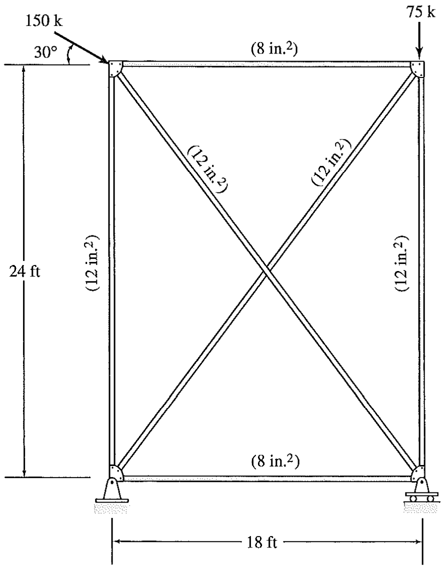
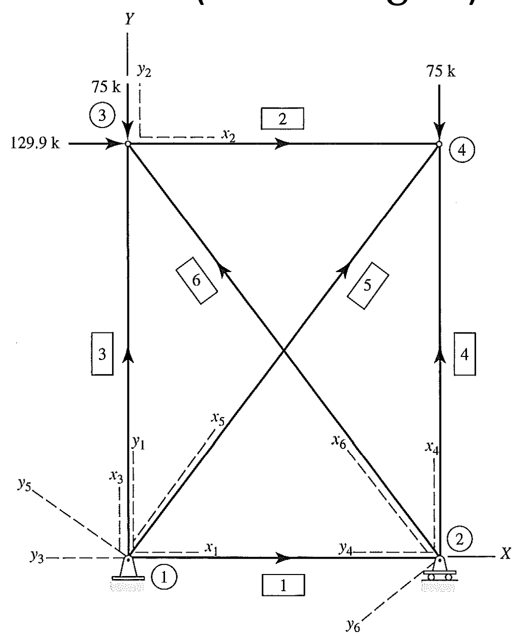
Chapter 4. Matrix Analysis of Plane Trusses
4.1. Global and Local Coordinates (Kassimali §3.1)



E = 29,000 ksi

(a) Actual Truss

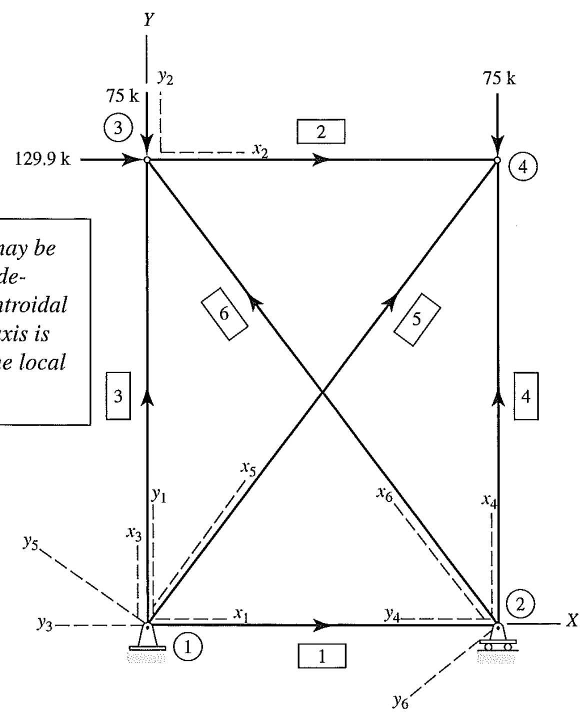


(b) Analytical Model Showing Global and Local Coordinate Systems

The global coordinate system used in this text is a right-handed XYZ coordinate system with the plane structure lying in the XY plane.

The origin of the local xyz coordinate system for a member may be arbitrarily located at one of the ends of the member in its undeformed state, with the x axis directed along the member's centroidal axis in the undeformed state. The positive direction of the y axis is defined so that the coordinate system is right-handed, with the local z axis pointing in the positive direction of the global Z axis.

- Each member has a *beginning joint*, and an *end joint*.
- Arrows mark beginning-to-end directions.



(b) Analytical Model Showing Global and Local Coordinate Systems

4.2. Degrees of Freedom (Kassimali §3.2)

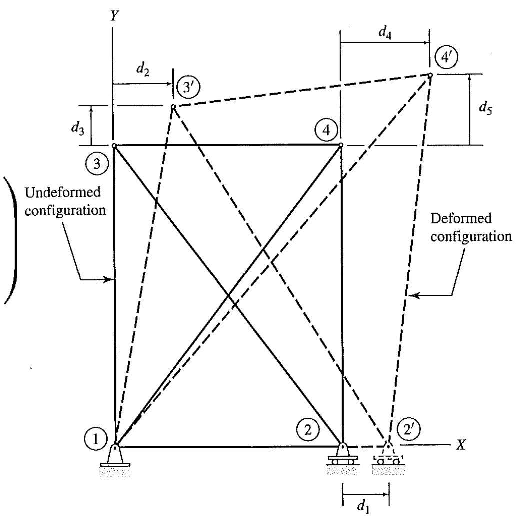
- Degrees of freedom: the independent joint displacements (translations and rotations) that are necessary to specify the deformed shape of the structure when subjected to an arbitrary loading.
- Truss structure: only joint translations provide the DOFs; no rotations.
- For this example, joint displacement vector contains five necessary and sufficient joint displacements to uniquely define an arbitrary deformed shape of the truss.

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} \qquad NDOF = NCJT\,(NJ) - NR$$

$$\begin{pmatrix} \text{number of} \\ \text{degrees of} \\ \text{freedom} \end{pmatrix} = \begin{pmatrix} \text{number of joint} \\ \text{displacements of} \\ \text{the unsupported} \\ \text{structure} \end{pmatrix} - \begin{pmatrix} \text{number of joint} \\ \text{displacements} \\ \text{restrained by} \\ \text{supports} \end{pmatrix}$$

$NCJT$: number of coordinates per joint
NJ: number of joints
NR: number of support restraints

$$\left.\begin{aligned} NCJT &= 2 \\ NDOF &= 2(NJ) - NR \end{aligned}\right\} \text{ for plane trusses}$$



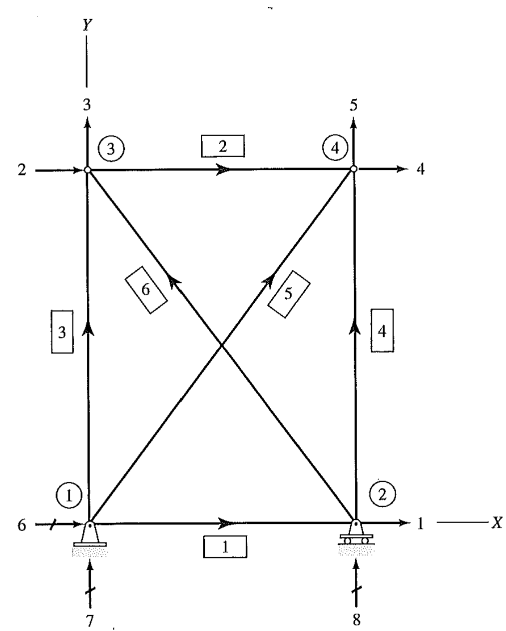
(c) Degrees of Freedom

Numbering the DOFs

- DOFs are numbered starting at the lowest-numbered joint that has a DOF, proceeding sequentially to higher-numbered joint.
- In case of more than one DOF at a joint, the translation along global X is numbered first, followed by the global Y.
- The first assigned DOF number is 1; the last assigned DOF number should be equal to $NDOF$.

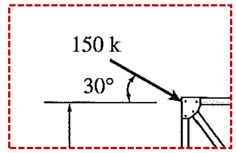
Numbering the Restrained Coordinates

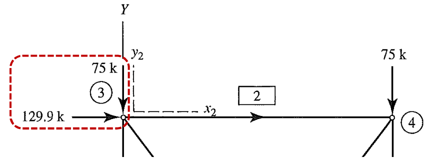
- After all DOFs are numbered, restrained coordinates are numbered in a similar manner, beginning with $NDOF+1$.
- The last restrained coordinate should be assigned with a number equal to $2(NJ)$.



(d) Degrees of Freedom and Restrained Coordinate Numbers

$$d_6 = d_7 = d_8 = 0$$



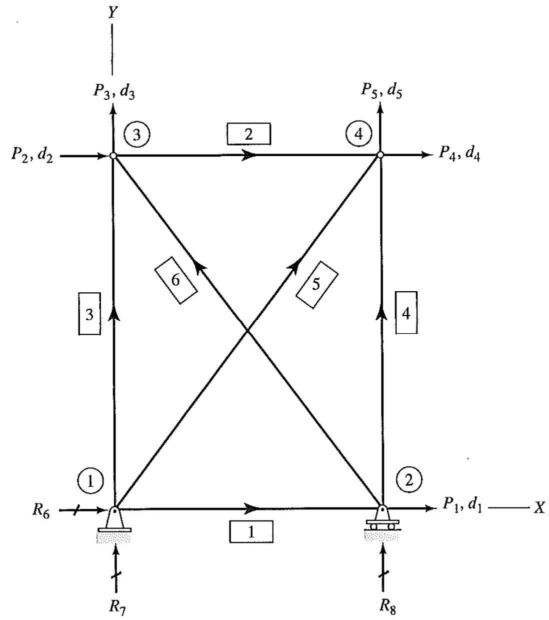


Joint Load Vector

$$\mathbf{P} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 129.9 \\ -75 \\ 0 \\ -75 \end{bmatrix} \text{k} \qquad \mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix}$$

Reaction Vector

$$\mathbf{R} = \begin{bmatrix} R_6 \\ R_7 \\ R_8 \end{bmatrix}$$



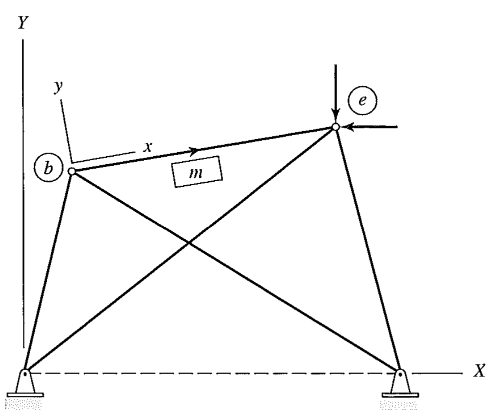
(e) Degrees of Freedom, Joint Loads, and Reactions

4.3. Member Stiffness Relations in Local Coordinates

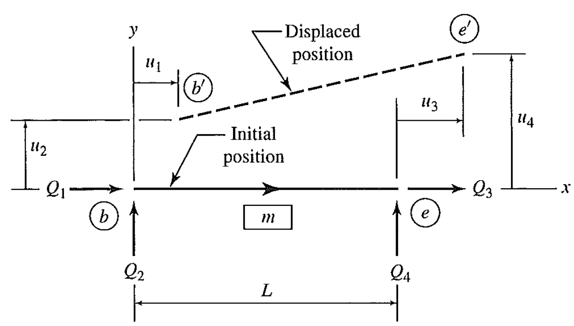
(Kassimali §3.3)

- Arbitrary member m: end displacements u_i, end forces Q_i (i = 1 ~ 4) .
- Positive directions of u_i and Q_i are along local positive x-y directions.
- Important to follow standard sequence : 1 -> 2-> 3 -> 4.
- The stiffness matrix for a member expresses the forces at the ends of the member as functions of the displacements of those ends.

$$\mathbf{Q} = \mathbf{k}\mathbf{u}$$



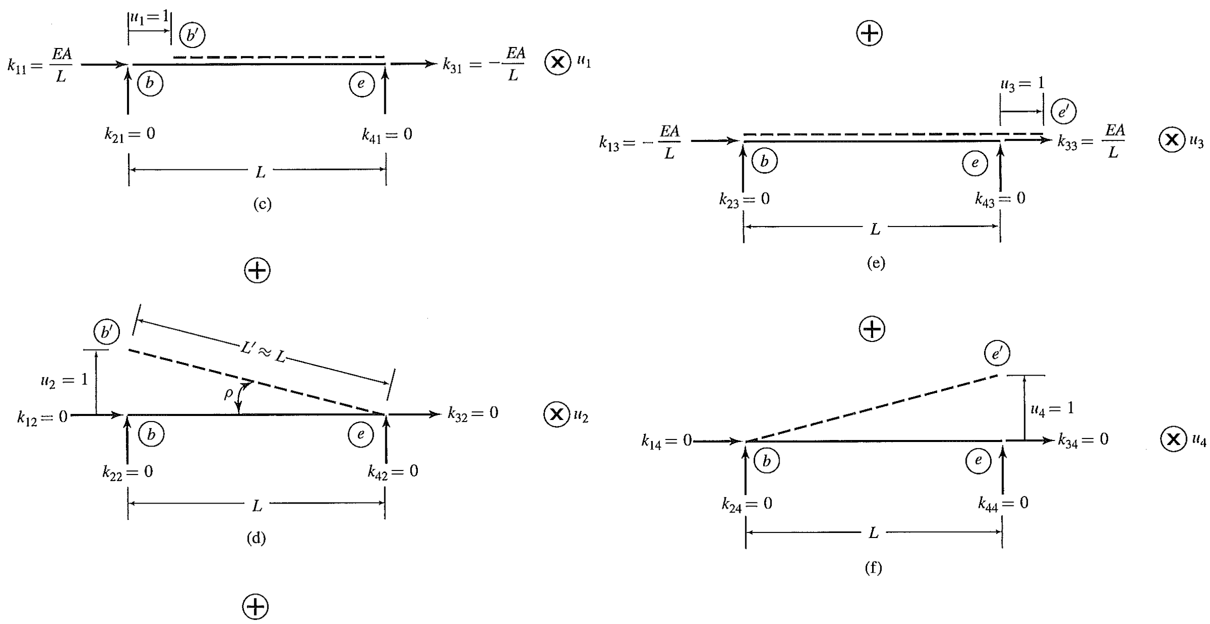
(a) Plane Truss



EA = constant

(b) Member Forces and Displacements in Local Coordinate System

- k_{ij} – force required along i in order to have unit displacement ONLY along j and ZERO displacement along other DOFs.



- Element stiffness matrix **k** containing all k_{ij} (i , j = 1 ~ 4).

$$\mathbf{k} = \begin{bmatrix} \frac{EA}{L} & 0 & -\frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{EA}{L} & 0 & \frac{EA}{L} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q_1 = k_{11}u_1 + k_{12}u_2 + k_{13}u_3 + k_{14}u_4$$
$$Q_2 = k_{21}u_1 + k_{22}u_2 + k_{23}u_3 + k_{24}u_4$$
$$Q_3 = k_{31}u_1 + k_{32}u_2 + k_{33}u_3 + k_{34}u_4$$
$$Q_4 = k_{41}u_1 + k_{42}u_2 + k_{43}u_3 + k_{44}u_4$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$$

or, symbolically, as

$$\boxed{\mathbf{Q} = \mathbf{k}\mathbf{u}}$$

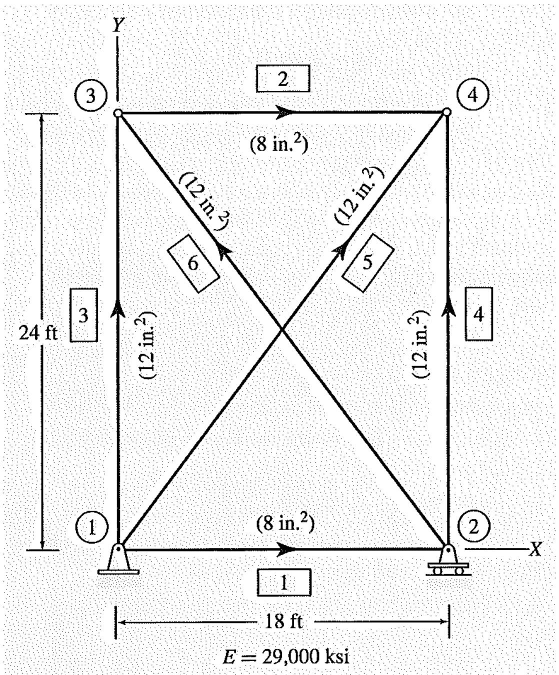


Fig. 3.4

Members 5 and 6 $E = 29{,}000$ ksi, $A = 12$ in.^{2},

$$L = \sqrt{(18)^2 + (24)^2} = 30 \text{ ft} = 360 \text{ in.}$$

$$\frac{EA}{L} = \frac{29{,}000(12)}{360} = 966.67 \text{ k/in.}$$

Thus,

$$\mathbf{k}_5 = \mathbf{k}_6 = \begin{bmatrix} 966.67 & 0 & -966.67 & 0 \\ 0 & 0 & 0 & 0 \\ -966.67 & 0 & 966.67 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{k/in.}$$