Computational Complexity

Mohammad Mahmoody

Session 13
27 Feb 2014

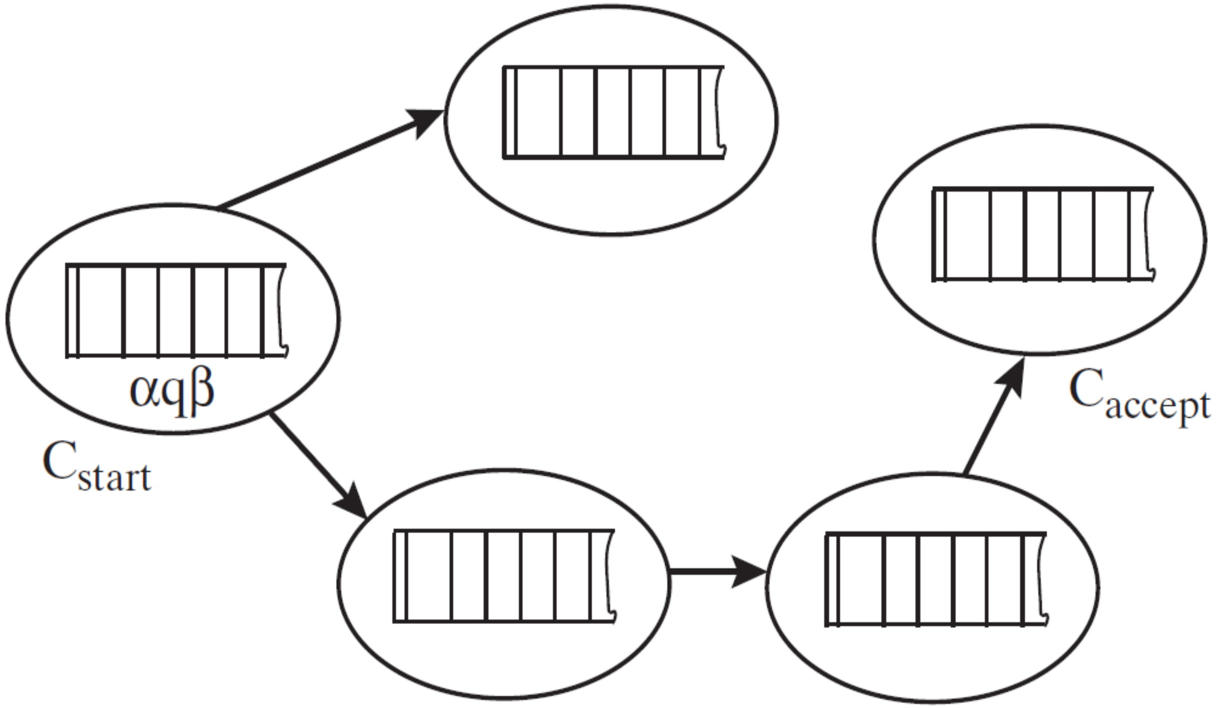
Today

- Will continue Space Complexity:

Recursive Algorithms in PSPACE and Savitch’s theorem

Bounding Space-Comp. by Time-Comp.

$$\mathbf{DTIME}(S(n)) \subseteq \mathbf{SPACE}(S(n)) \subseteq \mathbf{NSPACE}(S(n)) \subseteq \mathbf{DTIME}(2^{O(S(n))})$$



Interesting Space Complexity Classes

$$\mathbf{PSPACE} = \cup_{c>0}\mathbf{SPACE}(n^c)$$

$$\mathbf{NPSPACE} = \cup_{c>0}\mathbf{NSPACE}(n^c)$$

$$\mathbf{L} = \mathbf{SPACE}(\log n)$$

$$\mathbf{NL} = \mathbf{NSPACE}(\log n)$$

Interesting Problems in **NL**

$$\text{PATH} = \{\langle G, s, t\rangle : G \text{ is a directed graph in which there is a path from } s \text{ to } t\}$$

- Claim: $\text{PATH} \in \mathbf{NL}$.
- Proof: Do a non-deterministic walk of length n in starting from s.
  Accept iff reaching t in this time.

- Interesting:
  If one solves PATH in log space, then all **NL** is solvable in log space !

- Important: Is PATH in **L** as well?

Interesting problems in $\mathbf{PSPACE}$

- Trivial: $\mathbf{NP} \subseteq \mathbf{NSPACE}$
- We also saw: $\mathbf{NP} \subseteq \mathbf{EXP}$
- Can improve both by showing $\mathbf{NP} \subseteq \mathbf{PSPACE}$

Big Picture

$$\mathbf{L} \subseteq \mathbf{NL} \subseteq \mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} \subseteq \mathbf{EXP}$$

Non-deterministic vs Deterministic Space

Theorem 4.14 *(Savitch's Theorem* [Sav70]*)*
For any space-constructible $S : \mathbb{N} \to \mathbb{N}$ *with* $S(n) \geq \log n$, $\mathbf{NSPACE}(S(n)) \subseteq \mathbf{SPACE}(S(n)^2)$.