2.2 Let E and F be two events for which one knows that the probability that at least one of them occurs is 3/4. What is the probability that neither E nor F occurs? *Hint:* use one of DeMorgan's laws: $E^c \cap F^c = (E \cup F)^c$.

2.3 Let C and D be two events for which one knows that $\mathrm{P}(C) = 0.3, \mathrm{P}(D) = 0.4$, and $\mathrm{P}(C \cap D) = 0.2$. What is $\mathrm{P}(C^c \cap D)$?

2.4 ⊡ We consider events A, B, and C, which can occur in some experiment. Is it true that the probability that *only* A occurs (and not B or C) is equal to $\mathrm{P}(A \cup B \cup C) - \mathrm{P}(B) - \mathrm{P}(C) + \mathrm{P}(B \cap C)$?

2.5 The event $A \cap B^c$ that A occurs but not B is sometimes denoted as $A \setminus B$. Here $\setminus$ is the set-theoretic minus sign. Show that $\mathrm{P}(A \setminus B) = \mathrm{P}(A) - \mathrm{P}(B)$ if B implies A, i.e., if $B \subset A$.

2.6 When $\mathrm{P}(A) = 1/3$, $\mathrm{P}(B) = 1/2$, and $\mathrm{P}(A \cup B) = 3/4$, what is

a. $\mathrm{P}(A \cap B)$?
b. $\mathrm{P}(A^c \cup B^c)$?

2.7 ⊡ Let A and B be two events. Suppose that $\mathrm{P}(A) = 0.4$, $\mathrm{P}(B) = 0.5$, and $\mathrm{P}(A \cap B) = 0.1$. Find the probability that A or B occurs, but not both.

2.8 ⊞ Suppose the events D_1 and D_2 represent disasters, which are rare: $\mathrm{P}(D_1) \leq 10^{-6}$ *and* $\mathrm{P}(D_2) \leq 10^{-6}$. What can you say about the probability that at least one of the disasters occurs? What about the probability that they *both* occur?

2.9 We toss a coin three times. For this experiment we choose the sample space

$$\Omega = \{HHH, THH, HTH, HHT, TTH, THT, HTT, TTT\}$$

where T stands for tails and H for heads.

a. Write down the set of outcomes corresponding to each of the following events:

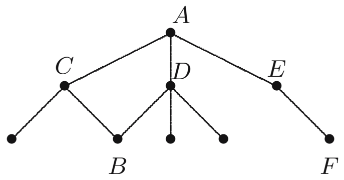
A : "we throw tails exactly two times."
B : "we throw tails at least two times."
C : "tails did not appear *before* a head appeared."
D : "the first throw results in tails."

b. Write down the set of outcomes corresponding to each of the following events: A^c, $A \cup (C \cap D)$, and $A \cap D^c$.

2.10 In some sample space we consider two events A and B. Let C be the event that A or B occurs, but not both. Express C in terms of A and B, using only the basic operations "union," "intersection," and "complement."

3.6 Exercises

3.1 ⊞ Your lecturer wants to walk from A to B (see the map). To do so, he first randomly selects one of the paths to C, D, or E. Next he selects randomly one of the possible paths at that moment (so if he first selected the path to E, he can either select the path to A or the path to F), etc. What is the probability that he will reach B after two selections?



3.2 ⊞ A fair die is thrown twice. A is the event "sum of the throws equals 4," B is "at least one of the throws is a 3."

a. Calculate $P(A \mid B)$.

b. Are A and B independent events?

3.3 ⊞ We draw two cards from a regular deck of 52. Let S_1 be the event "the first one is a spade," and S_2 "the second one is a spade."

a. Compute $P(S_1)$, $P(S_2 \mid S_1)$, and $P(S_2 \mid S_1^c)$.

b. Compute $P(S_2)$ by conditioning on whether the first card is a spade.

3.4 ⊡ A Dutch cow is tested for BSE, using Test A as described in Section 3.3, with $P(T \mid B) = 0.70$ and $P(T \mid B^c) = 0.10$. Assume that the BSE risk for the Netherlands is the same as in 2003, when it was estimated to be $P(B) = 1.3 \cdot 10^{-5}$. Compute $P(B \mid T)$ and $P(B \mid T^c)$.

3.5 A ball is drawn at random from an urn containing one red and one white ball. If the white ball is drawn, it is put back into the urn. If the red ball is drawn, it is returned to the urn together with two more red balls. Then a second draw is made. What is the probability a red ball was drawn on *both* the first and the second draws?

3.6 We choose a month of the year, in such a manner that each month has the same probability. Find out whether the following events are independent:

a. the events "outcome is an even numbered month" (i.e., February, April, June, etc.) and "outcome is in the first half of the year."

b. the events "outcome is an even numbered month" (i.e., February, April, June, etc.) and "outcome is a summer month" (i.e., June, July, August).

3.7 ⊞ Calculate

a. $P(A \cup B)$ if it is given that $P(A) = 1/3$ and $P(B \mid A^c) = 1/4$.

b. $P(B)$ if it is given that $P(A \cup B) = 2/3$ and $P(A^c \mid B^c) = 1/2$.

3.8 ⊞ Spaceman Spiff's spacecraft has a warning light that is supposed to switch on when the freem blasters are overheated. Let W be the event "the warning light is switched on" and F "the freem blasters are overheated." Suppose the probability of freem blaster overheating $P(F)$ is 0.1, that the light is switched on when they actually *are* overheated is 0.99, and that there is a 2% chance that it comes on when nothing is wrong: $P(W \mid F^c) = 0.02$.

a. Determine the probability that the warning light is switched on.

b. Determine the conditional probability that the freem blasters are overheated, given that the warning light is on.

3.9 ⊡ A certain grapefruit variety is grown in two regions in southern Spain. Both areas get infested from time to time with parasites that damage the crop. Let A be the event that region R_1 is infested with parasites and B that region R_2 is infested. Suppose $P(A) = 3/4$, $P(B) = 2/5$ and $P(A \cup B) = 4/5$. If the food inspection detects the parasite in a ship carrying grapefruits from R_1, what is the probability region R_2 is infested as well?

3.10 A student takes a multiple-choice exam. Suppose for each question he either knows the answer or gambles and chooses an option at random. Further suppose that if he knows the answer, the probability of a correct answer is 1, and if he gambles this probability is 1/4. To pass, students need to answer at least 60% of the questions correctly. The student has "studied for a minimal pass," i.e., with probability 0.6 he knows the answer to a question. Given that he answers a question correctly, what is the probability that he actually *knows* the answer?

3.11 A breath analyzer, used by the police to test whether drivers exceed the legal limit set for the blood alcohol percentage while driving, is known to satisfy

$$P(A \mid B) = P(A^c \mid B^c) = p,$$

where A is the event "breath analyzer indicates that legal limit is exceeded" and B "driver's blood alcohol percentage exceeds legal limit." On Saturday night about 5% of the drivers are known to exceed the limit.

a. Describe in words the meaning of $P(B^c \mid A)$.

b. Determine $P(B^c \mid A)$ if $p = 0.95$.

c. How big should p be so that $P(B \mid A) = 0.9$?

3.12 The events A, B, and C satisfy: $P(A \mid B \cap C) = 1/4$, $P(B \mid C) = 1/3$, and $P(C) = 1/2$. Calculate $P(A^c \cap B \cap C)$.

3.18 Suppose A and B are events with $0 < \mathrm{P}(A) < 1$ and $0 < \mathrm{P}(B) < 1$.

a. If A and B are disjoint, can they be independent?
b. If A and B are independent, can they be disjoint?
c. If $A \subset B$, can A and B be independent?
d. If A and B are independent, can A and $A \cup B$ be independent?

R Excercise:

Simulate 6000 dice rolls using R.
Count the number of 1's, 2's, . . . , 6's.
Display your results in form of a histogram.
How many realizations of each number have you expected?
How close was your result to what you expected?
About how often would you expect to get more than 1030 1's?
Run an R simulation to estimate the answer.