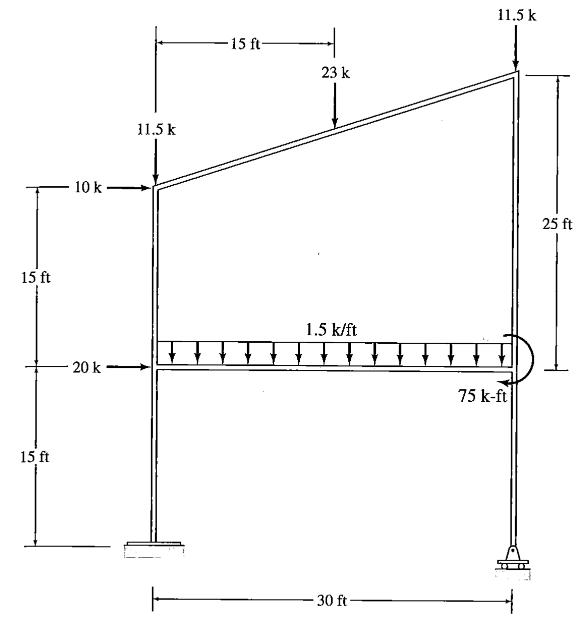
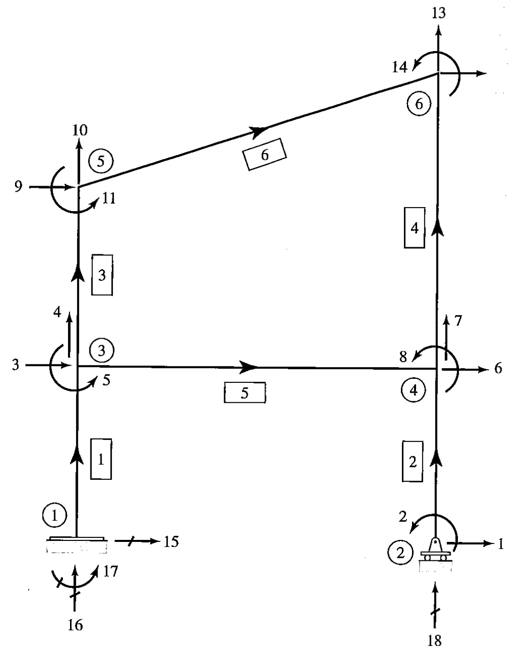
- Joint load vector **P** (containing both forces and moments) in kip-ft
- Different from MASTAN2: concentrated load is allowed on a member

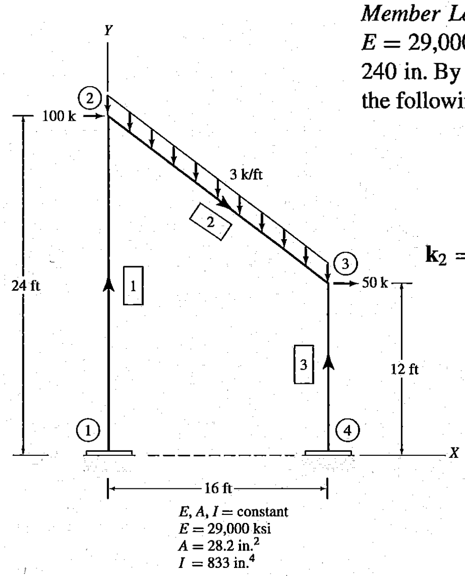
$$\mathbf{P} = \begin{bmatrix} 0 \\ 0 \\ 20 \\ 0 \\ 0 \\ 0 \\ 0 \\ -75 \\ 10 \\ -11.5 \\ 0 \\ 0 \\ -11.5 \\ 0 \end{bmatrix}$$

EXAMPLE 6.1





EXAMPLE 6.2

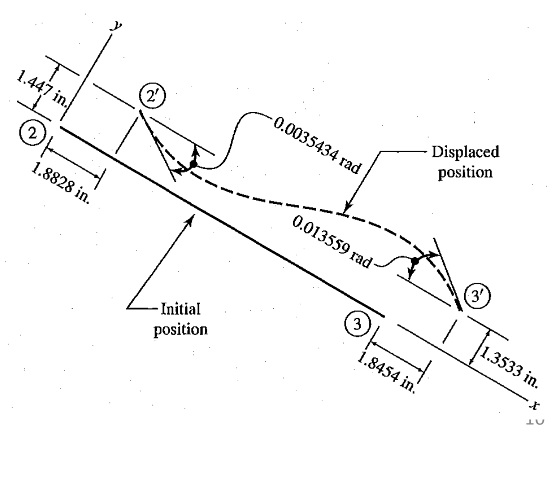


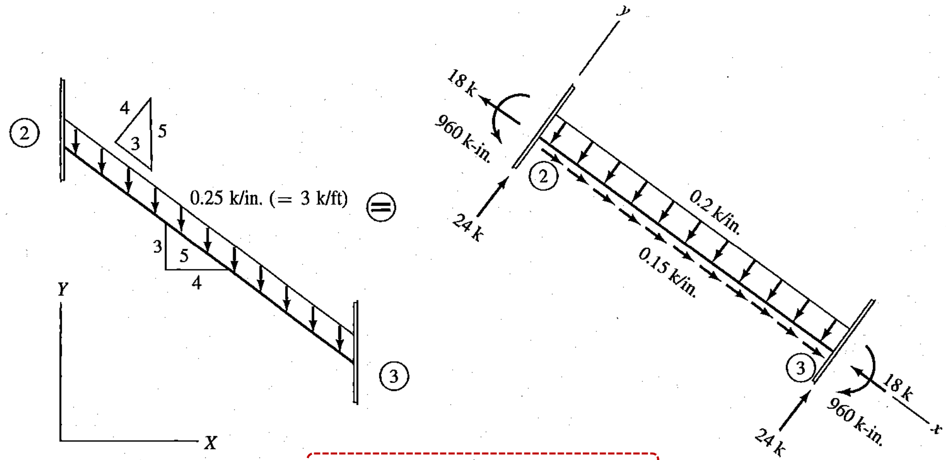
Member Local Stiffness Matrix: From Fig. 6.6(a), we can see that, for member 2, $E = 29{,}000$ ksi, $A = 28.2$ in.^{2}, $I = 833$ in.^{4}, and $L = \sqrt{(16)^2 + (12)^2} = 20$ ft $=$ 240 in. By substituting the numerical values of E, A, I, and L into Eq. (6.6), we obtain the following local stiffness matrix for member 2, in units of kips and inches.

$$\mathbf{k}_2 = \begin{bmatrix} 3{,}407.5 & 0 & 0 & -3{,}407.5 & 0 & 0 \\ 0 & 20.97 & 2{,}516.4 & 0 & -20.97 & 2{,}516.4 \\ 0 & 2{,}516.4 & 402{,}620 & 0 & -2{,}516.4 & 201{,}310 \\ -3{,}407.5 & 0 & 0 & 3{,}407.5 & 0 & 0 \\ 0 & -20.97 & -2{,}516.4 & 0 & 20.97 & -2{,}516.4 \\ 0 & 2{,}516.4 & 201{,}310 & 0 & -2{,}516.4 & 402{,}620 \end{bmatrix}$$

Member Local End Displacements:

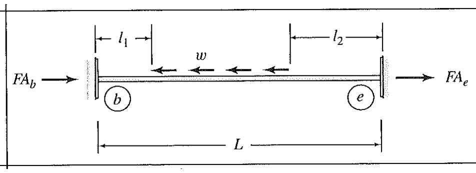
$$\mathbf{u}_2 = \begin{bmatrix} 1.8828 \\ 1.4470 \\ -0.0035434 \\ 1.8454 \\ 1.3533 \\ -0.013559 \end{bmatrix}$$





(c) Local Fixed-end Forces for Member 2

$$\mathbf{Q}_{f2} = \begin{bmatrix} \boxed{-18} \\ 24 \\ 960 \\ \boxed{-18} \\ 24 \\ -960 \end{bmatrix}$$

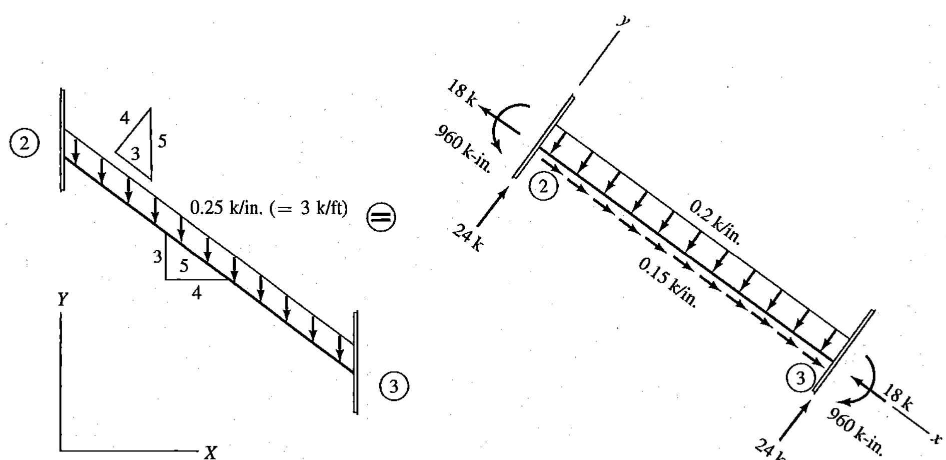
6.		$FA_b = \dfrac{w}{2L}(L - l_1 - l_2)(L - l_1 + l_2)$ $FA_e = \dfrac{w}{2L}(L - l_1 - l_2)(L + l_1 - l_2)$

$$w_x = -\frac{3}{5}(0.25) = -0.15 \text{ k/in.}$$

$$w_y = \frac{4}{5}(0.25) = 0.2 \text{ k/in.}$$

By substituting $w = -0.15$ k/in., $L = 240$ in., and $l_1 = l_2 = 0$

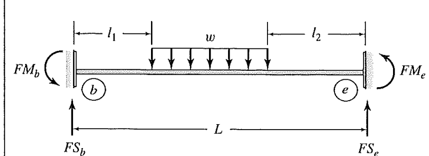
$$FA_b = FA_e = \frac{-0.15(240)}{2} = -18 \text{ k}$$



(c) Local Fixed-end Forces for Member 2

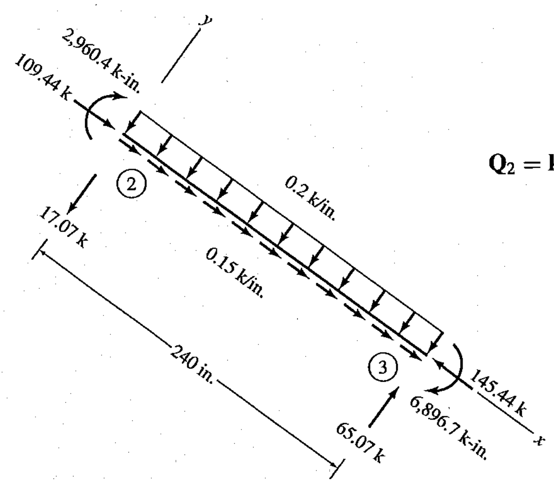
$$\mathbf{Q}_{f2} = \begin{bmatrix} -18 \\ 24 \\ 960 \\ -18 \\ 24 \\ -960 \end{bmatrix}$$

$w = 0.2$ k/in., $L = 240$ in., and $l_1 = l_2 = 0$

3.		$FS_b = \dfrac{wL}{2}\left[1 - \dfrac{l_1}{L^4}(2L^3 - 2l_1^2 L + l_1^3) - \dfrac{l_2^3}{L^4}(2L - l_2)\right]$ $FM_b = \dfrac{wL^2}{12}\left[1 - \dfrac{l_1^2}{L^4}(6L^2 - 8l_1 L + 3l_1^2) - \dfrac{l_2^3}{L^4}(4L - 3l_2)\right]$ $FS_e = \dfrac{wL}{2}\left[1 - \dfrac{l_1^3}{L^4}(2L - l_1) - \dfrac{l_2}{L^4}(2L^3 - 2l_2^2 L + l_2^3)\right]$ $FM_e = -\dfrac{wL^2}{12}\left[1 - \dfrac{l_1^3}{L^4}(4L - 3l_1) - \dfrac{l_2^2}{L^4}(6L^2 - 8l_2 L + 3l_2^2)\right]$

$$FS_b = FS_e = \frac{0.2(240)}{2} = 24 \text{ k}$$

$$FM_b = -FM_e = \frac{0.2(240)^2}{12} = 960 \text{ k-in.}$$



(d) Local End Forces for Member 2

$$\mathbf{Q}_2 = \mathbf{k}_2\mathbf{u}_2 + \mathbf{Q}_{f2} = \begin{bmatrix} 109.44 \text{ k} \\ -17.07 \text{ k} \\ -2{,}960.4 \text{ k-in.} \\ -145.44 \text{ k} \\ 65.07 \text{ k} \\ -6{,}896.7 \text{ k-in.} \end{bmatrix}$$

Equilibrium Check: To check whether the member is in equilibrium, we apply the three equations of equilibrium to the free body of the member shown in Fig. 6.6(d). Thus,

$+\searrow \sum F_x = 0 \quad 109.44 + 0.15(240) - 145.44 = 0$ **Checks**

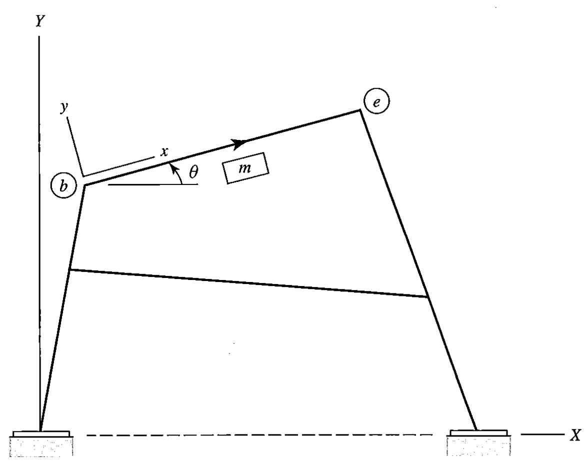
$+\nearrow \sum F_y = 0 \quad -17.07 - 0.2(240) + 65.07 = 0$ **Checks**

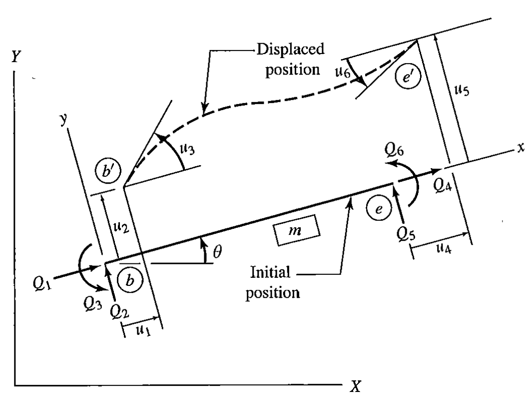
$+\circlearrowleft \sum M_{②} = 0 \quad -2{,}960.4 - 0.2(240)(120) - 6{,}896.7 + 65.07(240) = -0.3 \cong 0$

Checks

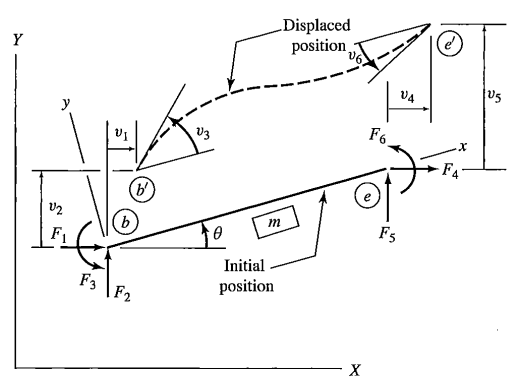
5.3. Coordinate Transformation

(Kassimali §6.3)





(b) Member End Forces and End Displacements in the Local Coordinate System



(c) Member End Forces and End Displacements in the Global Coordinate System

- Angle θ is considered positive when counterclockwise.
- In local coordinates:

$$\mathbf{Q} = \mathbf{k}\mathbf{u}$$

- Forces and displacements need to be transformed to global coordinates, so that stiffness of all members can be incorporated together.

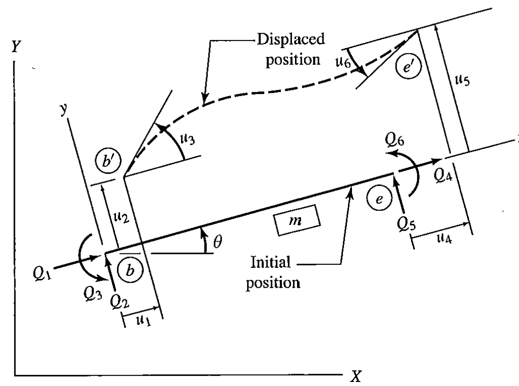
$$Q_1 = F_1 \cos\theta + F_2 \sin\theta$$
$$Q_2 = -F_1 \sin\theta + F_2 \cos\theta$$
$$Q_3 = F_3$$
$$Q_4 = F_4 \cos\theta + F_5 \sin\theta$$
$$Q_5 = -F_4 \sin\theta + F_5 \cos\theta$$
$$Q_6 = F_6$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$$

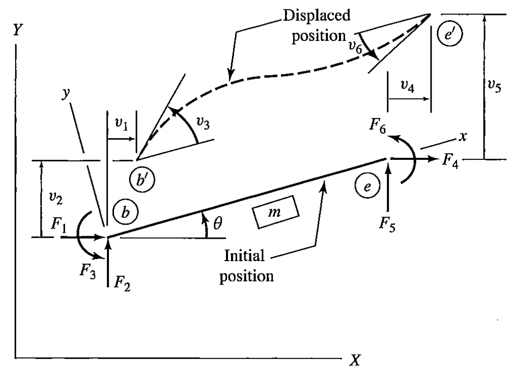
$$\mathbf{Q} = \boxed{\mathbf{T}}\mathbf{F}$$

- Likewise for displacements:

$$\mathbf{u} = \mathbf{T}\mathbf{v}$$



(b) Member End Forces and End Displacements in the Local Coordinate System



(c) Member End Forces and End Displacements in the Global Coordinate System

- Transformation from local to global is similar to truss structures.
- Because $\mathbf{T}^{-1} = \mathbf{T}^T$ (transformation matrix $\mathbf{T}$ is an orthogonal matrix):

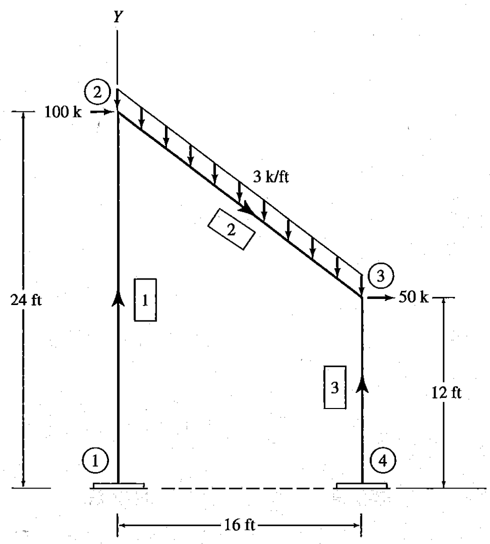
$$\mathbf{F} = \mathbf{T}^{-1}\mathbf{Q} = \mathbf{T}^T\mathbf{Q}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix}$$

- Crossing out 3rd and 6th rows and columns of $\mathbf{T}$, we get the transformation matrix $\mathbf{T}$ for truss structures.
- Likewise for displacements:

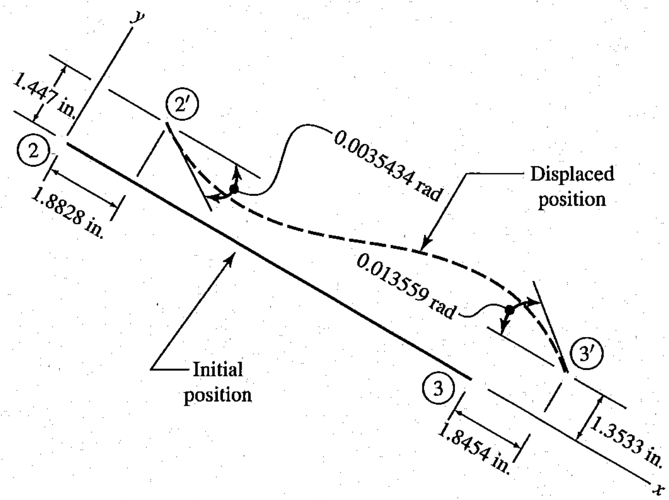
$$\mathbf{v} = \mathbf{T}^{-1}\mathbf{u} = \mathbf{T}^T\mathbf{u}$$

EXAMPLE 6.3



$E, A, I =$ constant
$E = 29{,}000$ ksi
$A = 28.2$ in.^{2}
$I = 833$ in.^{4}

$$\mathbf{u}_2 = \begin{bmatrix} 1.8828 \text{ in.} \\ 1.4470 \text{ in.} \\ -0.0035434 \text{ rad} \\ 1.8454 \text{ in.} \\ 1.3533 \text{ in.} \\ -0.013559 \text{ rad} \end{bmatrix}$$



(b) Displaced Position of Member 2

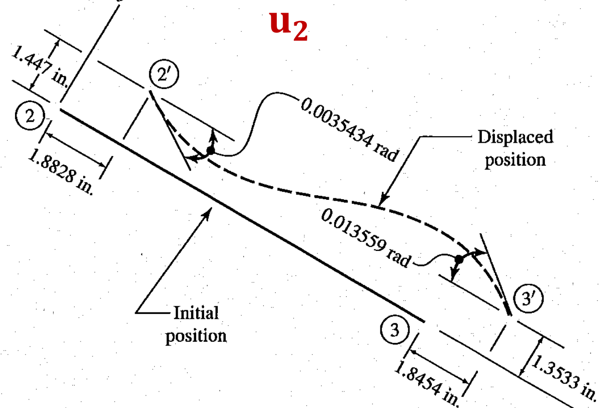
$$\cos\theta = \frac{X_3 - X_2}{L} = \frac{16 - 0}{20} = 0.8$$

$$\sin\theta = \frac{Y_3 - Y_2}{L} = \frac{12 - 24}{20} = -0.6$$

$$\mathbf{T}_2 = \begin{bmatrix} 0.8 & -0.6 & 0 & 0 & 0 & 0 \\ 0.6 & 0.8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8 & -0.6 & 0 \\ 0 & 0 & 0 & 0.6 & 0.8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

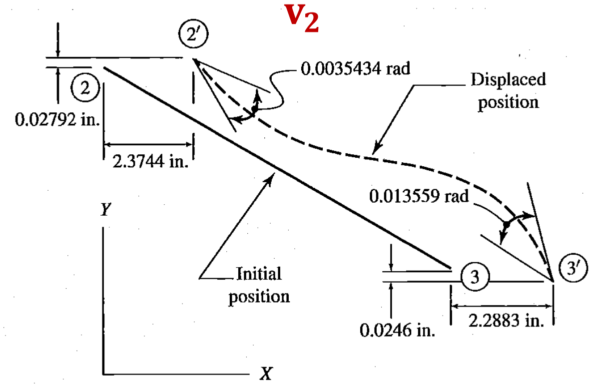
$$\mathbf{V}_2 = \mathbf{T}_2^T \mathbf{u}_2 = \begin{bmatrix} 2.3744 \text{ in.} \\ 0.02792 \text{ in.} \\ -0.0035434 \text{ rad} \\ 2.2883 \text{ in.} \\ -0.02460 \text{ in.} \\ -0.013559 \text{ rad} \end{bmatrix}$$

$\mathbf{u}_2$

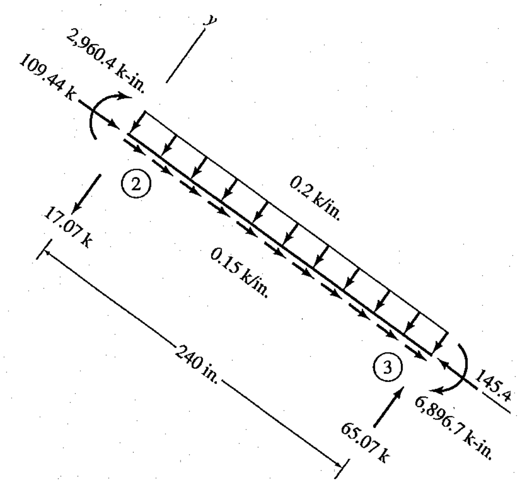


(b) Displaced Position of Member 2

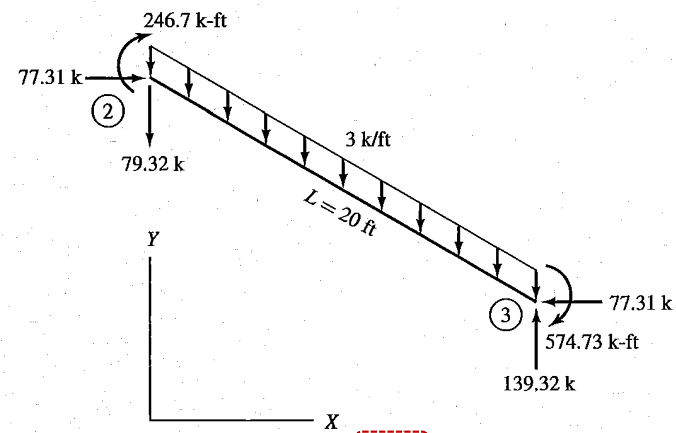
$\mathbf{V}_2$



(c) End Displacements in the Global Coordinate System for Member 2



(d) Local End Forces for Member 2



(d) End Forces in the Global Coordinate System for Member 2

$$\mathbf{Q}_2 = \mathbf{k}_2\mathbf{u}_2 + \mathbf{Q}_{f2} = \begin{bmatrix} 109.44 \text{ k} \\ -17.07 \text{ k} \\ -2{,}960.4 \text{ k-in.} \\ -145.44 \text{ k} \\ 65.07 \text{ k} \\ -6{,}896.7 \text{ k-in.} \end{bmatrix}$$

$$\mathbf{F}_2 = \mathbf{T}_2^T\mathbf{Q}_2 = \begin{bmatrix} 77.31 \text{ k} \\ -79.32 \text{ k} \\ \left\{ \begin{matrix} -2{,}960.4 \text{ k-in.} \\ (= -246.7 \text{ k-ft}) \end{matrix} \right\} \\ -77.31 \text{ k} \\ 139.32 \text{ k} \\ \left\{ \begin{matrix} -6{,}896.7 \text{ k-in.} \\ (= -574.73 \text{ k-ft}) \end{matrix} \right\} \end{bmatrix}$$