

ELE 471 / 571 SEZİM ve KESTİRİM TEORİSİ

ÖDEV 1

Teslim Tarihi: 2 Haziran 2014

1.1 Consider the detection problem

$$\begin{aligned}\mathcal{H}_0 : x[0] &= w[0] \\ \mathcal{H}_1 : x[0] &= 1 + w[0]\end{aligned}$$

where $w[0]$ is a zero mean Gaussian random variable with variance σ^2 . If the detector decides $\mathcal{H}_1$ if $x[0] > 1/2$, find the probability of making a wrong decision when $\mathcal{H}_0$ is true. To do so, determine the probability of deciding $\mathcal{H}_1$ when $\mathcal{H}_0$ is true or $P_0 = \Pr\{x[0] > 1/2; \mathcal{H}_0\}$. For this to be 10^{-3} what must σ^2 be?

1.2 Consider the detection problem

$$\begin{aligned}\mathcal{H}_0 : x[0] &= w[0] \\ \mathcal{H}_1 : x[0] &= 1 + w[0]\end{aligned}$$

where $w[0]$ is a uniformly distributed random variable on the interval $[-a, a]$ for $a > 0$. Discuss the performance of the detector that decides $\mathcal{H}_1$ if $x[0] > 1/2$ as a increases.

1.3 We observe the datum $x[0]$ where $x[0]$ is a Gaussian random variable with mean A and variance $\sigma^2 = 1$. We wish to test if $A = A_0$ or $A = -A_0$, where $A_0 > 0$. Propose a test and discuss its performance as a function of A_0 .

1.4 In Problem 1.1 now assume that the probability of $\mathcal{H}_0$ being true is $1/2$. If we decide $\mathcal{H}_1$ if $x[0] > 1/2$, find the *total* probability of error or

$$P_e = \Pr\{x[0] > 1/2 | \mathcal{H}_0\} \Pr\{\mathcal{H}_0\} + \Pr\{x[0] < 1/2 | \mathcal{H}_1\} \Pr\{\mathcal{H}_1\}.$$

Plot P_e versus σ^2 and explain your result.

1.5 In Problem 1.1 now assume that we have two samples on which to base our decision. We decide that a signal is present if

$$T = \frac{1}{2}(x[0] + x[1]) > \frac{1}{2}.$$

Determine all the values of $x[0]$ and $x[1]$ that will result in a decision that a signal is present. Plot these values in a plane. Also, plot the point $[E(x[0]), E(x[1])]^T$ assuming $\mathcal{H}_0$ is true and then assuming $\mathcal{H}_1$ is true. Comment on the results.