Computational Complexity

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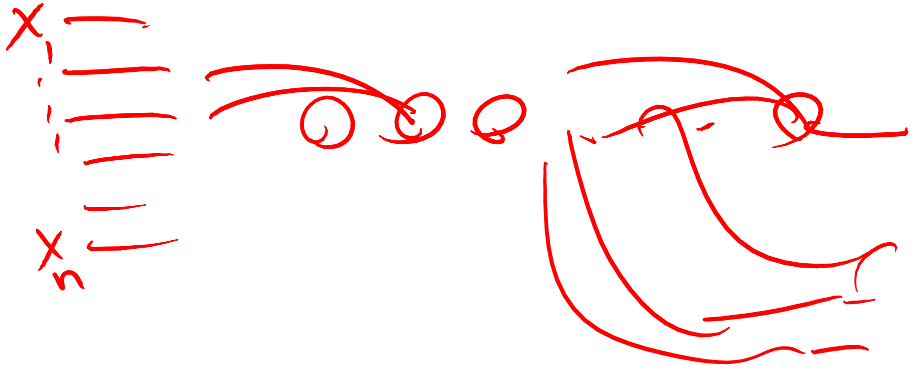
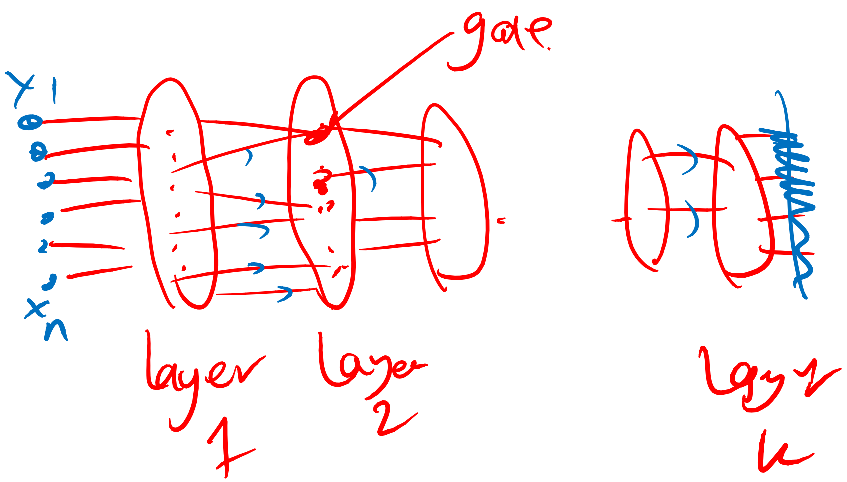
Depth of a Circuit as "parallel time"

Time — Size

Turing Machine — Circuit.





Thm: every directed graph with no cycle has a topological order

DAG — graph.

D: directed, A: acyclic

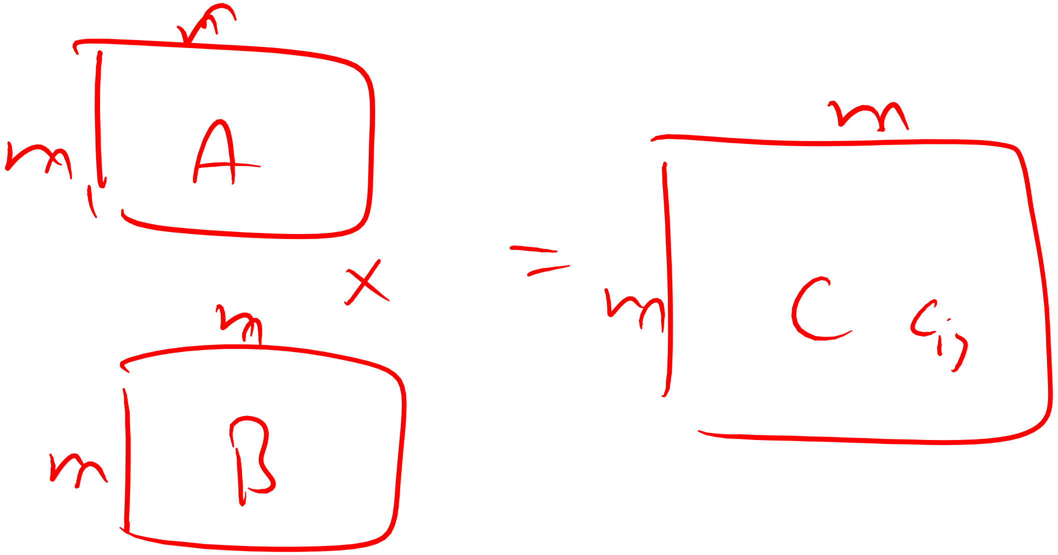
depth of a DAG G : length of longest path in G. : D_G

minimum # of layers G could be drawn : L_G

Thm $D_G = L_G$

Matrix multiplication

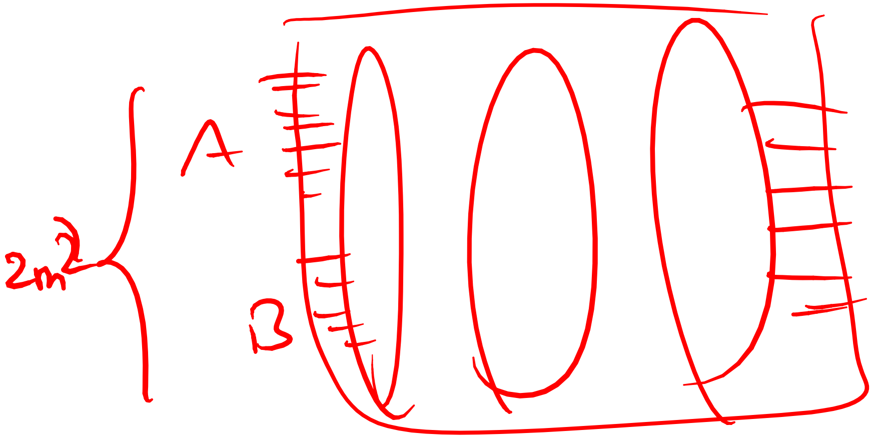
$n = \text{input len} = \Theta(m^2)$



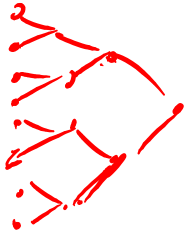
Most obvious Alg: $m^3 = n^{3/2}$

Strassen's Alg $m^{2.8..} = n^{<3/2}$

Compute c_{ij} indep from $c_{i'j'}$



$$c_{ij} = \sum_{k=1}^{m} a_{ik} \cdot b_{kj}$$



Computing $a_{ik} \cdot b_{kj}$ is indep of $a_{il} b_{lj}$

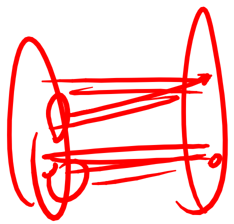
Thm: Comuting matrix multiplication in $\boxed{\lg(n)}$

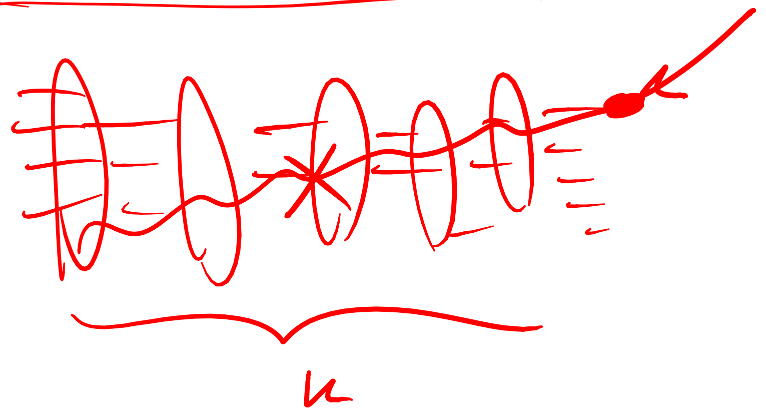
Other Circuit-Based Complexity Classes

$L \in NC_i$: if L can be computed by circuits $\{C_0, C_1, \ldots, C_n, \ldots\}$ such that $\text{Depth}(C_n) = O(\lg^i n)$

Thm: Computing matrix mult. over Boolean $(\times \equiv \wedge \quad + \equiv \oplus)$ is in NC_2

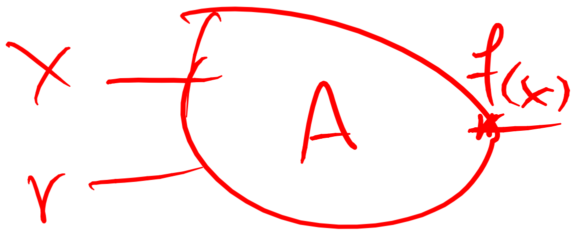
NC_0 = Constant depth circuits





New Topic: Randomized Computation

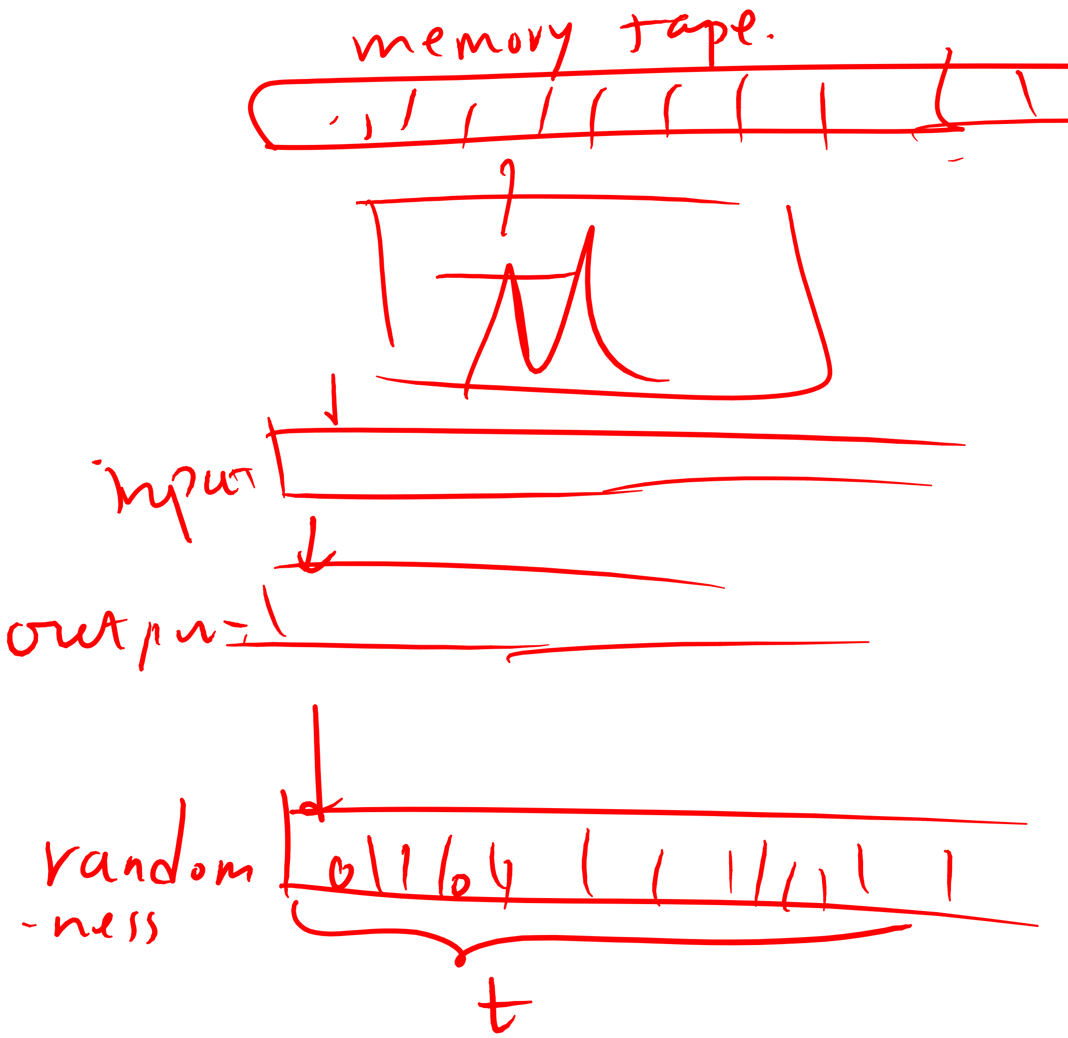
- Nature seems to provide randomness; can we use it?



- What if we find the "solution" with probability $1 - 2^{-100}$?

- Note: we do not assume the input to be random…

Formalizing Randomized Computation



if M runs in time t

when M starts to work. random Tape is written with a random sequence.

bounded error probabilistic poly time.

Complexity Class **BPP**

say that a PTM M decides L in time $T(n)$ if for every $x \in \{0,1\}^*$, M halts in $T(|x|)$ steps regardless of its random choices, and $\Pr[M(x) = L(x)] \geq \cancel{2/3}$.

$1-2^{-100}$

$\geq \frac{1}{2} + \frac{1}{n}$

We let $\mathbf{BPTIME}(T(n))$ be the class of languages decided by PTMs in $O(T(n))$ time and define $\mathbf{BPP} = \cup_c \mathbf{BPTIME}(n^c)$.