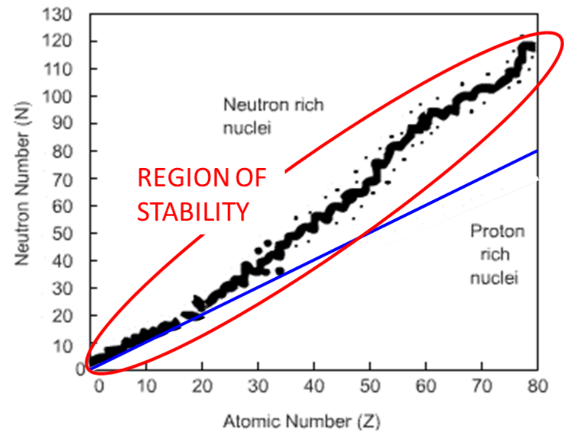
Decay Rate, Half Life, and Radiocarbon Dating

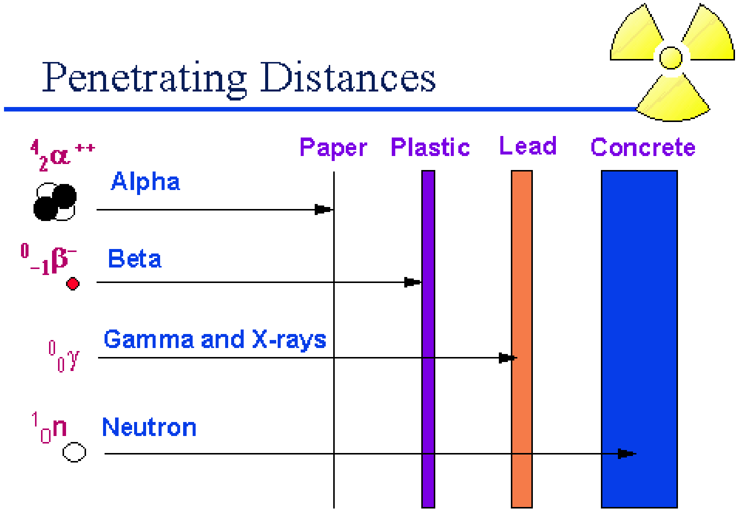
TELL ME,

Where half you been all my life?

Types of Radioactive Decay

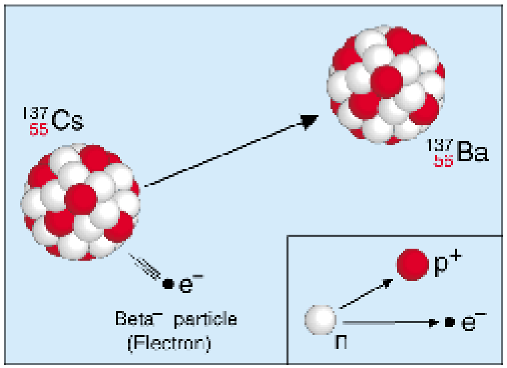
- Some nuclei are extremely stable, some are not.
- Unstable nuclei decay by emitting particles and turning into other nuclei. There are three main types of emission:
  - Alpha (α) particles, which are helium nuclei; heavy, positively charged.
  - Beta (β) particles, which electrons or positrons (lighter than alpha)
  - Gamma (γ) particles, which are high frequency light photons; massless



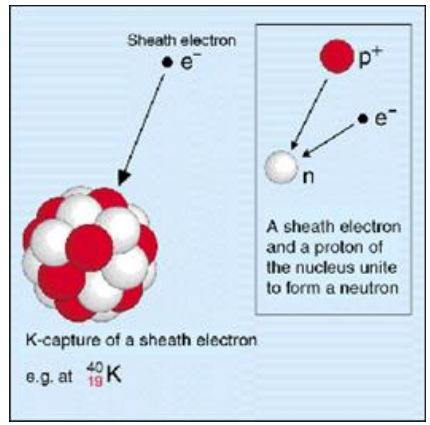


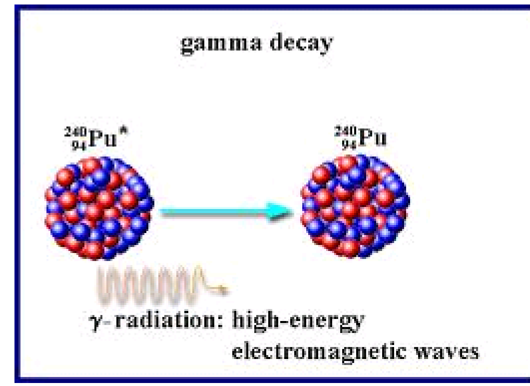
Radioactive Decay

Conservation of electric charge and nucleon number (also energy and momentum)

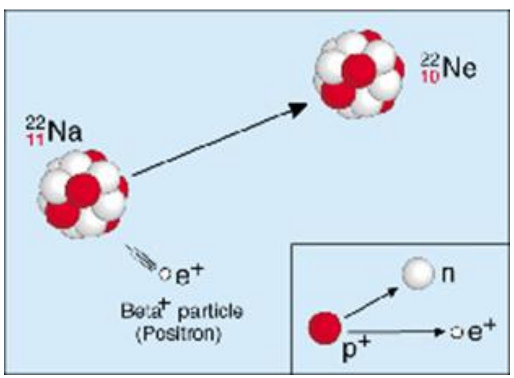


β^- decay: ${}^1_0\mathrm{n} \rightarrow {}^1_1\mathrm{p} + {}^0_0\mathrm{e}^-$

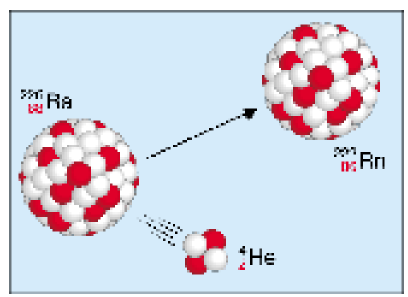




Electron capture: ${}^1_1\mathrm{p} + {}^0_0\mathrm{e}^- \rightarrow {}^1_0\mathrm{n}$



β^+ decay: ${}^1_1\mathrm{p} \rightarrow {}^1_0\mathrm{n} + {}^0_0\mathrm{e}^+$

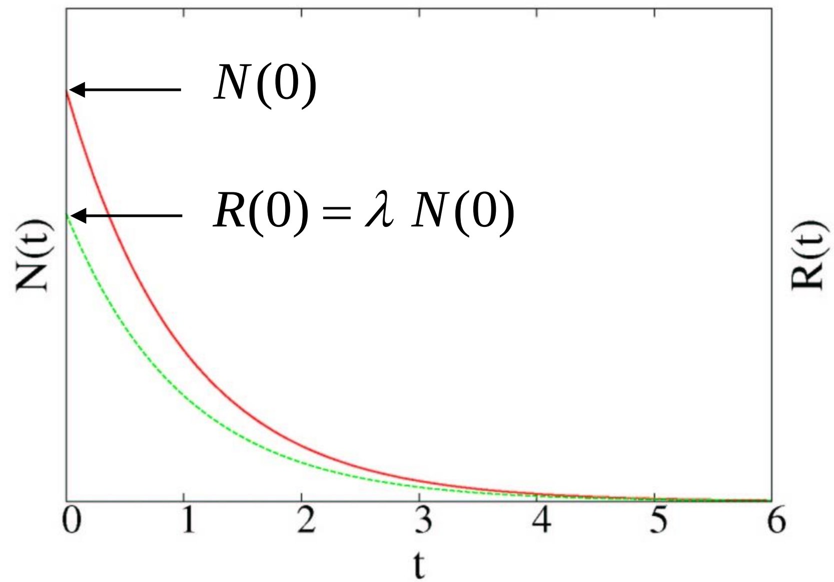


α decay: ${}^A_Z\mathrm{P} \rightarrow {}^{A-4}_{Z-2}D + {}^4_2\mathrm{He}$

Simulation: http://phet.colorado.edu/en/simulation/alpha-decay

Mathematics of Radioactive Decay

- The more nuclei that still remain, the more decays will occur in one second:
$$R(t) = \lambda N(t)$$
- λ is the decay constant measured in s^{-1}
$$\frac{\Delta N}{\Delta t} = -R(t) = -\lambda N(t)$$
- This implies an exponential decrease of $N(t)$ with time:
$$N(t) = N(0)e^{-\lambda t}$$
$$R(t) = R(0)e^{-\lambda t}$$
$$1 \text{ Curie (Ci)} = 3.70 \times 10^{10}\text{Bq}$$
$$1 \text{ Becquerel (Bq)} = 1 \text{ decay/s}$$



- $N(t)$ is the number of undecayed nuclei remaining at time t
- $N(0)$ is the initial number of undecayed nuclei
- $R(t)$ is the activity, i.e. the rate of decays per second

Quickuiz

Suppose you start with 10000 atoms of Barium-144, a radioactive isotope which undergoes β^- decay to turn into Lanthanum-144 with a decay constant $\lambda = 0.0582\ \mathrm{s}^{-1}$. How many of the Barium-144 atoms remain (as Barium-144) after 30.0 seconds?

A. 333

B. 582

C. 1744

D. 4520

$$N(t) = N(0)e^{-\lambda t}$$
$$R(t) = R(0)e^{-\lambda t}$$
$$R(t) = \lambda N(t)$$
$$N(0) = 10000$$
$$N(30.0) = 10000e^{-0.0582\times 30.0} = 1744.7$$

Quickuiz

You again begin with 10000 atoms of Barium-144, with decay constant $\lambda = 0.0582\ \mathrm{s}^{-1}$. What will be the activity of the Barium sample after 30 seconds (in units of becquerel $= \mathrm{Bq} =$ decays/second) ?

A. $1.175\ \mathrm{Bq}$

$\boxed{B.\ 102\ \mathrm{Bq}}$

C. $582\ \mathrm{Bq}$

D. $1744\ \mathrm{Bq}$

$$N(t) = N(0)e^{-\lambda t}$$
$$R(t) = R(0)e^{-\lambda t}$$
$$R(t) = \lambda N(t)$$
$$R(30.0) = \lambda N(30) = 101.542\ \mathrm{Bq}$$

Half-Life

- In exponential decay, it always takes the same amount of time for a quantity to drop to half its present value
- This amount of time is called the **half-life** of the decay process: $t_{1/2}$
- Relationship of half-life $t_{1/2}$ to decay constant λ:

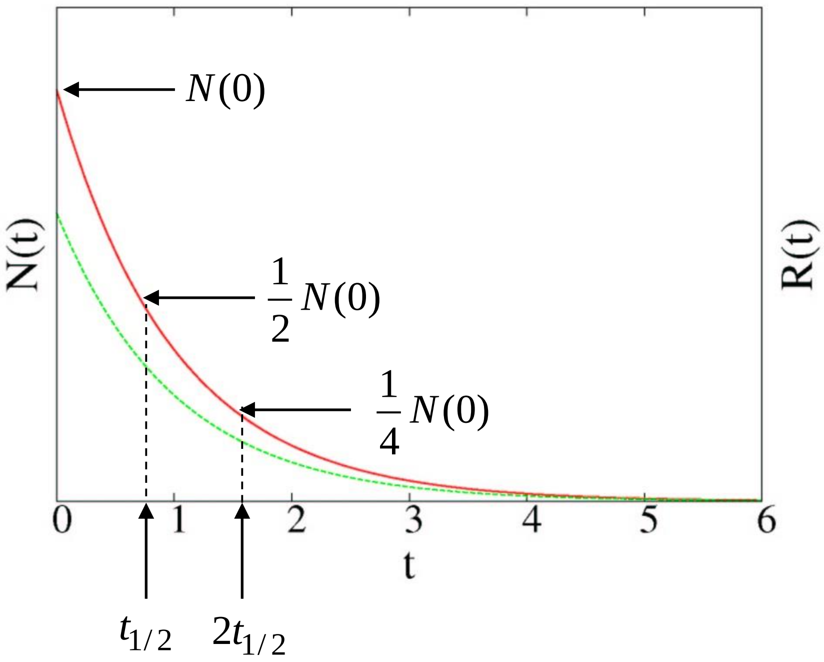
$$N(t) = N(0)e^{-\lambda t}$$

$$\frac{N\left(t_{t_{1/2}}\right)}{N(0)} = \frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$-\ln(2) = -\lambda t_{1/2}$$

$$\Rightarrow \boxed{t_{1/2} = \frac{\ln(2)}{\lambda}}$$

By contrast, $\tau \equiv 1/\lambda$ is a measure of average life time.



- Also useful:

$$N(t) = N(0)\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$R(t) = R(0)\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

Quickuiz

Suppose radioactive a nuclide has a half life of 15 years. If you have a sample of 80.0 g of that nuclide now, how much of it will be left after 30 years?

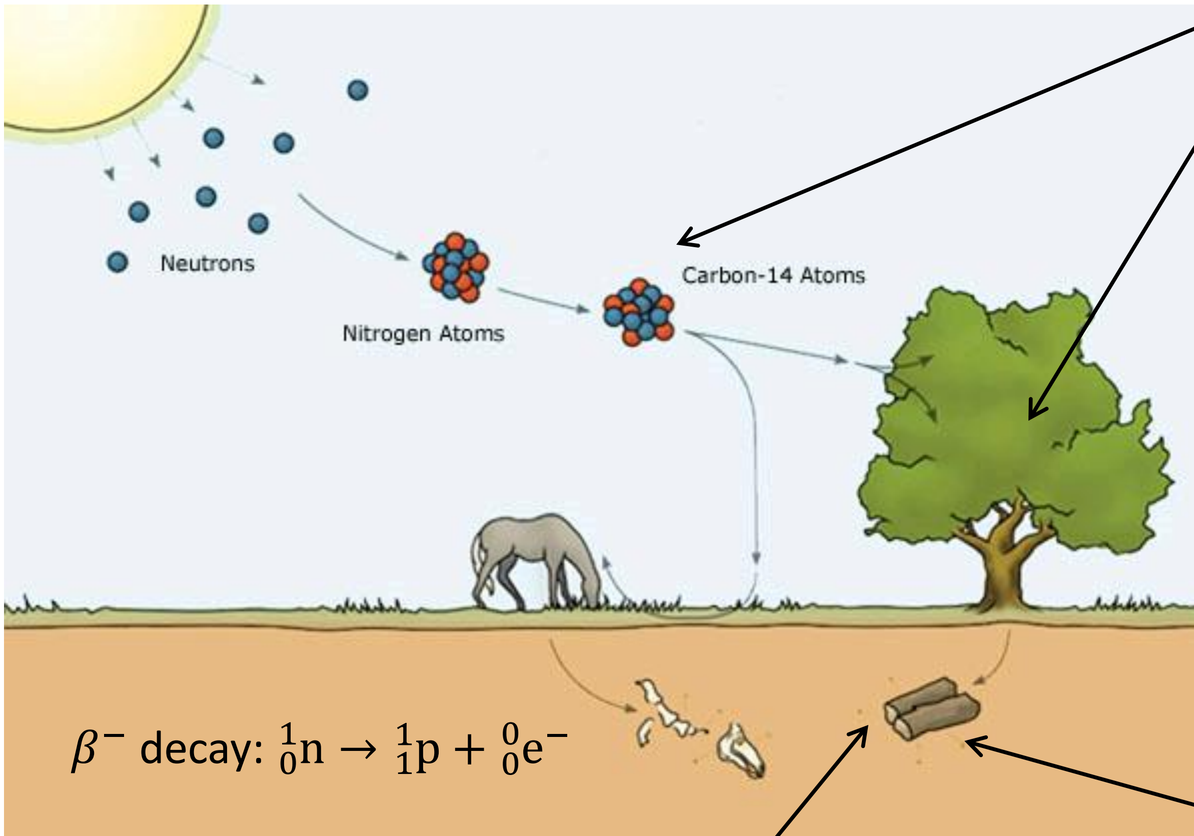
A. 50.0 g
B. 40.0 g
C. $\boxed{20.0 \text{ g}}$
D. 10.0 g

$$N(t) = N(0)\left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$N(t) = N(0)\left(\frac{1}{2}\right)^{30\text{yr}/15\text{yr}} = N(0)\left(\frac{1}{2}\right)^{2} = N(0)\left(\frac{1}{4}\right)$$

$$\Rightarrow m(30\text{yr}) = m(0)\left(\frac{1}{4}\right) = \frac{80.0\text{ g}}{4} = 20.0\text{ g}$$

Carbon-14 in the biosphere, living tissue and dead tissue



$\beta^- \text{ decay: } {}^1_0\text{n} \rightarrow {}^1_1\text{p} + {}^0_0\text{e}^-$

Activity of Carbon-14 in biosphere and living tissue:

16 decays/min per g of Carbon

Activity of Carbon-14 in dead tissue:

Less than 16 decays/min per g of Carbon

Carbon-14 is not replenished/cycled in dead tissue: half-life = 5730 years.

Simulation: http://phet.colorado.edu/en/simulation/radioactive-dating-game

Decay Constant for C-14

- Given the half-life $t_{1/2}$ of C-14, we want to find the corresponding decay constant λ:

$$t_{1/2} = \frac{\ln(2)}{\lambda}$$

$$\Rightarrow \lambda = \frac{\ln(2)}{t_{1/2}} = \frac{\ln(2)}{5730\ \mathrm{yr}} = 1.12 \times 10^{-4}\mathrm{yr}^{-1}$$

Example

- A sample of bone found in an Egyptian archaeological dig has a mass of 300.0 g. Assume that the percentage of the bone's mass contributed by Carbon atoms is approximately 9.50%. The activity of the sample is measured to be 5.10 Becquerels (Bq) above the natural background level.
- What is the activity of an equal mass of Carbon atoms in living tissue?

$$R = (28.5 \text{ g})\left(\frac{16 \text{ decays/mins}}{\text{g of Carbon}}\right) = (456 \text{ decays/min})\left(\frac{1 \text{ min}}{60\text{s}}\right) = 7.60 \text{ decays/s} = 7.60 \text{ Bq}$$

- What is the age of the Egyptian bone in years?

$$t = -\frac{t_{12}\ln\big(R(t)/R(0)\big)}{\ln 2} = -\frac{t_{12}\ln\big(R(0)/R(t)\big)}{\ln 2} = \frac{(5730 \text{ yr})\ln\left(\frac{7.60\text{Bq}}{5.10\text{Bq}}\right)}{\ln 2} = 3300 \text{ yr}$$

- What percentage of the Carbon-14 atoms originally present in the Egyptian bone have decayed?

$$\frac{N(t = 3300\ yr)}{N(0)} = \exp(-1.12 \times 10^{-4}\text{yr}^{-1} \times 3300\ yr) = 0.67 = 67\%$$

- Decayed percentage is $100\% - 67\% = 33\%$