Computational Complexity

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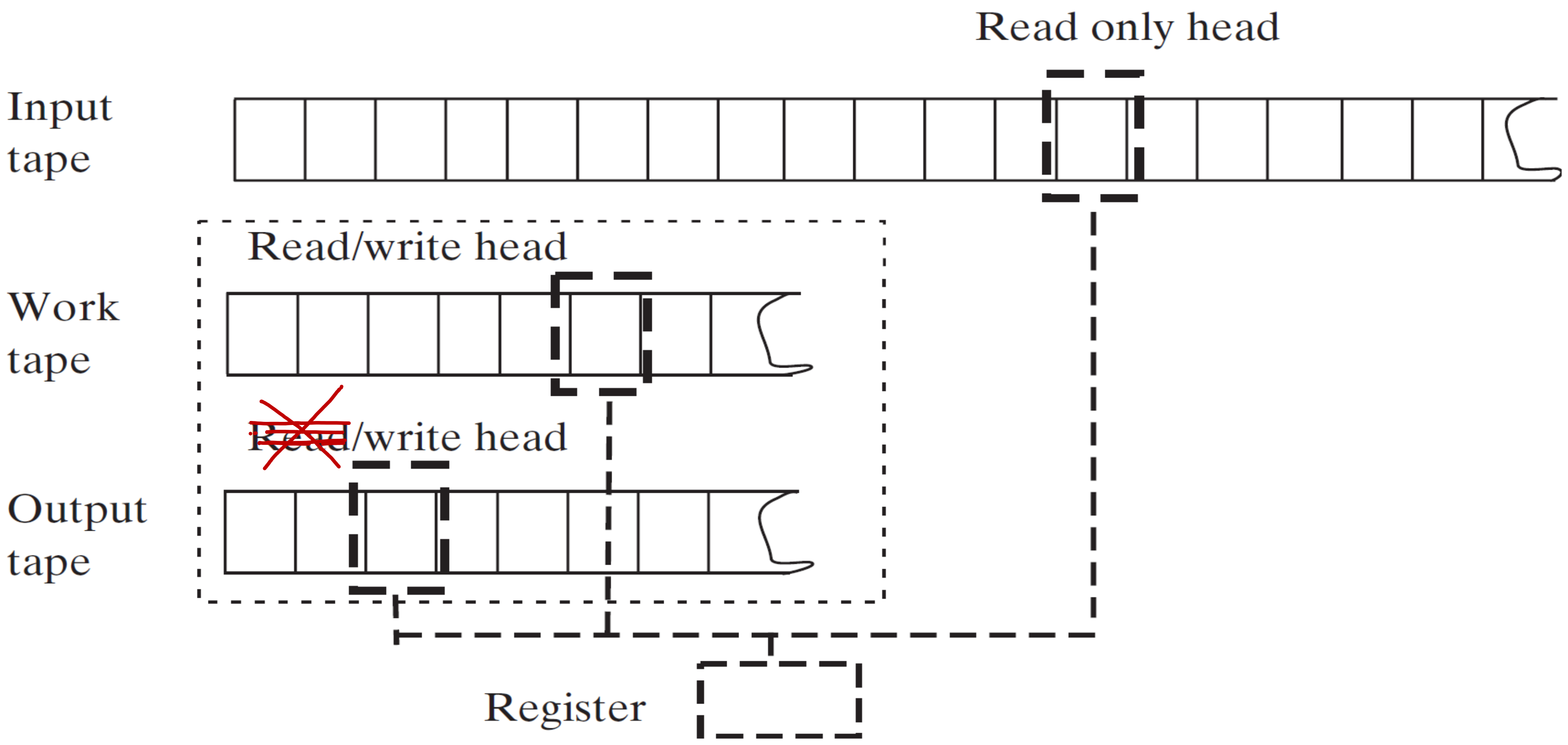
Today

- Will continue Space Complexity:

How much **memory** do we need to solve a problem?

transition function $\delta : Q \times \underbrace{\Gamma \times \Gamma} \to Q \times \underbrace{\Gamma \times \Gamma} \times \{L, R\}^3$



Time vs Space

- Time-based classes are inside space-based classes:

$$\mathbf{DTIME}\big(\mathrm{S(n)}\big) \subseteq \mathbf{SPACE}\big(\mathrm{S(n)}\big)$$
$$\mathbf{NTIME}\big(\mathrm{S(n)}\big) \subseteq \mathbf{NSPACE}\big(\mathrm{S(n)}\big)$$

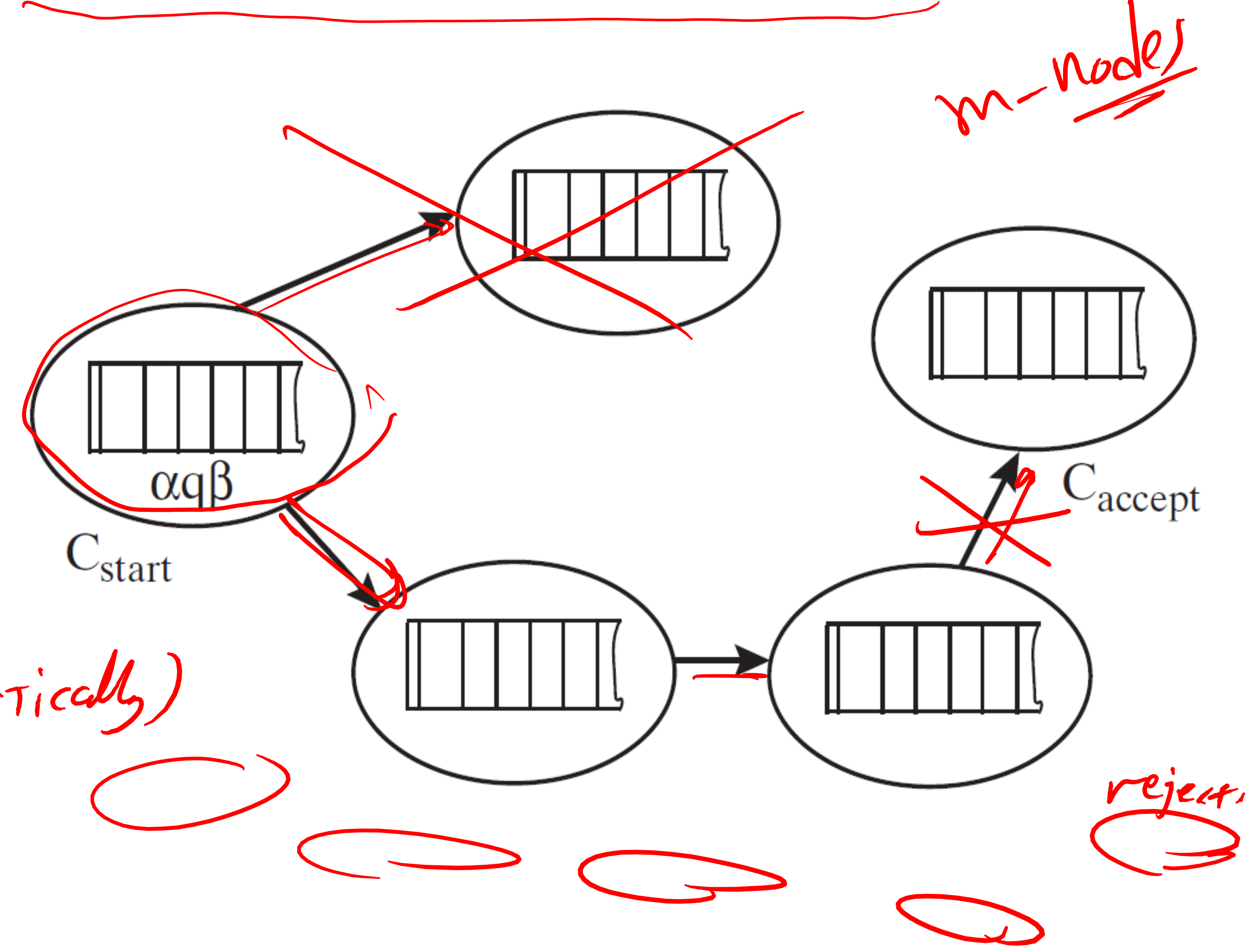
- Space-based classes are inside time-based classes

$$\mathbf{SPACE}(S(n)) \subseteq \mathbf{NSPACE}(S(n)) \subseteq \mathbf{DTIME}(2^{O(S(n))})$$

Bounding Space-Comp. by Time-Comp.

$$\mathbf{DTIME}(S(n)) \subseteq \mathbf{SPACE}(S(n)) \subseteq \mathbf{NSPACE}(S(n)) \subseteq \mathbf{DTIME}(2^{O(S(n))})$$

Let fix $s(n)$-space machine M.
for every input of length n
$\exists$ directed Graph G of Size $2^{O(S(n))}$
Such that G has a special "Start" node.
and some accept or reject node.
and. $M(x)$ "accept" (non-deterministically)
iff $\exists$ path in G from "start" to one of accepts.



Interesting Space Complexity Classes

$$\mathbf{PSPACE} = \cup_{c>0}\mathbf{SPACE}(n^c) \subseteq \mathrm{EXP} = \cup \mathrm{Time}(2^{n^c})$$

$$\mathbf{NPSPACE} = \cup_{c>0}\mathbf{NSPACE}(n^c) \subseteq \mathrm{EXP}.$$

$$\mathbf{L} = \mathbf{SPACE}(\log n) \subseteq \mathrm{P}$$

$$\mathbf{NL} = \mathbf{NSPACE}(\log n) \subseteq \mathrm{P}$$

Examples of "Low Space Complexity" (L)

$$\mathsf{EVEN} = \{x : x \text{ has an even number of 1s}\} \in L \subseteq P$$

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$\downarrow$

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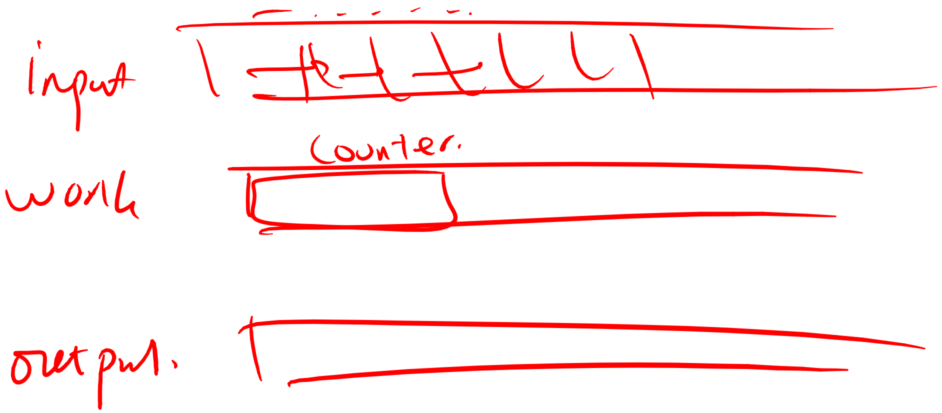
$NP \subseteq P$



$L \overset{?}{\neq} NP$

Path is NL-hard ??
PATH is NL-Complete ??

every moment we are in a node (u)

Interesting Problems in **NL**

$L \in NTime(g(n)) \to \exists$ graph G_x s.t. $|G_x| = poly(n)$ and to know $x \in L \Leftrightarrow$ G_x has path from start $\to$ accept

PATH = $\{\langle G, s, t\rangle :$ G is a directed graph in which there is a path from s to $t\}$

- Claim: PATH $\in$ **NL**

M
input tape (G, s, t)
work tape $p(g_n)$
output yes/No.

Psudo-Code of $M \in NL$:

at any time keep index of a node (u)
transition: choose one of v_1, v_2 non-deterministicly: $u \to v_1$, $u \to v_2$
if at any time $u = t \Rightarrow$ accept.

Is PATH in **L** as well? important