ECON 628 FINAL EXAM

SPRING 2014

(1) Assume $\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{e}$, $\mathrm{E}[\boldsymbol{e}|\boldsymbol{X}] = \boldsymbol{0}$, the first and second moments of the variables are finite, and the second moment of the regressors is invertible.
  (a) Derive the OLS estimator of $\boldsymbol{\beta}$. (p. 102-107)
  (b) Show that the OLS estimator of $\boldsymbol{\beta}$ is unbiased. (p.151-155)
  (c) Derive the variance of the OLS estimator of $\boldsymbol{\beta}$. Do not yet assume homoskedasticity! (p.155-157)
  (d) From now on, you can assume that the errors are homoskedastic. How does the variance of the OLS estimator of $\boldsymbol{\beta}$ change when you assume homoskedasticity? (p. 157)
  (e) The variance of the OLS estimator of $\boldsymbol{\beta}$ depends on the variance of unobserved errors, and therefore cannot be calculated directly. How would you *estimate* the variance of the OLS estimator of $\boldsymbol{\beta}$? (p. 162-163, 166-167)
  (f) How would you obtain the standard error of the OLS estimator of $\boldsymbol{\beta}$? (p.172-173)

(2) Assume $\boldsymbol{y} = \boldsymbol{x}\beta + \boldsymbol{e}$, the first and second moments of the variables are finite, and the second moment of the regressors is invertible.
  (a) Show that the OLS estimator of β is consistent. (p. 235-238)
  (b) Find the asymptotic distribution of the OLS estimator of β. Show how you obtain its asymptotic variance. Is it the same as the finite sample variance? Explain. (p. 238-243, 251-253, 256-257)
  (c) How would you test the hypothesis $\mathrm{H}_0 : \beta = 0$? (p. 263, 266-267, 320-326)
  (d) Describe what we mean by type I error, type II error, and the power of a test. You can support your discussion with figures. (p. 323-327, 357, Dougherty's slides)
  (e) What is a p-value? Would you use a p-value in reporting your results? Why? (p. 329-334, Dougherty's slides)
  (f) What is a confidence interval used for? How would you obtain one, given your result in (2b)? (p. 263, 268-270, 355)

(3) Consider the OLS regression of the $n \times 1$ vector $\boldsymbol{y}$ on the $n \times k$ matrix $\boldsymbol{X}$. Consider an alternative set of regressors $\boldsymbol{Z} = \boldsymbol{X}\boldsymbol{C}$, where $\boldsymbol{C}$ is a $k \times k$ non-singular matrix. Thus, each column of $\boldsymbol{Z}$ is a mixture of some of the columns of $\boldsymbol{X}$. Compare the OLS estimates and residuals from the regression of $\boldsymbol{y}$ on $\boldsymbol{X}$ to the OLS estimates from the regression of $\boldsymbol{y}$ on $\boldsymbol{Z}$.

(4) Using matrix algebra, show $\boldsymbol{X}'\hat{\boldsymbol{e}} = \boldsymbol{0}$. Show that $\frac{1}{n}\sum_{i=1}^{n} \hat{y}_i = \bar{y} \equiv \frac{1}{n}\sum_{i=1}^{n} y_i$ when $\boldsymbol{X}$ contains a constant. (p. 110-113)

(5) Let $\hat{\boldsymbol{e}}$ be the OLS residual from a regression of $\boldsymbol{y}$ on $\boldsymbol{X}$. Find the OLS coefficient from a regression of $\hat{\boldsymbol{e}}$ on $\boldsymbol{X}$. Let $\hat{\boldsymbol{y}} = \boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$. Find the OLS coefficient from a regression of $\hat{\boldsymbol{y}}$ on $\boldsymbol{X}$. Explain your results. Show how you can decompose $\boldsymbol{y}$ into two orthogonal components, $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{e}}$ using the projection matrix, $\boldsymbol{P}$, and orthogonal projection matrix, $\boldsymbol{M}$. Show that $\hat{\boldsymbol{y}}$ and $\hat{\boldsymbol{e}}$ are orthogonal to each other. (p.114-117, 119)

(6) Explain the insight behind the Frisch-Waugh-Lowell theorem. Why is it useful? (p. 123-125)

(7) A weighted sample mean takes the form $\bar{y}^* = \frac{1}{n}\sum_{i=1}^{n} w_i y_i$ for some non-negative constants satisfying $\frac{1}{n}\sum_{i=1}^{n} w_i = 1$. Show that $\bar{y}^*$ is unbiased for $\mu = \mathrm{E}[y_i]$. Calculate $\mathrm{Var}[\bar{y}^*]$.

(8) What is the rationale behind the Delta Method? Suppose $\sqrt{n}(\hat{\mu} - \mu) \xrightarrow{d} N(0, v^2)$ and set $\beta = \mu^2$ and $\hat{\beta} = \hat{\mu}^2$. Use the Delta Method to obtain an asymptotic distribution for $\sqrt{n}(\hat{\beta} - \beta)$. (p. 211-214, 259-262)

(9) Take the model $y_i = \boldsymbol{x}_{1i}'\boldsymbol{\beta}_1 + \boldsymbol{x}_{2i}'\boldsymbol{\beta}_2 + e_i$ with $\mathrm{E}[\boldsymbol{x}_i e_i] = \mathbf{0}$. Suppose that $\boldsymbol{\beta}_1$ is estimated by regressing y_i on $\boldsymbol{x}_{1i}$ only. Find the probability limit of this estimator. In general, is it consistent for $\boldsymbol{\beta}_1$? If not, under what conditions is this estimator consistent for $\boldsymbol{\beta}_1$? (see also p. 64-65)

(10) Of the variables $(y_i^*, y_i, \boldsymbol{x}_i)$ only the pair $(y_i, \boldsymbol{x}_i)$ are observed. In this case, we say that y_i^* is a latent variable. Suppose

$$y_i^* = \boldsymbol{x}_i'\boldsymbol{\beta} + e_i, \quad \mathrm{E}[\boldsymbol{x}_i e_i] = 0, \quad y_i = y_i^* + u_i,$$

where u_i is a measurement error satisfying

$$\mathrm{E}[\boldsymbol{x}_i u_i] = 0, \quad \mathrm{E}[y_i^* e_i] = 0.$$

Let $\hat{\boldsymbol{\beta}}$ denote the OLS coefficient from the regression of y_i on $\boldsymbol{x}_i$.

(a) Is $\boldsymbol{\beta}$ the linear projection of y_i on $\boldsymbol{x}_i$ (i.e. is $\hat{\boldsymbol{\beta}}$ unbiased)? Note the homework solution of this question is not exactly correct: you need to check if $\mathrm{E}[\hat{\boldsymbol{\beta}}]$=$\boldsymbol{\beta}$! (see also p. 54-55)

(b) Is $\hat{\boldsymbol{\beta}}$ consistent for $\boldsymbol{\beta}$ as $n \to \infty$?

(c) Find the asymptotic distribution of $\sqrt{n}(\hat{\boldsymbol{\beta}} - \boldsymbol{\beta})$ as $n \to \infty$.