Fall 2013-MATH 141 Section FC02
CALC II
Exam I. 09/30/13

Answer each problem on a separate sheet of paper. Use the back side if necessary. Perfect score is 100 pts. There are some very challenging problems on this test. Do the problems that you are confident about first, then attack the more difficult problems.

1. Find the limits of:
(i) (**10 pts.**) $\lim_{x\to 0} \frac{e^{(x^2)}-1}{e^x-1}$ (ii) (**5 pts.**) $\lim_{x\to 1} \frac{4^x-3^x-1}{x-1}$
(iii) (**10 pts.**) $\lim_{x\to 0^+} x^{\sin x}$ (iv) (**5 pts.**) $\lim_{x\to 0^+} \sin x \ln(\sin x)$.
Hint: For (iv) you might want to think of an easier function.
Answers should be either a real number, $\pm\infty$, or D.N.E.

2. For (i), (ii), and (iii) determine whether the series diverges, converges conditionally, or converges absolutely.
(i) (**5 pts.**) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2+1}$
(ii) (**5 pts.**) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$
(iii) (**10 pts.**) $\sum_{n=1}^{\infty} (-1)^n \frac{(\ln n)^p}{n}$, where p is any positive integer
(iv) (**5 pts.**) Find the sum of the series $\sum_{n=0}^{\infty} \frac{2^n+5^n}{2^n 5^n}$.

3. Determine the radius of convergence of the following power series.
(i) (**5 pts.**) $\sum_{n=2}^{\infty} (\ln n) x^n$ (ii) (**10 pts.**) $\sum_{n=1}^{\infty} \frac{1^2\cdot 3^2\cdot 5^2 \ldots (2n-1)^2}{2^2\cdot 4^2\cdot 6^2 \ldots (2n)^2} x^{2n}$.
(iii) (**10 pts.**)For what values of x does $\sum_{n=1}^{\infty} nx^n$ converge? Show that for the x values that you found, $\sum_{n=1}^{\infty} nx^n = \frac{x}{(1-x)^2}$.

4. (i) (**10 pts.**) Find the Taylor expansion of $f(x) = \cos^2(x)$. (**Hint**: Use the trigonometric identity $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$)
(ii) (**10 pts.**) Use the **Lagrange Remainder formula** (i.e. the remainder formula for Taylor series) to prove that the series converges for all values of x.