1 Practice Exercises. Probability, utility and decision making

Exercise 1. *Consider a set of $n = 2$ weather stations. Your prior belief that the i-th station is correct is $P(H_i) = 1/n$. You assume that only one station is the correct one, i.e. that $P(H_i \cap H_j) = 0$ for any two different stations $i \neq j$.*

Now assume that the stations are making the following predictions for the next two days. Let A denote the event of rain on Saturday. Let B denote the event of rain on Sunday. The first station predicts that there will be rain on Saturday with probability 10%, i.e. $P(A \mid H_1) = 0.1$ and rain on Sunday with probability 50%, .e. $P(B \mid H_1) = 0.5$. The second station that there will be rain with probability on Saturday with probability 20% and on Sunday with probability 20% again, i.e. $P(A \mid H_2) = 0.2$ and $P(B \mid H_2) = 0.2$.

1. *What is the marginal probability of rain on Saturday, $P(A)$?*
2. *What about on Sunday, $P(B)$?*

Exercise 2. *Assume that you need to travel over the weekend. You wish to decide whether to take the train or take the car. Assume that the train and car trip cost exactly the same amount of money. The train trip takes 2 hours. If it does not rain, then the car trip takes 1.5 hour. However, if it rains the road becomes both more slippery and more crowded and so the average trip time is 2.5 hours. Assume that your utility function is equal to the negative amount of time spent travelling: $U(t) = -t$.*

1. *Let it be Friday. What is the expected utility of taking the car on Sunday? What is the expected utility of taking the train on Sunday? What is the Bayes-optimal decision, assuming you will travel on Sunday?*
2. *Let it be a rainy Saturday, i.e. that A holds. What is your posterior probability over the two weather stations, given that it has rained, i.e. $P(H_i \mid A)$? What is the new marginal probability of rain on Sunday, i.e. $P(B \mid A)$? What is now the expected utility of taking the car versus taking the train on Sunday? What is the Bayes-optimal decision?*

Exercise 3. *It is possible for the utility function to be nonlinear.*

1. *One example is $U(t) = 1/t$, which is a* convex *utility function. How would you interpret the utility in that case? Without performing the calculations, can you tell in advance whether your optimal decision can change? Verify your answer by calculating the expected utility of the two possible choices.*
2. *How would you model a problem where the objective involves arriving in time for a particular appointment?*

2 Feedback

Finally, some questions about this unit:

1. Did you find the material interesting?
2. Did you find it potentially useful?

3. How much did you already know?
4. How much had you already seen but did not remember in detail?
5. How much have you seen for the first time?
6. Which aspect did you like the most?
7. Which aspect did you like the least?
8. Did the exercises help you to understand the material?
9. Feel free to add any further comments.