Local and Global Axes

- Kassimali, §8.3 Space Frames. Global axes: X, Y, Z
- Axial direction: local x

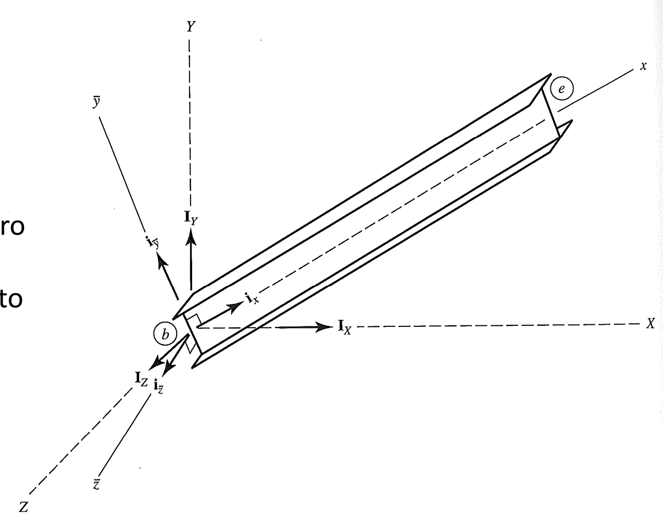
$$L = \sqrt{(X_e - X_b)^2 + (Y_e - Y_b)^2 + (Z_e - Z_b)^2}$$

$$r_{xX} = \cos\theta_{xX} = \frac{X_e - X_b}{L}$$

$$r_{xY} = \cos\theta_{xY} = \frac{Y_e - Y_b}{L}$$

$$r_{xZ} = \cos\theta_{xZ} = \frac{Z_e - Z_b}{L}$$

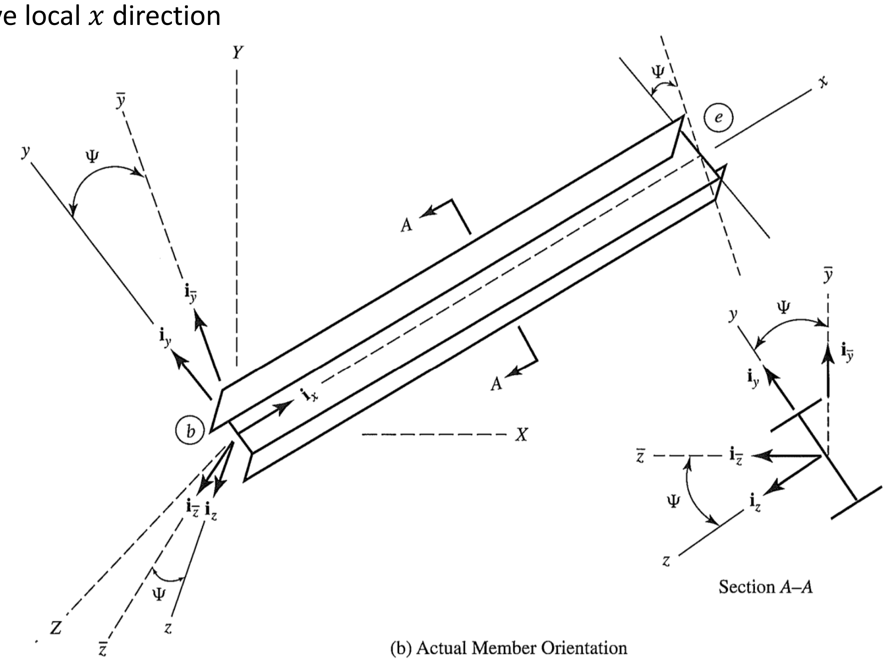
- Location directions ($\bar{y}$ and $\bar{z}$) with zero angle of roll are chosen such that x and $\bar{y}$ form a vertical plane (parallel to global Y)
- As a result, $\bar{z}$ is horizontal (perpendicular to global Y)



(a) Member Orientation with $x\bar{y}$ Plane Vertical

Fig. 8.18

- Angle of roll Ψ (called Beta in Mastan2 when defining an element): rotation angle about local x axis s.t. $\bar{y}$ reaches actual local y direction (and $\bar{z}$ reaches actual local z direction)
- Right-hand rule: positive Ψ is rotation from from $\bar{y}$ to y with thumb pointing at positive local x direction



(b) Actual Member Orientation

Fig. 8.18 (*continued*)

- Local axes with zero angle of roll: x , $\bar{y}$, and $\bar{z}$
- Transformation between local unit vectors and global unit vectors:

$$\begin{bmatrix} \mathbf{i}_x \\ \mathbf{i}_{\bar{y}} \\ \mathbf{i}_{\bar{z}} \end{bmatrix} = \begin{bmatrix} r_{xX} & r_{xY} & r_{xZ} \\ -\dfrac{r_{xX} r_{xY}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} & \sqrt{r_{xX}^2 + r_{xZ}^2} & -\dfrac{r_{xY} r_{xZ}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} \\ -\dfrac{r_{xZ}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} & 0 & \dfrac{r_{xX}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_X \\ \mathbf{I}_Y \\ \mathbf{I}_Z \end{bmatrix} \qquad \textbf{(8.62)}$$

- Recall that:

$$r_{xX} = \cos\theta_{xX} = \frac{X_e - X_b}{L}$$
$$r_{xY} = \cos\theta_{xY} = \frac{Y_e - Y_b}{L}$$
$$r_{xZ} = \cos\theta_{xZ} = \frac{Z_e - Z_b}{L}$$

- In Mastan2, with default zero angle of roll (called Beta in Mastan2 when defining an element), apply distributed load w_y or w_z to see the local axes.


```
clear

% Vector from beginning to end joint
Vbe = [0 0 1]';

rxX = Vbe(1) / norm(Vbe);
rxY = Vbe(2) / norm(Vbe);
rxZ = Vbe(3) / norm(Vbe);

tmp21 = -rxX * rxY / sqrt(rxX^2 + rxZ^2);
tmp22 = sqrt(rxX^2 + rxZ^2);
tmp23 = -rxY * rxZ / sqrt(rxX^2 + rxZ^2);
tmp31 = -rxZ / sqrt(rxX^2 + rxZ^2);
tmp33 = rxX/sqrt(rxX^2 + rxZ^2);

[   rxX     rxY     rxZ;
    tmp21   tmp22   tmp23;
    tmp31     0     tmp33]
```

$$r_{xX} = \cos\theta_{xX} = \frac{X_e - X_b}{L}$$
$$r_{xY} = \cos\theta_{xY} = \frac{Y_e - Y_b}{L}$$
$$r_{xZ} = \cos\theta_{xZ} = \frac{Z_e - Z_b}{L}$$

$$\begin{bmatrix} r_{xX} & r_{xY} & r_{xZ} \\ -\dfrac{r_{xX} r_{xY}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} & \sqrt{r_{xX}^2 + r_{xZ}^2} & -\dfrac{r_{xY} r_{xZ}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} \\ -\dfrac{r_{xZ}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} & 0 & \dfrac{r_{xX}}{\sqrt{r_{xX}^2 + r_{xZ}^2}} \end{bmatrix}$$

Vertical Members

- Try Vbe `= [0 1 0]';`
- Eq. (8.62) does not apply to vertical members (local x axis along global Y axis)
- For vertical members, r_{xX}=____ r_{xZ} =____.
- In this special case, following transform can be adopted (but not always).

$$\begin{bmatrix} \mathbf{i}_x \\ \mathbf{i}_{\bar{y}} \\ \mathbf{i}_{\bar{z}} \end{bmatrix} = \begin{bmatrix} 0 & r_{xY} & 0 \\ -r_{xY} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_X \\ \mathbf{I}_Y \\ \mathbf{I}_Z \end{bmatrix}$$

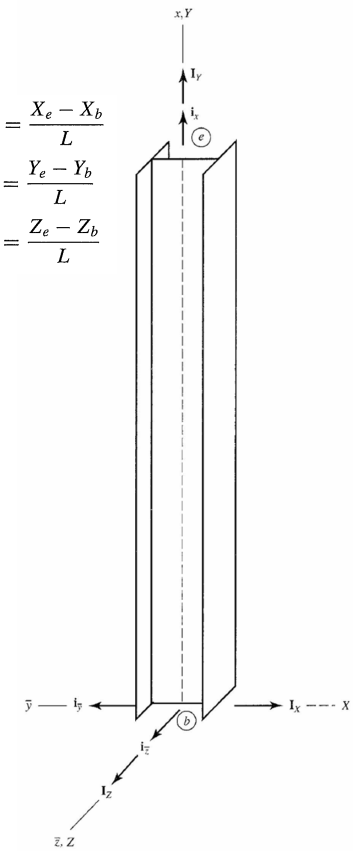
- Mastan2 uses the transform when beginning point is lower (i.e. θ_{xY}=0 and r_{xY}= 1 as shown in the figure). But when beginning point is higher, i.e. θ_{xY}=180° and r_{xY}= -1, Mastan2 uses following transform:

$$\begin{bmatrix} \mathbf{i}_x \\ \mathbf{i}_{\bar{y}} \\ \mathbf{i}_{\bar{z}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_X \\ \mathbf{I}_Y \\ \mathbf{I}_Z \end{bmatrix}$$

$$r_{xX} = \cos\theta_{xX} = \frac{X_e - X_b}{L}$$

$$r_{xY} = \cos\theta_{xY} = \frac{Y_e - Y_b}{L}$$

$$r_{xZ} = \cos\theta_{xZ} = \frac{Z_e - Z_b}{L}$$



(a) Orientation of Vertical Member with z̄ Axis Parallel to Global Z Axis