Computational Complexity

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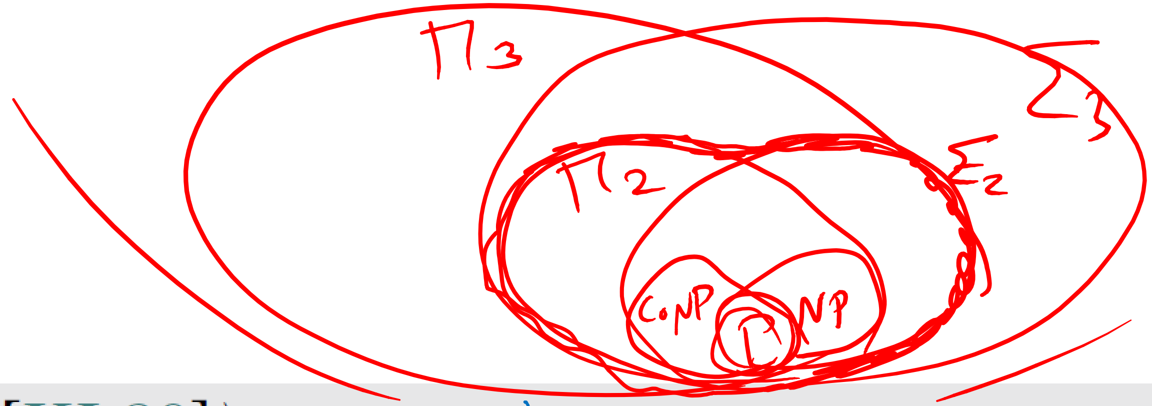
$X = \text{co-}Y \qquad X \le Y \Longrightarrow Y \le X$



Is **NP** $\subseteq$ **P**/*poly* ?

Theorem 6.19 *(Karp-Lipton Theorem* [KL80]*)*
If $\mathbf{NP} \subseteq \mathbf{P}_{/\mathrm{poly}}$, *then* $\mathbf{PH} = \Sigma_2^p$. $\Longleftrightarrow$ $\Pi_2 \subseteq \Sigma_2$

for every n $\exists$ circuit C_n $|C_n| \le poly(n)$
if you give formula F
of m variables and size n
to $C_n \Longrightarrow C_n(f)=1$ iff
f is satisfiable.

$L \in \Sigma_2$ $\quad \exists M : x \in L$ iff $\exists w_1 \forall w_2 \; M(w_1, w_2, x) = 1$

$L \in \Pi_2$ $\quad \exists T \; x \in L$ iff $\forall w_1 \exists w_2 \; T(w_1, w_2, x) = 1$

$L \in PH$ $\quad \exists M$

$x \in L$ iff $\forall w_1 \exists w_2 \forall_3 \; \exists \; Q \; Q' \; M(x, w_1, \ldots) = 1$

merge $\forall \; \exists \quad \exists \; \forall \; M'$

$L \in \Pi_2$: $\exists_M$: $x \in L \Longleftrightarrow \forall w_1 \underbrace{\exists w_2 \quad M(x, w_1, w_2) = 1}$

goal: $\exists_{T}$: $x \in L \Longleftrightarrow \exists_{u_1} \forall_{u_2} \; T(u_1, u_2, x) = 1$

$L \in \Sigma_2$ because:

$x \in L \Longleftrightarrow \exists \underbrace{\text{circuit } C_{\text{poly}(n)}}_{u_1} \; \forall \underbrace{w_1}_{u_2} \; T(C_{\text{poly}(n)}, w_1, x) = 1$

?

$\underbrace{\exists w_2 \; M(w_1, w_2, x) = 1}$

Cook-Levin.

T: is $\varphi^{x}_{M_{w_1}}$ satisfiable?

find answer by $C_{\text{poly}(n)}(\varphi^{x}_{w}) = ?$

if $C(\cdot) =$ yes ask for z verify it.

Correction: Use $\underline{\text{SAT solver } C_n}$

$C(\varphi)$ either gives z : $\varphi(z) = 1$
or says φ is NOT satisfiable.

if $NP \in P/poly \Longrightarrow \exists\ C_1 C_2 \cdots C_n \cdots$

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if you give $|\varphi| = n$ to C_n : $C_n(\varphi)$

$\Rightarrow$ if φ is Not satisfiable $C_n(\varphi) = NO$

if φ is satisfiable $C_n(\varphi) = (yes, z)$ s.t $\varphi(z) = 1$

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proof: use the search to decision reduction for SAT