Homework 1: Putting the "Fun" in Fundamentals

CSE 20 Introduction to Discrete Mathematics

Due 11am Monday July 7, 2014

The folowing exercises are taken from The Book of Proof

Exercises for Section 1.1 Introduction to Sets

A. Write each of the following sets by listing their elements between braces.

1.1.6. $\{x \in \mathbb{R} : x^2 = 9\}$

1.1.7. $\{x \in \mathbb{R} : x^2 + 5x = -6\}$

1.1.12. $\{x \in \mathbb{Z} : |2x| < 5\}$

B. Write each of the following sets in set-builder notation.

1.1.17. $\{2, 4, 8, 16, 32, 64, \ldots\}$

1.1.18. $\{0, 4, 16, 36, 64, 100, \ldots\}$

1.1.25. $\{\ldots \frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8, \ldots\}$

C. Find the following cardinalities

1.1.33. $|\{x \in \mathbb{Z} : |x| < 10\}| =$

1.1.34. $|\{x \in \mathbb{N} : |x| < 10\}| =$

1.1.35. $|\{x \in \mathbb{Z} : x^2 < 10\}| =$

1.1.36. $|\{x \in \mathbb{N} : x^2 < 10\}| =$

1.1.37. $|\{x \in \mathbb{N} : x^2 < 0\}| =$

Exercises for Section 1.2 The Cartesian Product

A. Write out the indicated sets by listing their elements between braces.

1.2.1. Suppose $A = \{1, 2, 3, 4\}$ and $B = \{a, c\}$.

(a) $A \times B$

(b) $B \times A$

(c) $A \times A$

B. Sketch these Cartesian products on the x-y plane $\mathbb{R}^2$

1.2.10. $\{-1, 0, 1\} \times \{1, 2, 3\}$

1.2.18. $\mathbb{Z} \times \mathbb{Z}$

Exercises for Section 1.3 Subsets

A. List all the subsets of the following sets.

1.3.1. $\{1, 2, 3, 4\}$

1.3.2. $\{1, 2, \emptyset\}$

B. Write out the following sets by listing their elements between braces.

1.3.9. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 2\}$

1.3.12. $\{X : X \subseteq \{3, 2, a\} \text{ and } |X| = 1\}$

C. Decide if the following statements are true or false. Explain.

1.3.13. $\mathbb{R}^3 \subseteq \mathbb{R}^3$

1.3.14. $\mathbb{R}^2 \subseteq \mathbb{R}^3$

Exercises for Section 1.4 Power Sets

A. Find the indicated sets

1.4.2. $\mathscr{P}(\{1, 2, 3, 4\})$

1.4.5. $\mathscr{P}(\mathscr{P}(\{2\}))$

B. Suppose that $|A| = m$ and $|B| = n$. Find the following cardinalities.

1.4.13. $|\mathscr{P}(\mathscr{P}(\mathscr{P}(A)))|$

1.4.15. $|\mathscr{P}(A) \times \mathscr{P}(B)|$

Exercises for Section 1.5 Union, Intersection, Difference

1.5.2. Suppose $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7\}$ and $C = \{2, 8, 4\}$. Find:

1.5.2(a). $A \cup B$

1.5.2(b). $A \cap B$

1.5.2(d). $A - C$

1.5.2(g). $B \cap C$

Exercises for Section 1.6 Complement

1.6.2. Let $A = \{0, 2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$ have universal set $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$. Find:

1.6.2(a). $\overline{A}$

1.6.2(c). $A \cap \overline{A}$

1.6.2(d). $A \cup \overline{A}$

1.6.2(g). $\overline{A} \cup \overline{B}$

1.6.2(h). $\overline{A \cap B}$

1.6.3. Sketch the set $X = [1, 3] \times [1, 2]$ on the plane $\mathbb{R}^2$. On separate drawings shade in the sets $\overline{X}$ and $\overline{X} \cap ([0, 2] \times [0, 3])$

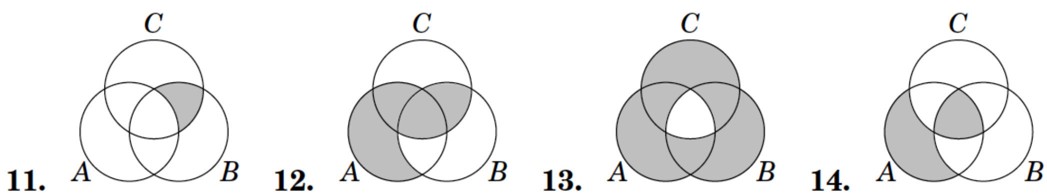
Exercises for Section 1.7 Venn Diagrams

1.7.1. Draw a Venn Diagram for $\overline{A}$.

1.7.2. Draw a Venn Diagram for $B - A$.

1.7.8. Suppose sets A and B are in a universal set U. Draw Venn Diagrams for $\overline{A \cap B}$ and $\overline{A} \cup \overline{B}$. Based on your drawings, do you think it's true that $\overline{A \cap B} = \overline{A} \cup \overline{B}$?

Following are Venn diagrams for expressions involving sets A,B and C. Write the corresponding expression.



1.7.11.

1.7.12.

1.7.13.

1.7.14.

Exercises for Section 1.8 Indexed Sets

Suppose $A_1 = \{a, b, d, e, g, f\}$, $A_2 = \{a, b, c, d\}$, $A_3 = \{b, d, a\}$ and $A_4 = \{a, b, h\}$.

1.8.1(a). $\cup_{i=1}^{4} A_i =$

1.8.1(b). $\cap_{i=1}^{4} A_i =$

For each $n \in \mathbb{N}$, let $A_n = \{0, 1, 2, 3, \ldots, n\}$.

1.8.3(a). $\cup_{i \in \mathbb{N}} A_i =$

1.8.3(b). $\cap_{i \in \mathbb{N}} A_i =$

Exercises for Section 2.1 Statements

Decide whether or not the following are statements. In the case of a statement, say if it is true or false, if possible.

2.1.1. Every real number is an even integer.

2.1.2. Every even integer is a real number.

2.1.7. The derivative of any polynomial of degree 5 is a polynomial of degree 6.

2.1.9. $\cos(x) = -1$

2.1.10. $(\mathbb{R} \times \mathbb{N}) \cap (\mathbb{N} \times \mathbb{R}) = \mathbb{N} \times \mathbb{N}$

Exercises for Section 2.2 And, Or, Not

Express each statement or open sentence in one of the forms $P \wedge Q$, $P \vee Q$, or $\sim P$. *Be sure to also state exactly what the statements P and Q stand for.*

2.2.1. The number 8 is both even and a power of 2.

2.2.2. The matrix A is not invertible.

2.2.6. There is a quiz scheduled for Wednesday or Friday.

2.2.7. The number x equals zero, but the number y does not.

2.2.13. Human beings want to be good, but not too good, and not all the time. (George Orwell)

Exercises for Section 2.3 Conditional Statements

Without changing their meanings, convert each of the following sentences into a sentence having the form *"If P then Q."*

2.3.1. A Matrix is invertable provided that its determinant is nonzero.

2.3.2. For a function to be continuous, it is sufficient that it is differentiable.

2.3.3. For a function to be integrable, it is nessary that it is continuous.

2.3.4. A function is rational if it is a polynomial.

2.3.5. An integer is divisable by 8 only if if is divisable by 4.

2.3.7. A series converges whenever it converges absolutely.

2.3.8. A geometric series with ratio r converges if $|r| < 1$.