Computational Complexity

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Session 10

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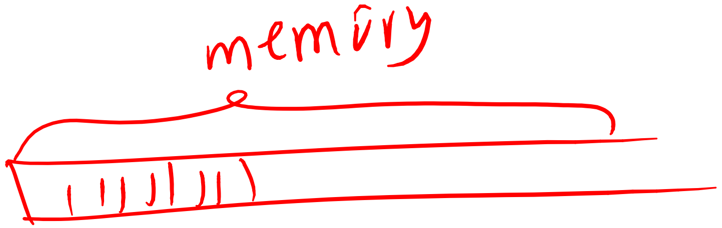
Today

- Space Complexity:

How much **memory** do we need to solve a problem?

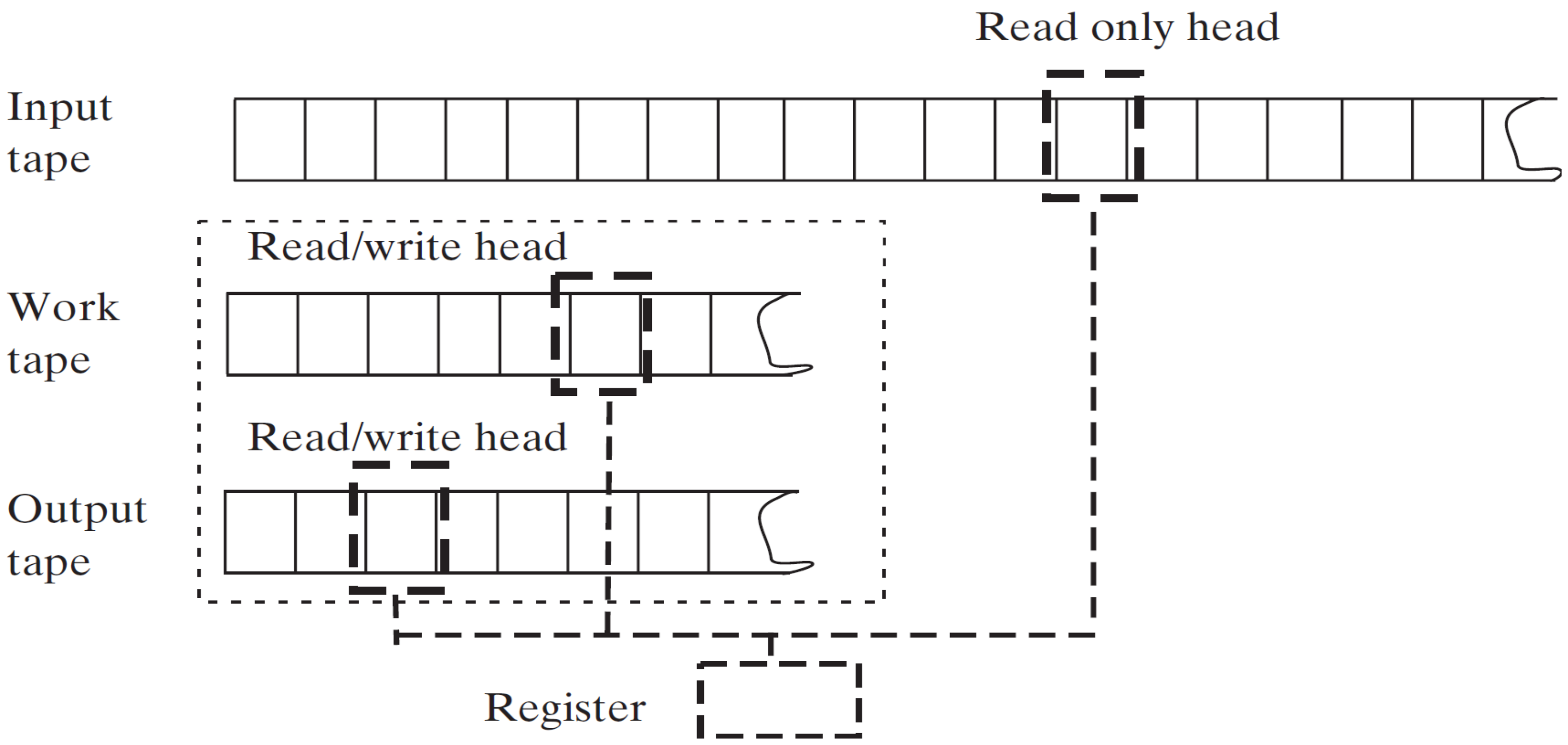
Space bounded by Time ?



- If an algorithm runs in time $T(n)$ how much memory $m(n)$ can it use?
- If $T(n)$ counts "steps" of Turing machine $\Rightarrow$ $m(n) \leq T(n)$
- How about $T(n)$ be number of actual seconds?
- A Terabyte of data can be written in 1000 seconds.
- It is OK to run an algorithm for a week, but not to use 500 Terabyte …

We need a theory of Memory Complexity

- Which memory cells should we count?

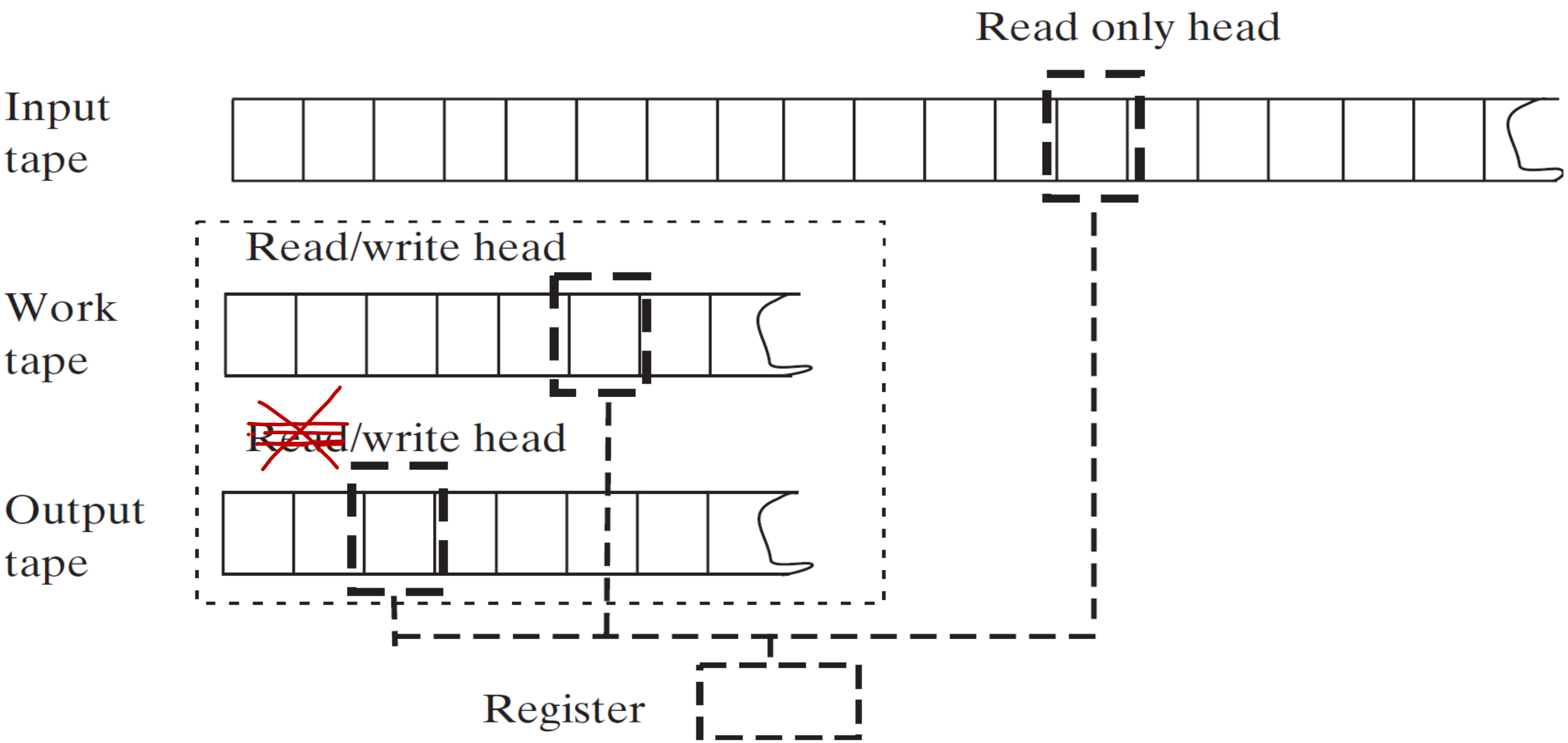


- Which memory cells should we count?

- Suppose data D is streaming from the Internet. We compute $f(D)$.
  Shall we count $|D|$ as memory used?

- Suppose a node on the Internet forwards stream data D.
  How much memory the node needs?

transition function $\delta : Q \times \Gamma \times \Gamma \to Q \times \Gamma \times \Gamma \times \{L,R\}^3$



Definition 4.1 *(Space-bounded computation)*
Let $S : \mathbb{N} \to \mathbb{N}$ and $L \subseteq \{0, 1\}^*$. We say that $L \in \mathbf{SPACE}(s(n))$ if there is a constant c and a TM M deciding L such at most $c \cdot s(n)$ locations on M's work tapes (excluding the input tape) are ever visited by M's head during its computation on every input of length n.

Similarly, we say that $L \in \mathbf{NSPACE}(s(n))$ if there is an NDTM M deciding L that never uses more than $c \cdot s(n)$ nonblank tape locations on length n inputs, regardless of its nondeterministic choices.

Time vs Space

memory complexity
time complexity

- Bounding Time by Space

$$\mathbf{DTIME}(\mathrm{S(n)}) \subseteq \mathbf{SPACE}(\mathrm{S(n)})$$

$$\mathbf{NTIME}(\mathrm{S(n)}) \subseteq \mathbf{NSPACE}(\mathrm{S(n)})$$

- Bounding Space by Time

$$\mathbf{SPACE}(S(n)) \subseteq \mathbf{NSPACE}(S(n)) \subseteq \mathbf{DTIME}(2^{O(S(n))})$$

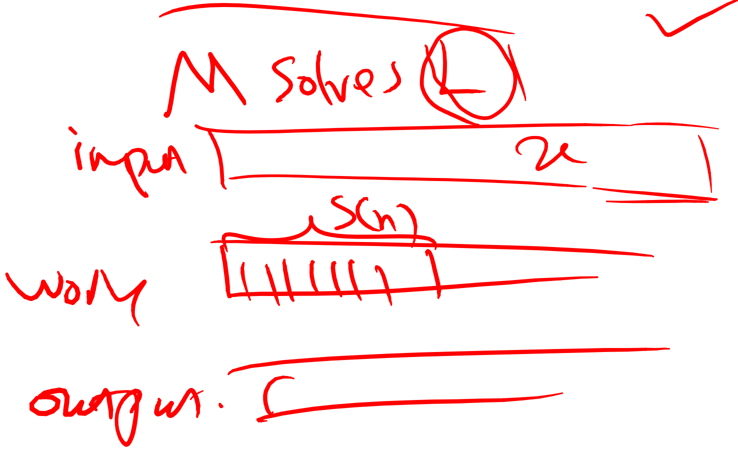
Bounding Space by Time

$S(n) \geq \log(n)$:

if $S(n) \leq \log(n)$? P

$$\mathbf{DTIME}(S(n)) \subseteq \mathbf{SPACE}(S(n)) \subseteq \mathbf{NSPACE}(S(n)) \subseteq \mathbf{DTIME}(2^{O(S(n))})$$

$2^{c \cdot \log(n)} = (2^{\log n})^c = n^c$



possibility $\leq 2^{S(n)}$

Total State: | q, work tape, header's location on input tape, work tape | $\leq 2^{S(n)} \times |Q| \times n \times S(n) \leq 2^{4 \cdot S(n)}$

Claim: if Total state repeats itself ⇒ in loop: Total (i) = Total (j) ⇒ Total (i+1) = Total (j+1)