
# Computational Complexity

Mohammad Mahmoody

Session 14
4 March 2014

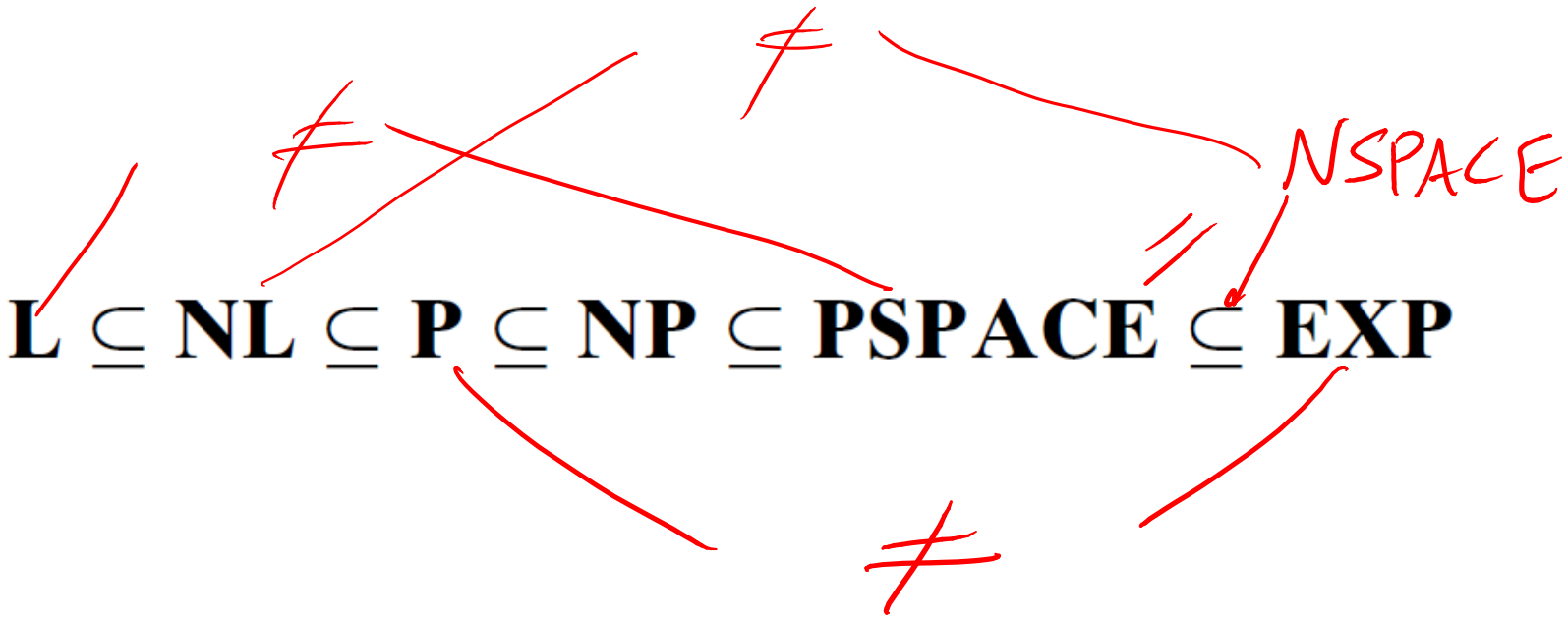
# Today

- Savitch’s theorem

# Interesting problems in $\mathbf{PSPACE}$

- Trivial: $\mathbf{NP} \subseteq \mathbf{NSPACE}$
- We also saw: $\mathbf{NP} \subseteq \mathbf{EXP}$
- Can improve both by showing $\mathbf{NP} \subseteq \mathbf{PSPACE}$

# Big Picture



# Non-deterministic vs Deterministic Space

**Theorem 4.14** *(Savitch's Theorem [Sav70])*
*For any space-constructible* $S : \mathbb{N} \to \mathbb{N}$ *with* $S(n) \geq \log n$, $\mathbf{NSPACE}(S(n)) \subseteq \mathbf{SPACE}(S(n)^2)$.

given input $x$. have $NTM^A$ of space complexity $S(n)$.

if $x \in L \Rightarrow A$ "accepts" $x$ is "some" branch.

$x \notin L \Rightarrow A$ will Not "accept" $x$ in any branch.

Recall Conf. graph $G_{(x,A)}$ $\quad |G| \leq 2^{O(S(n))}$

if recursively go for path of length $k$ $\Rightarrow$ time $\leq 2^k$

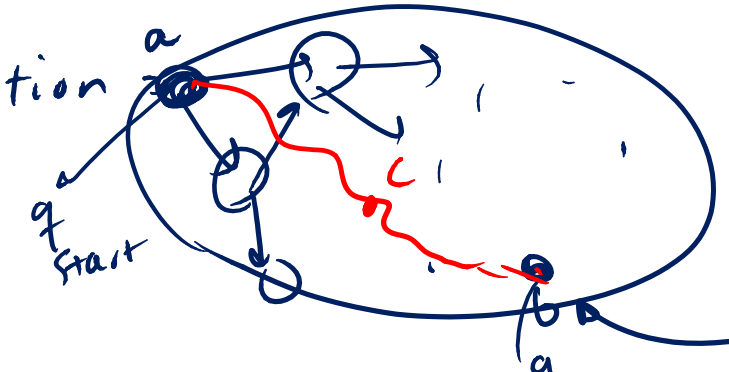

$\leq 2^{2^{S(n)}}$

Recursive algorithm. { Starts by knowing NDM $\underline{A}$ of space $S(n)$
{ given input $|\underline{x}| = n$

Know: if a path of acceprace exists $\Rightarrow$ $|P| \leq 2^{\frac{t}{O(S(n))}}$

$\leq 2^t$

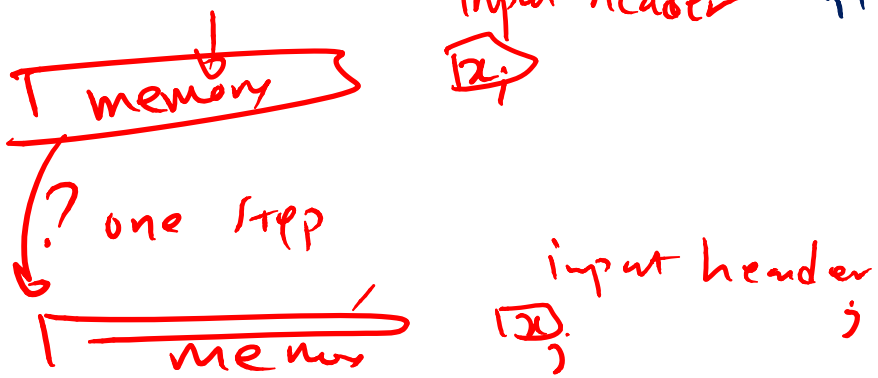
Start Configration




Claim: $\exists$ path $|P| \leq 2^t$ from a to b iff

$\exists$ node c: $a \xrightarrow{|P_1| \leq 2^{t-1}} c \xrightarrow{|P_2| \leq 2^{t-1}} b$

input header




Code

PathFind (t, a, b):

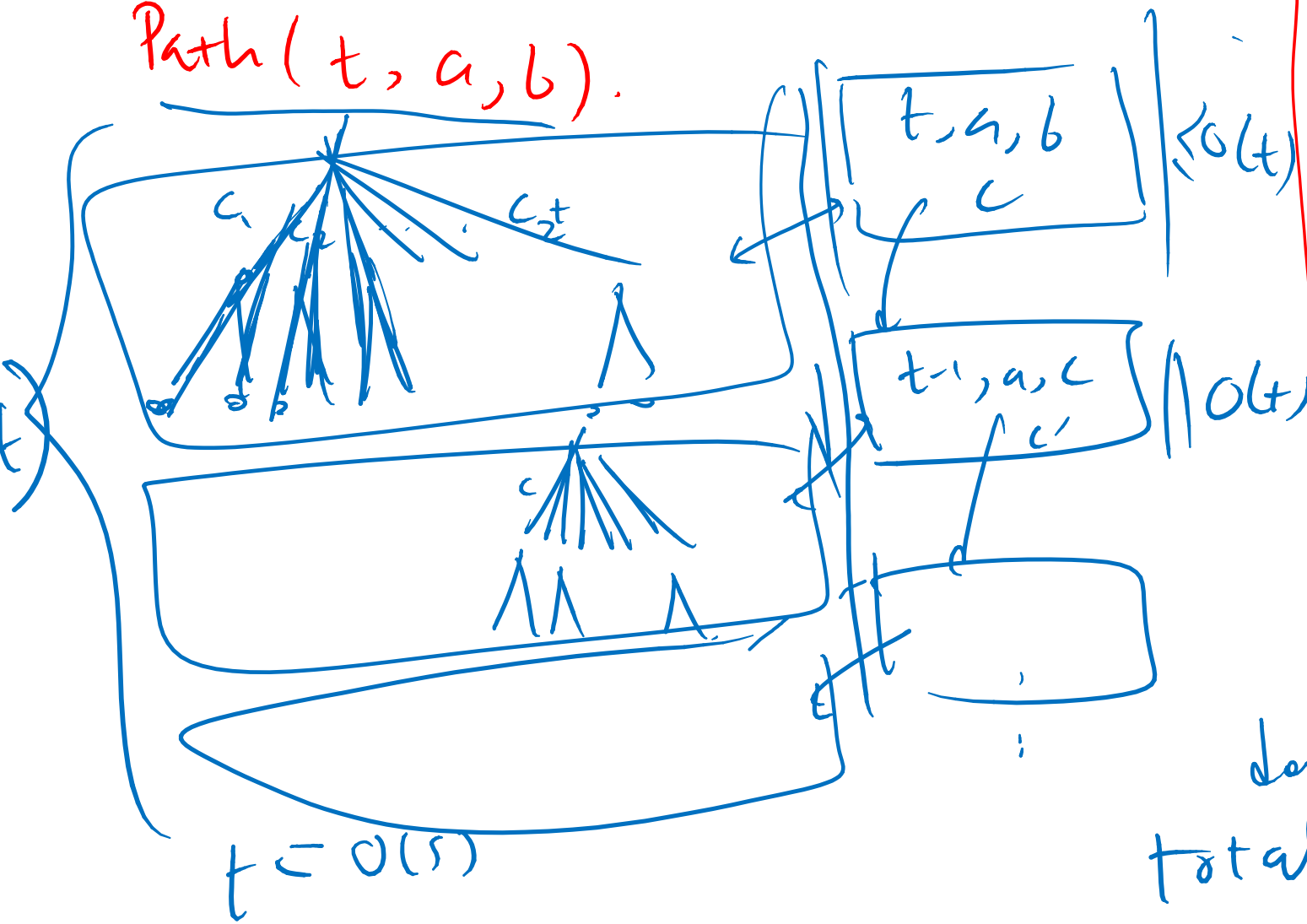
is there a path of length $2^t$ from a $\rightarrow$ b

if t=0:
simple check using A's code.

if t $\geq$ 1 then:
for all c if
Path($t-1$, a, c) $\wedge$ Path($t-1$, c, b)
return True.
return False.

# Space Complexity of our Alg :

What is Space used by Path(t, a, b).




Path (i, a, b)

if i=0 — solve easily

if i≥1 ⇒

for all node c

if Path(i-1, a, c) ∧ Path(i-1, c, b)

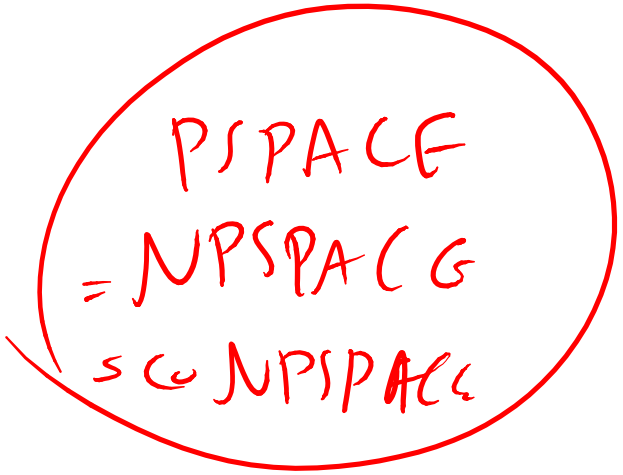
return True.

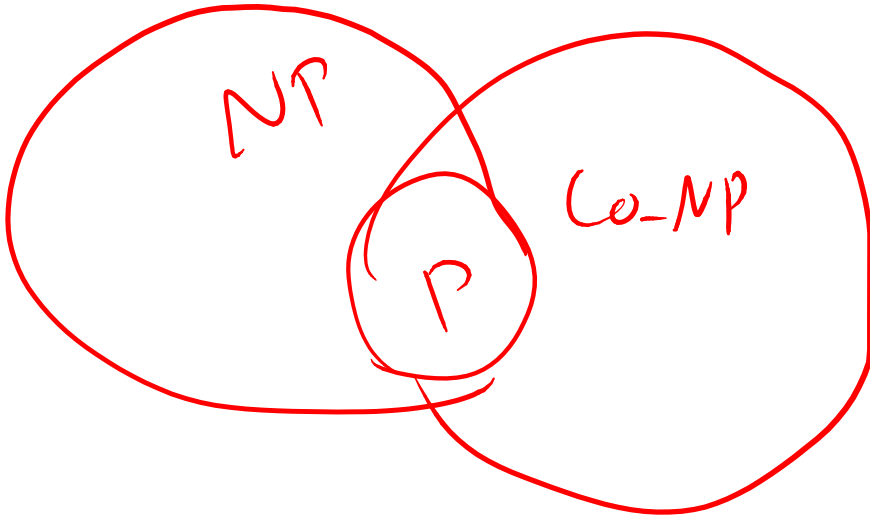
return False.

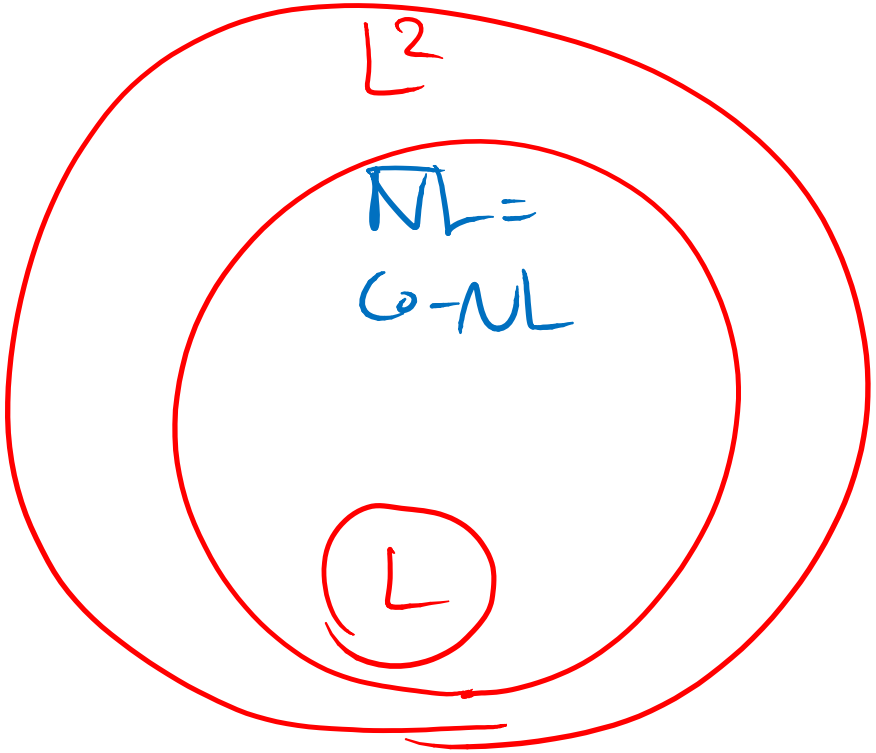
Graph $|G| \leq 2^t$

depth of execution is (t)

total space $\leq O(t) \cdot t \leq O(t^2)$




$NL = co\text{-}NL$

Immerman Theorem.