CS635: Tools and Environments for Optimization. Lecture 10

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New Problem!

- Given network $G = (N, A)$
- Each node $i \in N$ has a "supply" b_i
- $b_i < 0 \Rightarrow$ node i "demands" an amount b_i.
- For feasibility, we must have that $\sum_{i \in N} b_i = 0$. (Supply = Demand)
- Arcs $a \in A$ may have costs c_a or capacities u_a

Min-Cost Network Flow

Find a minimum cost flow of the commodity from supply nodes to demand nodes without exceeded the arc capacity

MCNF: Mathematical Formulation

$$\min \sum_{(i,j) \in A} c_{ij} x_{ij}$$

$$\begin{aligned} \sum_{j \mid (i,j) \in A} x_{ij} - \sum_{j \mid (i,j) \in A} x_{ji} &= b_i \quad \forall i \in N \\ 0 \le x_{ij} &\le u_{ij} \quad \forall (i,j) \in A \end{aligned}$$

GAMS time: `mincost.gms`, `mincost2.gms`

- `abort`
- Important: Note use of dynamic set `arc`

mincost2.gms

- You can loop over sets in GAMS. `loop`
- Important GAMS attributes: `modelstat`, `solvestat`

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Attributes Controlled by the User

- iterlim iteration limit
- limcol number of columns displayed for each block of variables
- limrow number of rows displayed for each block of equations
- optfile option file usage
- reslim time limit for solver. Usually in CPU seconds
- solprint solution print option

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Attributes Controlled by the Solver

- iterusd: number of iterations used
- modelstat: model status (cf. Section 10.5.4)
- numequ: number of single equations generated
- numvar: number of single variables generated
- resusd: resource units (in CPU seconds) used to solve model
- solvestat: solver status (cf. Section 10.5.4)

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Model Status (10.5.4)

1. OPTIMAL
2. LOCALLY OPTIMAL
3. UNBOUNDED
4. INFEASIBLE
5. LOCALLY INFEASIBLE
6. INTERMEDIATE INFEASIBLE
7. INTERMEDIATE NONOPTIMAL
8. INTEGER SOLUTION
9. INTERMEDIATE NON-INTEGER
10. INTEGER INFEASIBLE

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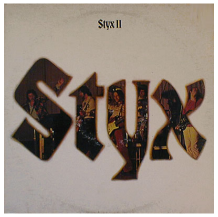
Solver Status (10.5.5)

1. Normal Completion
2. Iteration Interrupt (option iterlim)
3. Resource Interrupt (option redlim)
4. Terminated By Solver
5. Evaluation Error Limit
6. Unknown

Come Sail Away, Come Sail Away



- Sailco manufactures sailboats. During the next 4 months the company must meet the following demands for their sailboats:
  - 40, 60, 75, 25
- At the beginning of Month 1, Sailco has 10 boats in inventory.
- Each month it must determine how many boats to produce.

Sailco, Cont.

- During any month, Sailco can produce up to 40 boats with regular labor and an unlimited number of boats with overtime labor.
  - Boats produced with regular labor cost $400 each to produce,
  - while boats produced with overtime labor cost $450 each.
- It costs $20 to hold a boat in inventory from one month to the next. Find the production and inventory schedule that minimizes the cost of meeting the next 4 months' demands on time.

Sailco

- Applications that are not obviously network problems can be modeled as such.

Model as a network

- One node per month – given demand at each month.
- Inflow to each month node from a "regular labor" node, with capacity 40 and cost 400
- Inflow to each month node from a "overtime labor" node, with unlimited capacity and cost 450
- Month-to-month arcs representing inventory, (cost 20)
- Balance constraints applied only at month nodes.

- GAMS Time: `sailco2.gms`

Max Flow

Maximum Flow Problem

Given a network capacitated $G = (N, A)$, with capacities $u \in \mathbb{R}_+^{|A|}$, a source node $s \in N$, and a sink node $t \in V$, what is the maximum flow that can be sent from s to t.

- Model "what goes in = what come out".

$$\sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = 0$$

- This constraint holds forall nodes except the source and sink: $(\forall i \in N \setminus \{s\} \setminus \{t\})$

Max Flow Problem

$$\max \sum_{j:(s,j)\in A} x_{sj}$$

$$\begin{aligned} \text{s.t.} \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} &= 0 \quad \forall i \in N \setminus \{s\} \setminus \{t\} \\ x_{ij} &\le u_{ij} \quad \forall (i,j) \in A \\ x_{ij} &\ge 0 \quad \forall (i,j) \in A \end{aligned}$$

- Note that due to flow balance, objective is the same as

$$\sum_{j:(j,t)\in A} x_{jt}$$

Let's Have a Picnic!

- The Hatfields, Montagues, McCoys and Capulets are going on their annual family picnic.
- Four cars are available to transport the families to the picnic.
- The cars can carry the following numbers of people: car 1, 4; car 2, 3; car 3, 3; car 4, 4.
- There are four people in each family, and no car can carry more than two people from any one family.
- Determine the maximum number of people that can be transported to the picnic.

Max Flow Problem

- Let's try and model this is a max flow problem

The $0.0001 Question

- Who Can Make This A Max Flow Problem?

GAMS Break

SPP

Shortest Path Problem

Find the shortest path through a network from a specified origin to a specified destination.

- This is a special case of min cost flow, where costs are distances, there is a single supply source with supply +1 and a single demand (sink)

- There are specialized algorithms for shortest path that do not exploit the relationship with min-cost flow and in fact are more efficient from the complexity viewpoint: $O(|A| + |N| \log |N|)$

Examples

- `short1.gms`
  - Note: `alias` statement
  - Note: `sqr` and `sqrt`
- `short2.gms`
  - Note: `option seed=`
  - Note: `loop` (This is cool)
  - Note: difference between assignments to arcs(i,j) and arcs(j,i). Use of the `alias` is essential!
  - Note: `modelstat`

Count the CPU Time:

- `totalSolveTime = totalSolveTime + short.resusd;`

More Goodness

Homework #4

- Due Friday @11AM.
- Three problems.