Assignment 2: Computational Complexity

Due date: Thursday 20 Feb in Class.
You can return it in class or during office hours.

1. (5) Read Theorem 2.2, understand the proof, and write the (full) proof in your own words. The technique "padding" used here is a very useful one and we might use it later.

2. Read assignment 2.6 for the definition of *Nondeterministic* Universal Turing Machine N-UTM (you don't have to solve assignment 2.6, but I assume you can at least sketch the proof of part (a) assuming UTM for deterministic case). Now suppose we try to adapt the proof of Theorem 3.1 to prove a time hierarchy theorem for non-deterministic computation as follows: we try to use N-UTM instead of UTM. Namely, we define language L as follows. Given input x, let M_x be the Nondeterministic TM encoded by x and let $\overline{M}_x$ be another non-deterministic Turing machine that in *every* possible branch of computation (out of all exponentially many executions) does the same as M_x till the very last step but then suddenly "flips" its final answer (changes yes to no and vice versa). We define $x \in L$ if and only if $\overline{M}_x$ "accepts" x (for this you need to recall when we say that a nondeterministic machine accepts an input). If we define L this way, then:

   - (5) Is it that $L \in Ntime(n^2)$?
   - (10) How about $L \notin Ntime(n)$? Why?

   The above shows why the straightforward adaptation of the proof of Theorem 3.1 to non-deterministic case (to prove Theorem 3.2) does not work. So now you might have a good motivation to read the beautiful proof of Theorem 3.2.

3. (10) We already know that if $L \leq_p L'$ (i.e. there is a Karp reduction from L to L'), then $L' \in P$ implies $L \in P$. Prove that $L \leq_p L'$ also shows that: $L' \in NP \Rightarrow L \in NP$.

4. Show that:

   - (5) $TAUTOLOGY \leq_T SAT$ and $TAUTOLOGY \geq_T SAT$ where $\leq_T$ denotes Turing reductiosn[1].
   - (10) If either of the following holds: $SAT \leq_P TAUTOLOGY$ or $SAT \geq_P TAUTOLOGY$, then $NP = coNP$.

5. (5) Prove that if $P = NP$ then $NP = coNP$. (Hint: use first part of previous question).

6. Suppose $X \leq_T Y$ where X and Y are two functions[2]. Then show that:

   - (10) $P^X \subseteq P^Y$ and $NP^X \subseteq NP^Y$.
   - (10) Can you also show that $(Dtime(n^2))^X \subseteq (Dtime(n^2))^Y$? Why?

[1] The book calls Turing reductions: Cook reductions.

[2] Recall that the Turing reduction can be defined between any two functions or even search problems and not only the decision problems

7. (10) Assignment 2.27.
8. (10) Extra Credit: Assignment 2.11