec{E}_P$$

$$|\vec{E}_P| = \frac{kq}{r^2} = \frac{(9 \times 10^9)(10^{-9})}{(0.5 \times 10^{-1})^2} = 3.6 \times 10^3\ \frac{N}{C}$$

$$\left|\vec{E}_{net}\right| = \left|\vec{E}_r + \vec{E}_P\right| = 3.6 \times 10^3 + 2.0 \times 10^3$$

$$= 5.6 \times 10^3\ \frac{N}{C}\ \text{pointing left}$$

Point charge:

$$\left|\vec{E}\right| = kq/r^2$$
$$V = kq/r$$

Rod:

$$\left|\vec{E}\right| = const.$$
$$V = \left|\vec{E}\right|\Delta r$$
$$EPE = qV$$

# Example

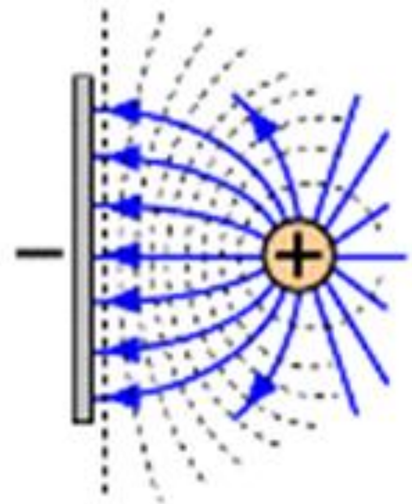
- A very long negatively charged rod extends along the $y$-axis with a field intensity $|\vec{E}_r| = 2.0 \times 10^3\ \mathrm{N/C}$. At point $P = (10.0\ \mathrm{cm}, 0)$ there is a positive charge $q = 1.0\ \mathrm{nC}$.
- What is the electric potential difference between $M = (5.0\ \mathrm{cm}, 0)$ and $T = (7.5\ \mathrm{cm}, 0)$ $\Delta V_{MT} = V_M - V_T$?

$$\text{due to rod: } \Delta V_{r,MT} = (2.0 \times 10^3)(0.05 - 0.075) = -50\ \mathrm{V}$$

$$\text{due to charge: } \Delta V_{P,MT} = \frac{kq}{r_M} - \frac{kq}{r_T}$$

$$= \frac{(9 \times 10^9)(10^{-9})}{5.0 \times 10^{-2}} - \frac{(9 \times 10^9)(10^{-9})}{2.5 \times 10^{-2}} = -180\ \mathrm{V}$$

$$\Delta V_{MT} = V_M - V_T = \Delta V_{r,MT} + \Delta V_{P,MT} = -230\ \mathrm{V}$$



Point charge:

$|\vec{E}| = kq/r^2$

$V = kq/r$

Rod:

$|\vec{E}| = const.$

$V = |\vec{E}|\Delta r$

$EPE = qV$