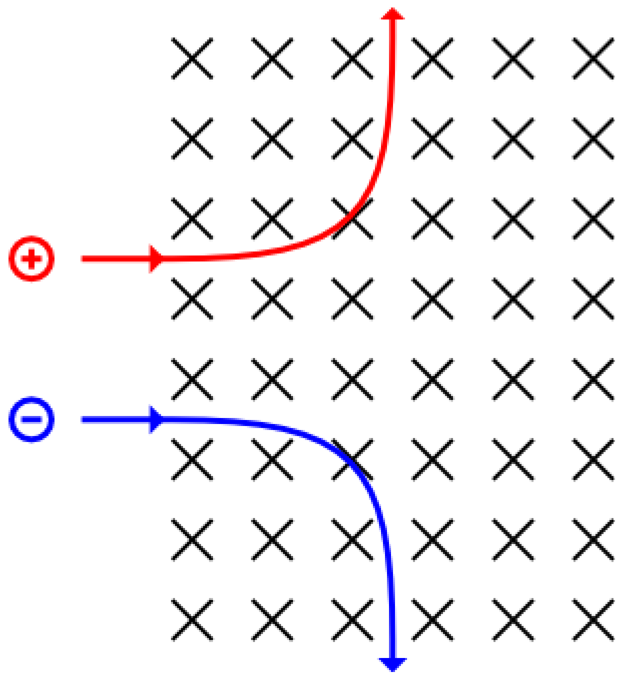
Charged Particles in Magnetic Fields



Quickuiz

A particle with positive charge $+q$ is moving with speed v in the $+x$ direction within a region in which a uniform magnetic field with magnitude B points in the $-x$ direction. Which of the following changes will result in the greatest increase in the magnitude of the magnetic force on the particle?

A. Doubling the speed v

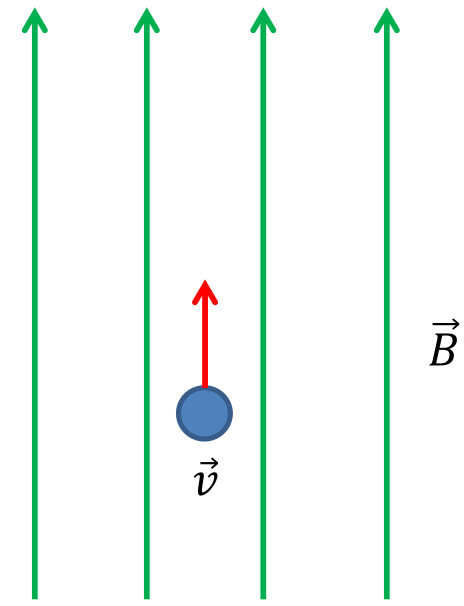
B. Halving the charge on the particle to $+q/2$

C. Rotating the magnetic field to point in the $+y$ direction

D. Tripling the magnitude of the magnetic field

Paths of Charged Particles in Uniform Magnetic Field Case #1

- Velocity of particle is parallel to the magnetic field, i.e. $\theta = 0°$:
  $\left|\vec{F}_m\right| = |q||\vec{v}|\left|\vec{B}\right||\sin 0°| = 0$
- The particle will move in a straight line at constant velocity, i.e. no force acts on it.



Paths of Charged Particles in Uniform Magnetic Field Case #2

- Velocity of particle is perpendicular to the magnetic field, i.e. $\theta = 90°$:

$$\left|\vec{F}_m\right| = |q||\vec{v}|\left|\vec{B}\right||\sin 90°| = |q||\vec{v}|\left|\vec{B}\right|$$

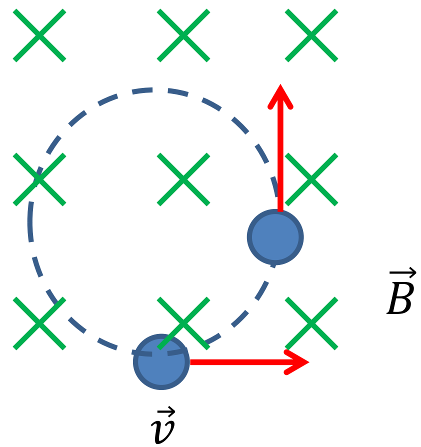
- The particle will move in a circles experiencing centripetal acceleration:

$$a_c = |\vec{v}|^2/r$$

- Therefore:

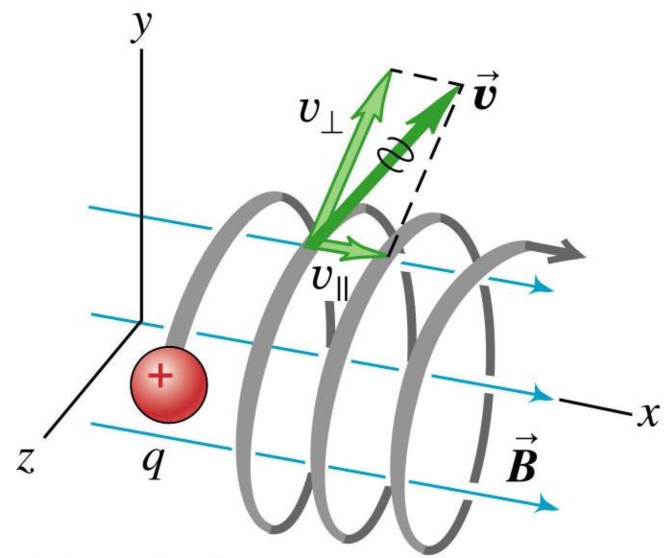
$$F_{net} = ma_c \Rightarrow |q||\vec{v}|\left|\vec{B}\right| = \frac{m|\vec{v}|^2}{r}$$

$$\Rightarrow \boxed{r = \frac{m|\vec{v}|}{|q|\left|\vec{B}\right|}}$$

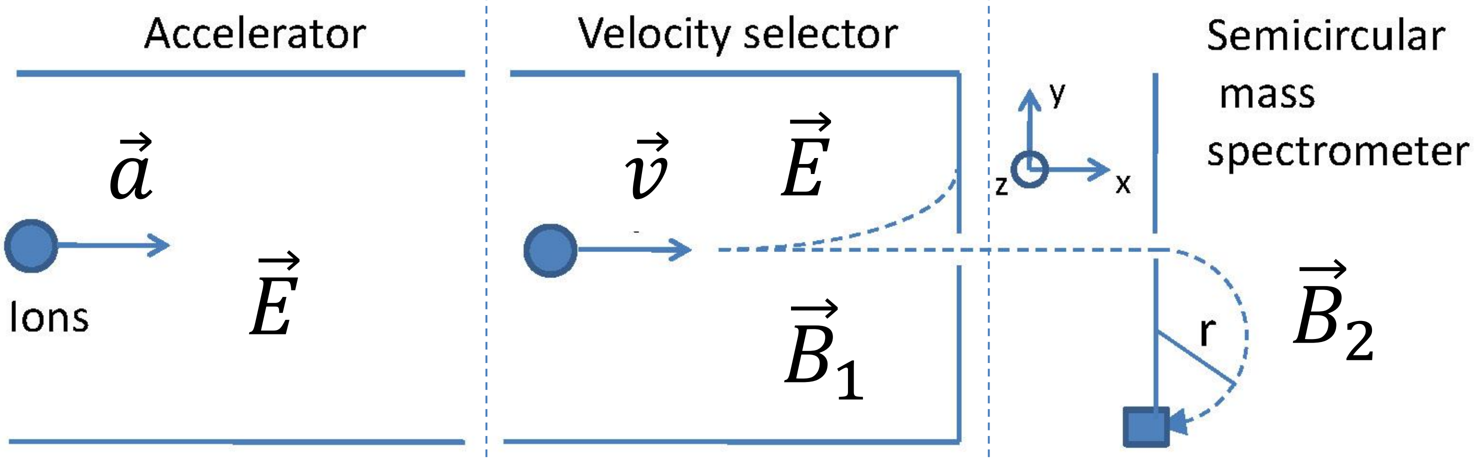


Paths of Charged Particles in Uniform Magnetic Field Case #3

- If $\vec{v}$ has components both parallel to and perpendicular to $\vec{B}$, the parallel component of $\vec{v}$ is unaffected, while the perpendicular component executes circular motion in the plane perpendicular to $\vec{B}$, giving **helical motion**



Mass Spectrometry



Particle enters electric field:

$$\vec{F} = \vec{F}_e = q\vec{E} = m\vec{a}$$

Only particles with a given velocity selected

$$q\vec{E} = q\vec{v} \times \vec{B}_1$$

$$\Rightarrow |\vec{v}| = \frac{|\vec{E}|}{|\vec{B}_1|}$$

Radius of curvature measured

$$\frac{m|\vec{v}|^2}{r} = q|\vec{v}||\vec{B}_2|$$

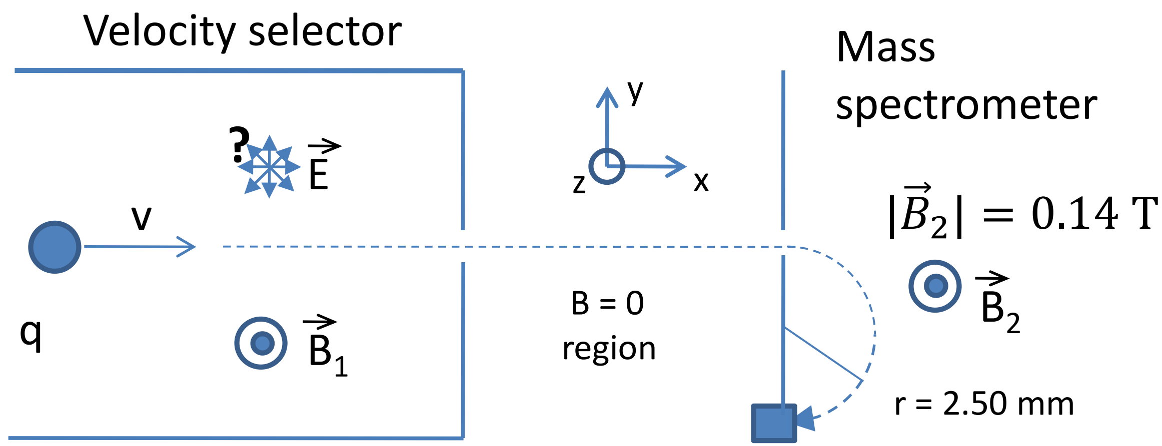
$$\Rightarrow r = \frac{m|\vec{v}|}{q|\vec{B}_2|}$$

Demo Problem

A charged particle moving in the positive x-direction enters a velocity selector in which B points in the positive z-direction with magnitude $0.070\ \mathrm{T}$. There is also a uniform electric field inside the velocity selector with magnitude $E = 600.0\ \mathrm{V/m}$. The particle is undeflected and so passes through the velocity selector. It then enters a mass spectrometer in which the magnetic field is twice as strong. As a result of the magnetic force in the mass spectrometer, the particle begins to curve downward (in the $-y$ direction). It follows a clockwise semicircular arc of radius $r = 2.55\ \mathrm{mm}$ in the xy-plane, eventually entering one of the detectors in the mass spectrometer.

What direction must the electric field in the velocity selector be pointing?

Positive y-direction



$$|\vec{v}| = \frac{|\vec{E}|}{|\vec{B}_1|} \qquad r = \frac{m|\vec{v}|}{q|\vec{B}_2|}$$

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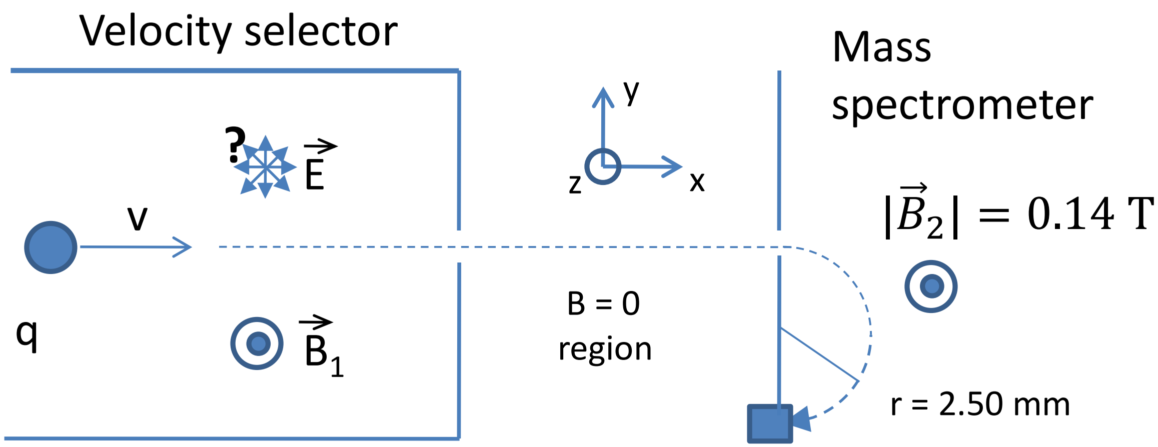
What is the speed of the particle in the velocity selector?

$$v = \frac{E}{B_1} = \frac{600.0\ \mathrm{V/m}}{0.07\ \mathrm{T}}$$

$$= 8570\,\frac{\mathrm{m}}{\mathrm{s}}$$



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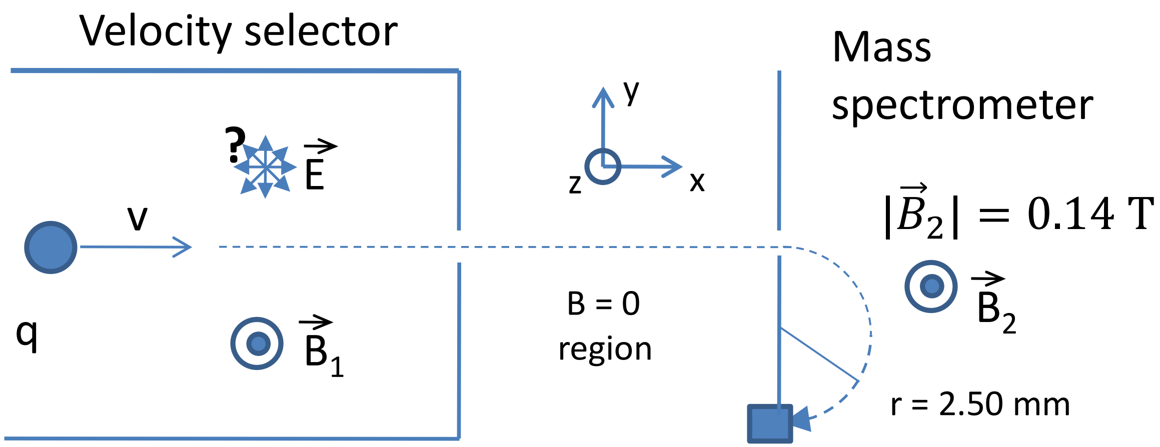
Find the ratio of charge to mass (q/m) for this particle.

$$r = \frac{mv}{qB_2} \Rightarrow$$

$$\frac{q}{m} = \frac{v}{B_2 r}$$

$$= \frac{8570\ \mathrm{m/s}}{(0.14\mathrm{T})(2.55 \times 10^{-3}\mathrm{m})}$$

$$= 2.40 \times 10^{7}\mathrm{C/kg}$$



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