PH325: Advanced Statistical Mechanics

Centre for Condensed Matter Theory, Physics Department, IISc Bangalore
Semester I, 2014–2015

PROBLEM SET 0, DUE: AUG. 14, 2014

A request: Please try to work out the problems, as far as possible, by yourself (discuss with friends, if necessary). Please do not give in to your urge to "google out the answer". Furthermore, the work you submit must be your own – in other words, there should be no consultation with others after you sit down to write out your answers.

A note on the problems: Problems are marked by C, T, C/T or E. C means that the problem is *conceptual*, i. e., it helps you to understand and use the conceptual ideas of the subject. Such problems, usually, will not involve long calculations. On the other hand, problems marked with a T have primarily *technical* content. They should help you familiarise with a calculational technique that has been either discussed in class, or is newly introduced in the problem. Such problems may also involve the use of MATHEMATICA, or writing a C/Fortran/Java/Python code etc. A suggestion to approach the T problems is to reason out what the answers *ought to be* by conceptual arguments, i. e., ALWAYS DO THE PHYSICS BEFORE YOU DO THE MATH. C/T problems involve both. E stands for *exploratory*, i. e., that will point you to material elsewhere or ask you a question to ponder and think about.

Reading: Chaikin and Lubensky, Chapters 1 and 2.

0/1. **(C) Free energy:** What is free about "free energy"? This is your opportunity to revise the laws of thermodynamics, ensembles and associated thermodynamic potentials.

0/2. **(C) Ideal gas chemical potential:** Prove or disprove the statement: The chemical potential of a classical ideal gas is an increasing function of temperature at all temperatures in all spatial dimensions.

0/3. **(C) Fermi gas in 3d:** Prove or disprove: At all temperatures, the pressure of an ideal gas of (spinless) fermions is always less that that of its classical counterpart (with the same density of particles).

0/4. **(C) Ideal spin-1 Bose gas:** Consider an ideal gas of spin-1 bosons of mass m with density ρ in three spatial dimensions.

   (a) The Bose-Einstein condensation (BEC) temperature T_c of this system larger than that of a corresponding system of spin-0 bosons.

   (b) If a Zeeman magnetic field B (such a magnetic field couples only to spin) is applied, the T_c increases from the value at $B = 0$.

   (c) Can you ever realize such a system, i. e., where you can apply a magnetic field that couples only to the spin degree of freedom of the particle? In other words, is this a "real" question?

0/5. **(T) Ideal spin-1 Bose gas:** In this problem you have to work out the math of problem 0/4, and confirm the physics you have discovered in that problem.

(a) Find the symmetries and scales of this system, and write out all the natural dimensionless quantities.

(b) Obtain the chemical potential of the system as a function of temperature T. Make a plot of this, along with the spin-0 result.

(c) Find the compressibility of the system and plot as a function of temperature, comparing with the spin-0 result.

(d) Apply a Zeeman field B and redo the calculations above. Plot the chemical potential, compressibility etc. in the presence of the magnetic field B.

(e) Are the results you find in agreement with your physical ideas? Explain.

0/6. **(C/T) Fermions pair to form bosons!** We have in three spatial dimensions a density ρ of equal number of $\uparrow$ and $\downarrow$ spin fermions of mass m. The fermions of opposite spin attract each other and form a tightly bound bosonic object ("Cooper pair") with *binding energy* E_b.

(a) Write out a simple Hamiltonian for this system assuming that all particles are non-interacting. (You might find this a bit strange (if they did not interact how can the fermions form Cooper pairs) and may even conclude that the instructor is demented[1])

(b) What are the energy scales in the system?

(c) Assuming that E_b is much larger than other scales in the problem, discuss the nature of the system at temperatures $T \ll E_b$ and $T \gg E_b$.

(d) Is there a phase transition in the system? Perform a detailed analysis.

0/7. **(C/T) String theory...by a Vainika:** Consider a Veena string of length L, mass per unit length μ, and tension Y. Note that this is a "continuous system". Let the string be stretched along the x-axis.

(a) **(C)** Introducing $\mathbf{u}(x)$ and $\boldsymbol{\pi}(x)$ the transverse displacement and momentum fields, write out the Hamitonian functional. Obtain the equations of motion of the string from this.

(b) **(C)** The string is kept in contact with a heat bath at temperature T. How does the root-mean-square(rms) displacement of the center of the string ($x = L/2$) depend on the temperature T? Write out an expression for the rms displacement.

(c) **(T)** *Calculate* the rms displacement discussed in the previous part.

Note that this problem involves many conceptual and technical ideas. Make sure that you are fully familiar with them – it may help list out the key conceptual and technical ideas of this problem.

0/8. **(E) Outstanding problems:** What are some key outstanding problems of statistical mechanics? Do a survey and report with references.

⋆

[1] This might actually be true!