PH325: Advanced Statistical Mechanics

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https://piazza.com/iisc.ernet.in/summer2014/ph325/home

Overview

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- What is this course about?

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- Course outline

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- Logistics

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The World Around (Outside?) Us

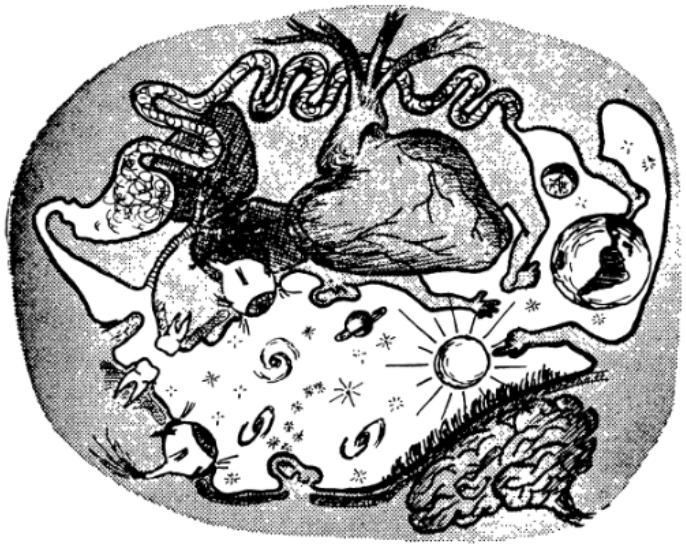

FIGURE 20

Inside-out universe. This surrealistic drawing represents a man walking on the surface of the Earth and looking up at the stars. The picture is transformed topologically according to the method indicated in Figure 19. Thus the Earth, sun, and stars are crowded in a comparatively narrow channel running through the body of the man, and surrounded by his internal organs.

(Gamow from One, Two, Three...Infinity)

- We know that our world is made of "constituents"!

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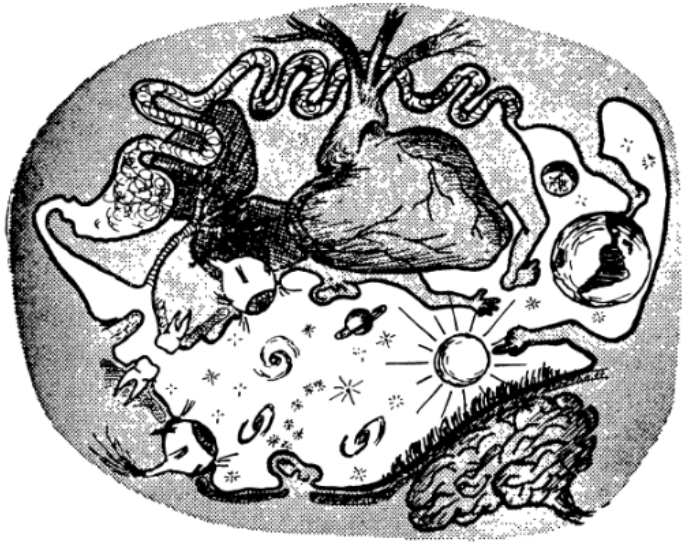

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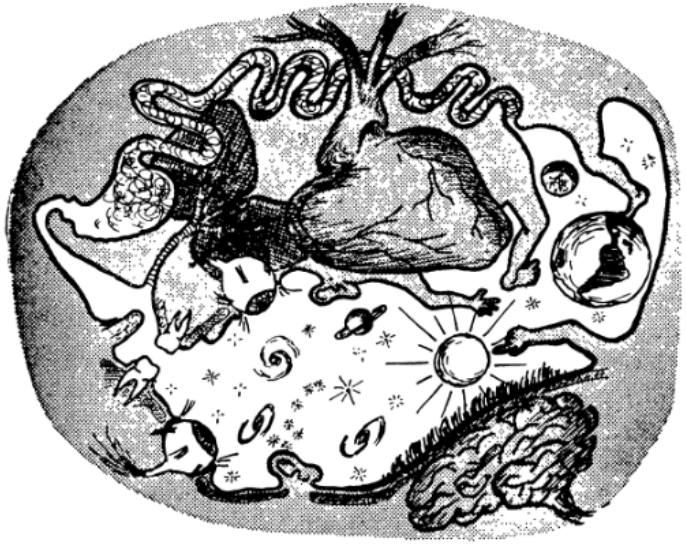

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- Our experiences about "agglomerate objects" are encapsulated in the laws of thermodynamics
- *Phenomenological* description of the world...all physics *is* this!

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- For a given set of "equilibrium macroscopic conditions" statistical mechanics provides a solution for ρ (both in classical and quantum)– these are the well known equilibrium ensembles

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- For a system with fixed number of particles, and at a temperature T,
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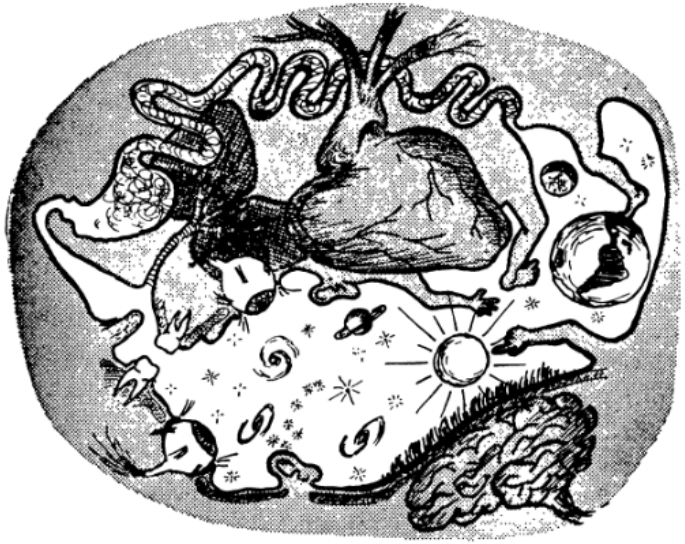
- Lets see whats out there...

The World Around Us...Again!



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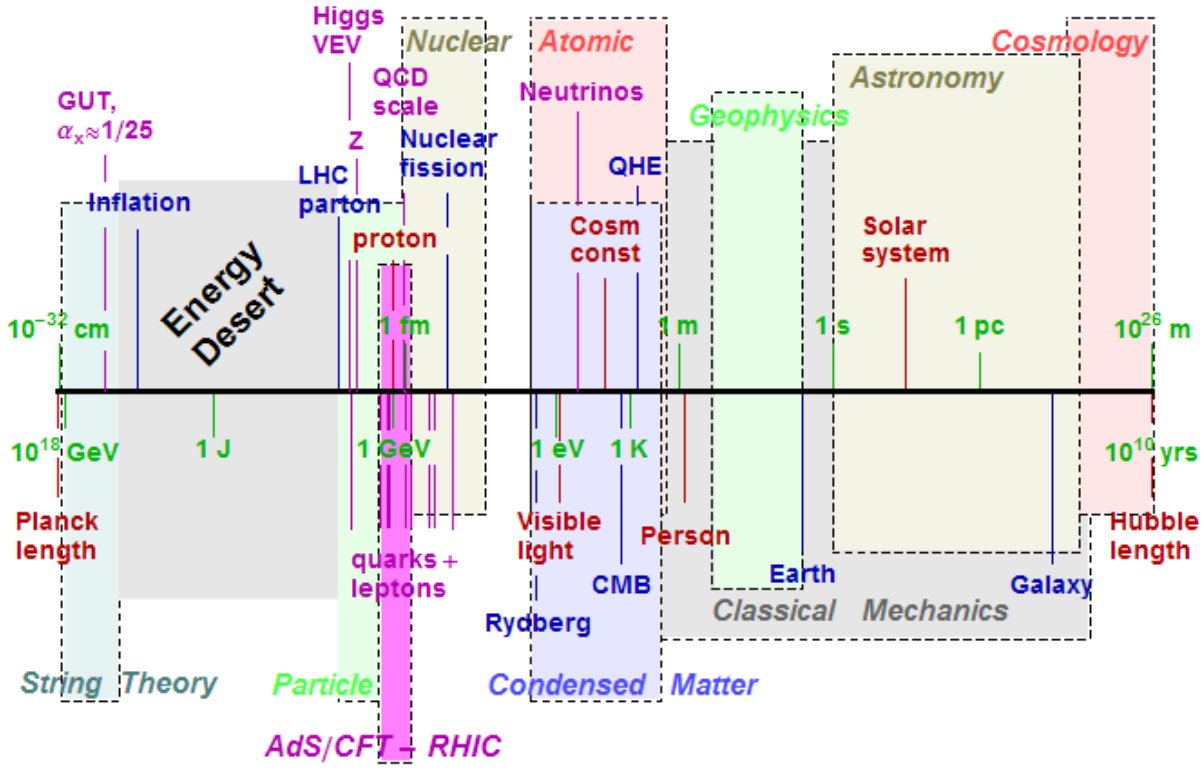
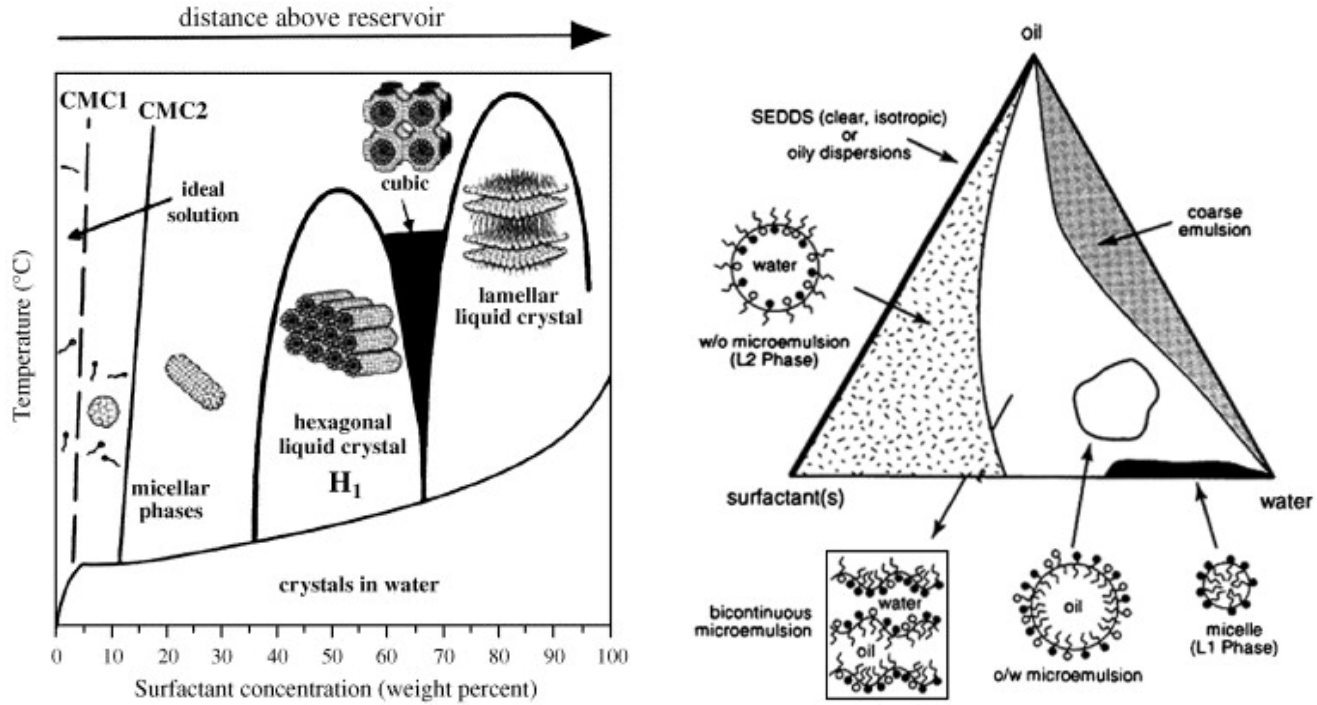


(Internet)

- Many phenomena over a large range of energy scales
- "Fundamental objects" at one scale become constituents of systems at larger scales..
- Things happen when we put lots of stuff together..

Phases of Micelles etc.



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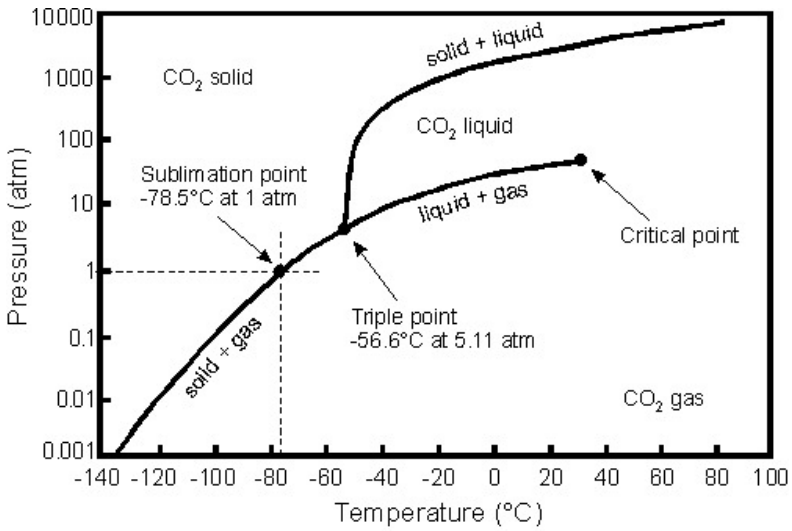
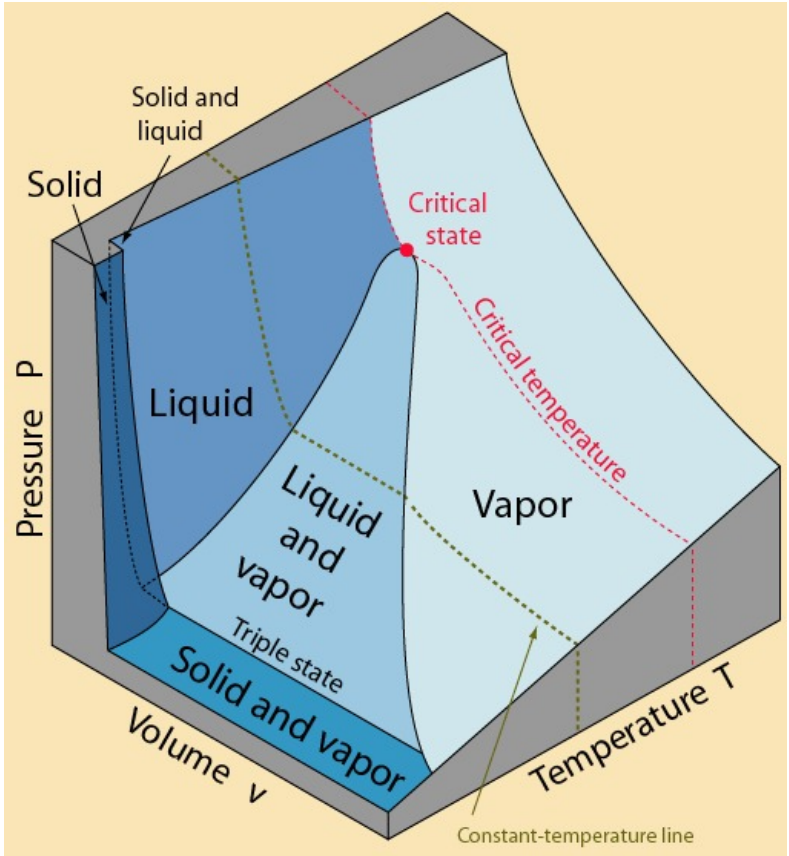
Key Point

- There are many "phases" and "phase transitions" in systems with many constituents!
- One can go from one phase to another by changing "macroscopic" parameters

...of molecules like CO_2 and H_2O...



Pressure-Temperature phase diagram for CO_2.



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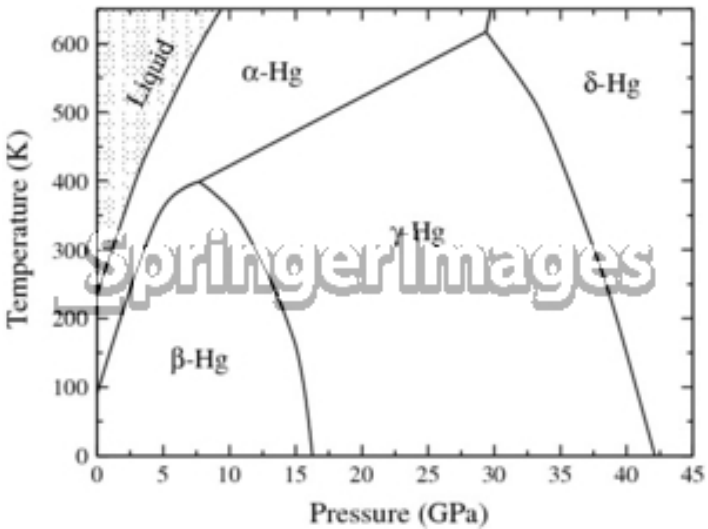
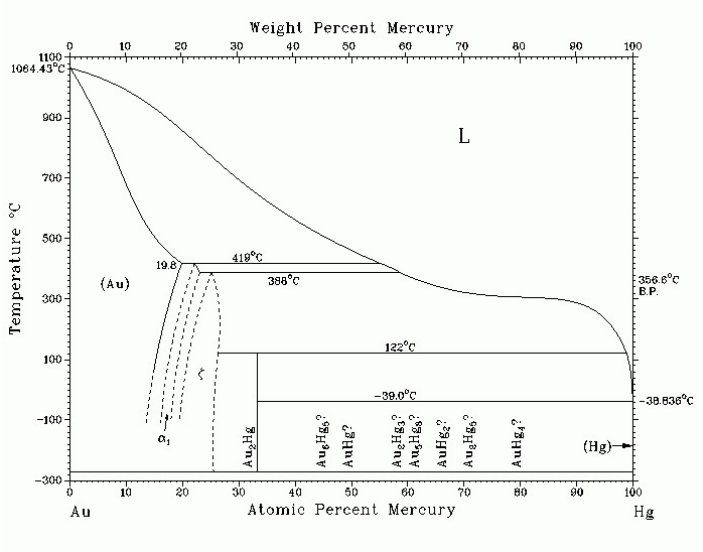
...and of elements!





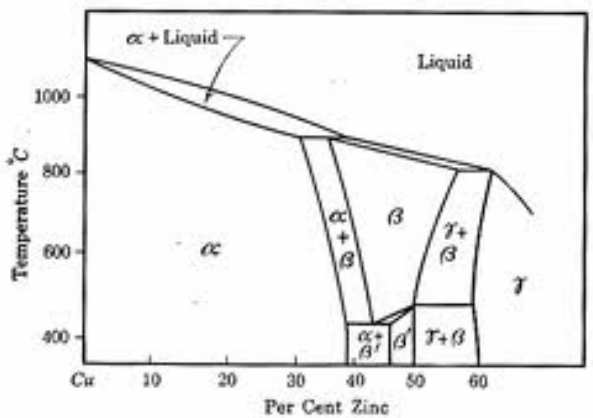
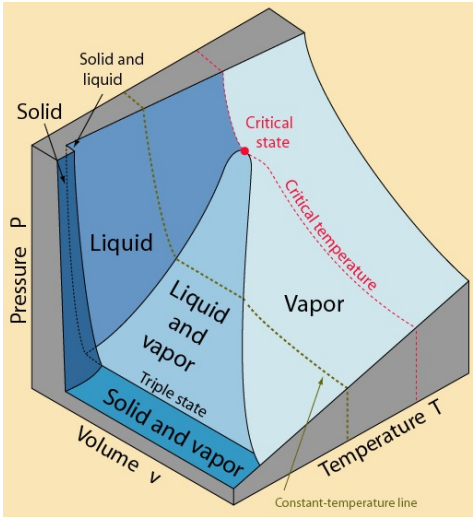


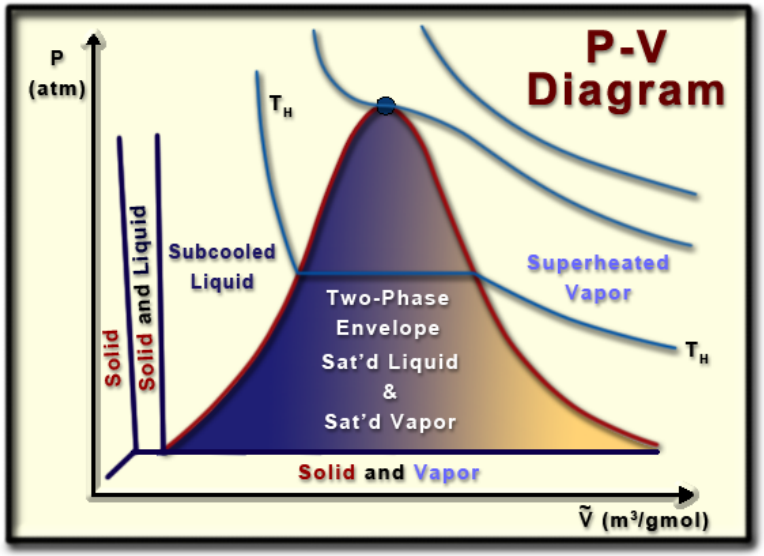
FIG. 57.—Copper-zinc diagram. (*Metals Handbook*, American Society for Metals, 1939 ed.)

(Internet)

..and their "mixtures"

First Attempt to Obtain The Phase Diagram





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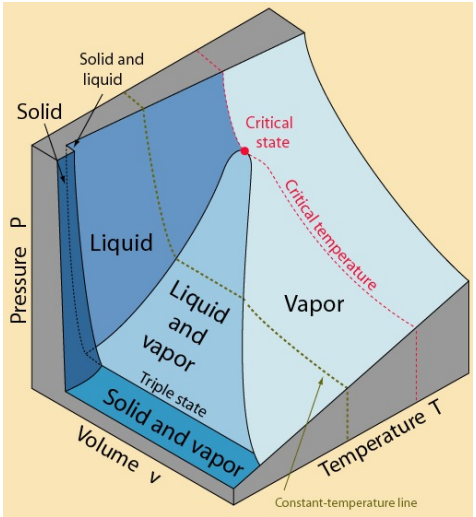
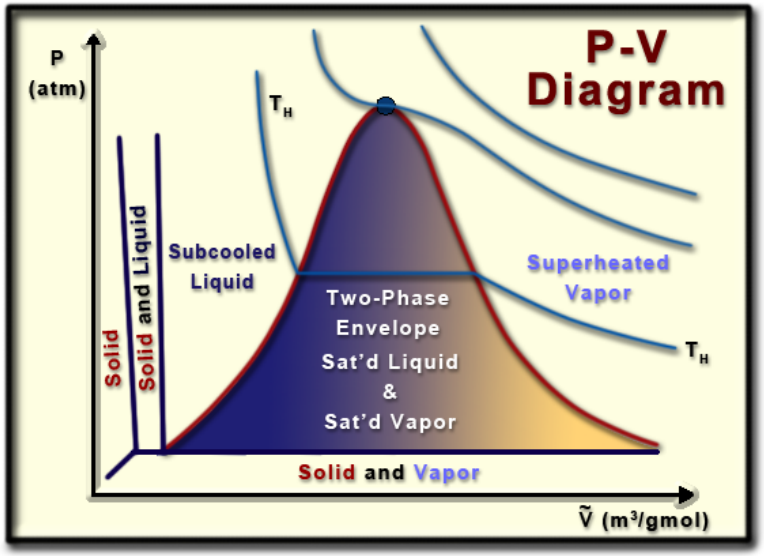
- Take a shot at obtaining this phase diagram...based on knowledge of PH202

First Attempt to Obtain The Phase Diagram





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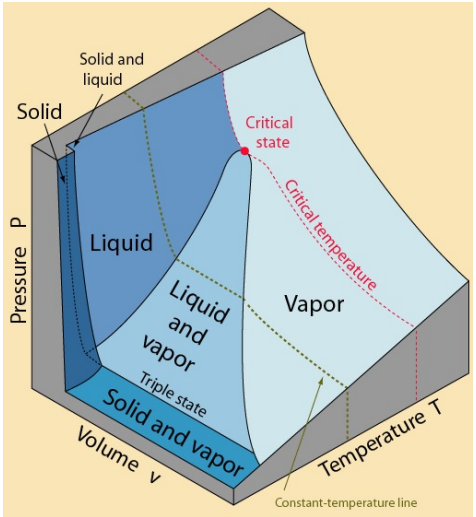

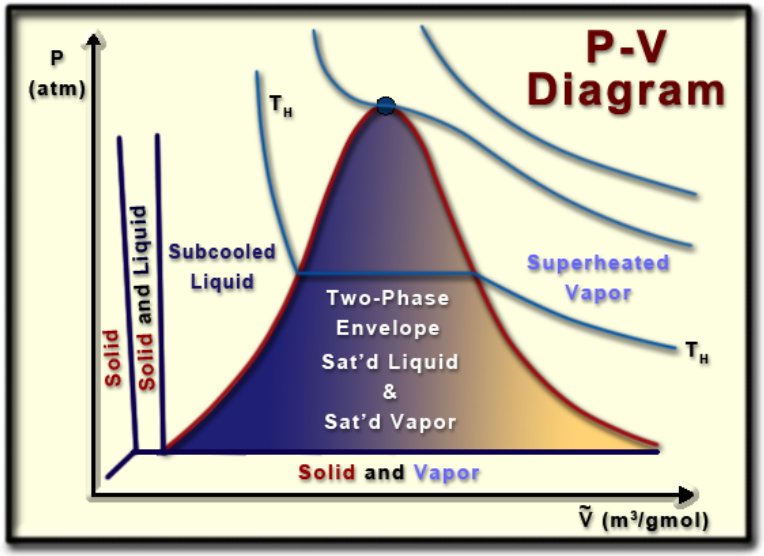

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- Take a shot at obtaining this phase diagram...based on knowledge of PH202
- ...
- and, fail miserably!

Key Point

- Interactions are *CRUCIAL*!

Key Degrees of Freedom

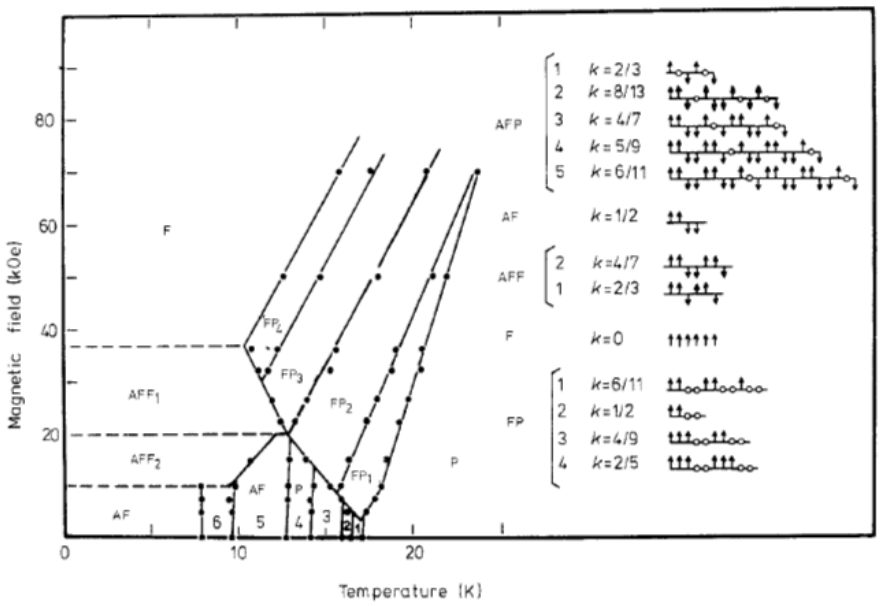


Figure 2. Phase diagram (H, T) of CeSb deduced from the specific heat peaks and determined in increasing temperature. Notation of the phases is the same as in Rossat-Mignod *et al* (1979a, b).

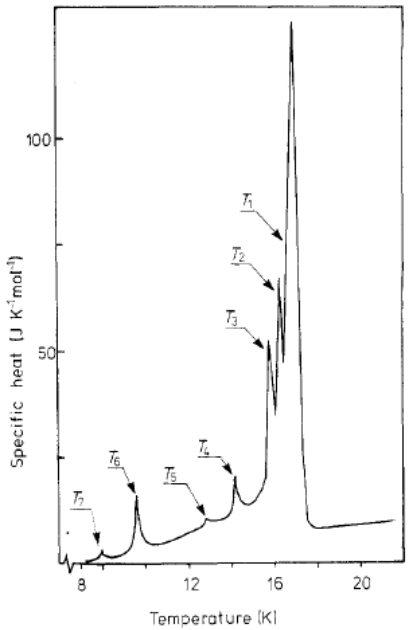
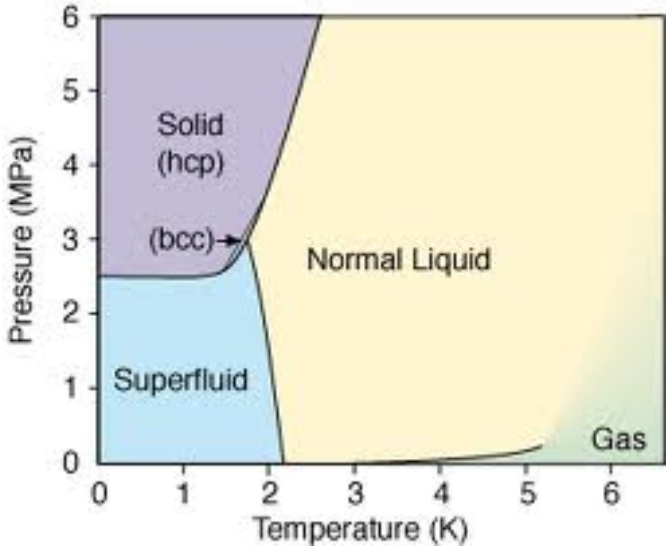


Figure 4. Specific heat of CeSb in zero applied magnetic field.

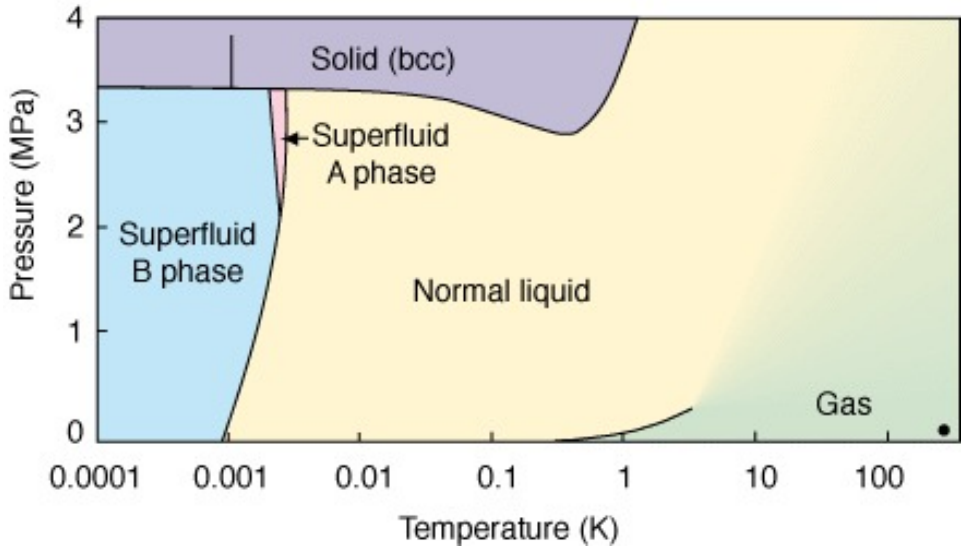
Key Point

- Only some degrees of freedom may be involved in a phase transition. Not all degrees of freedom may participate in a phase transition!

Further Examples



^{4}He



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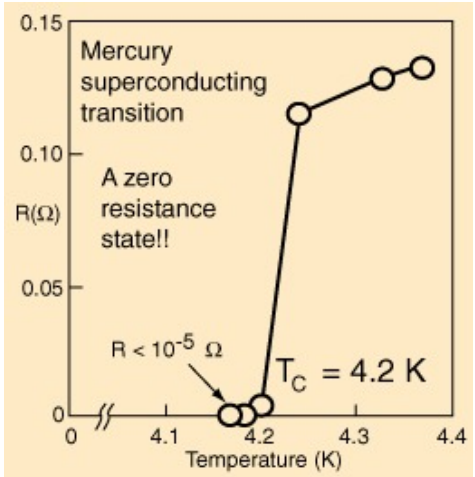
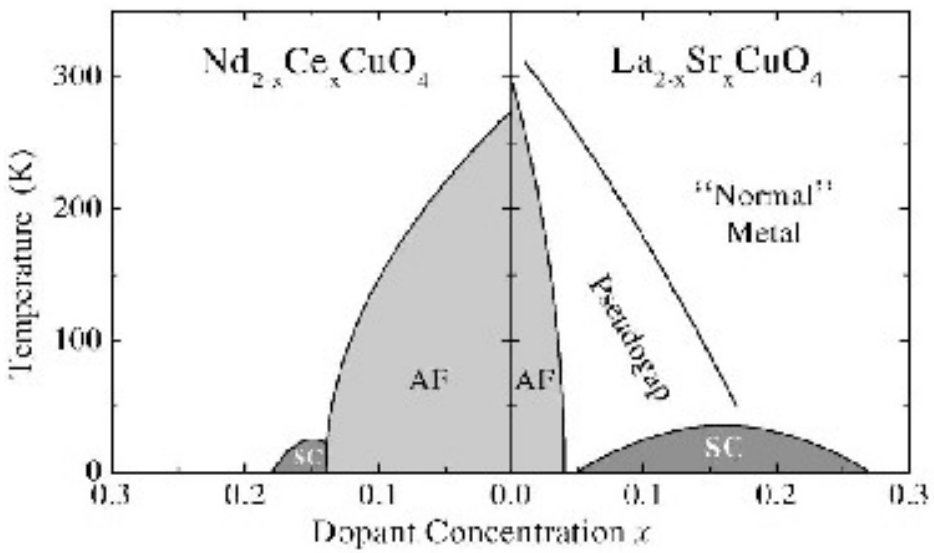
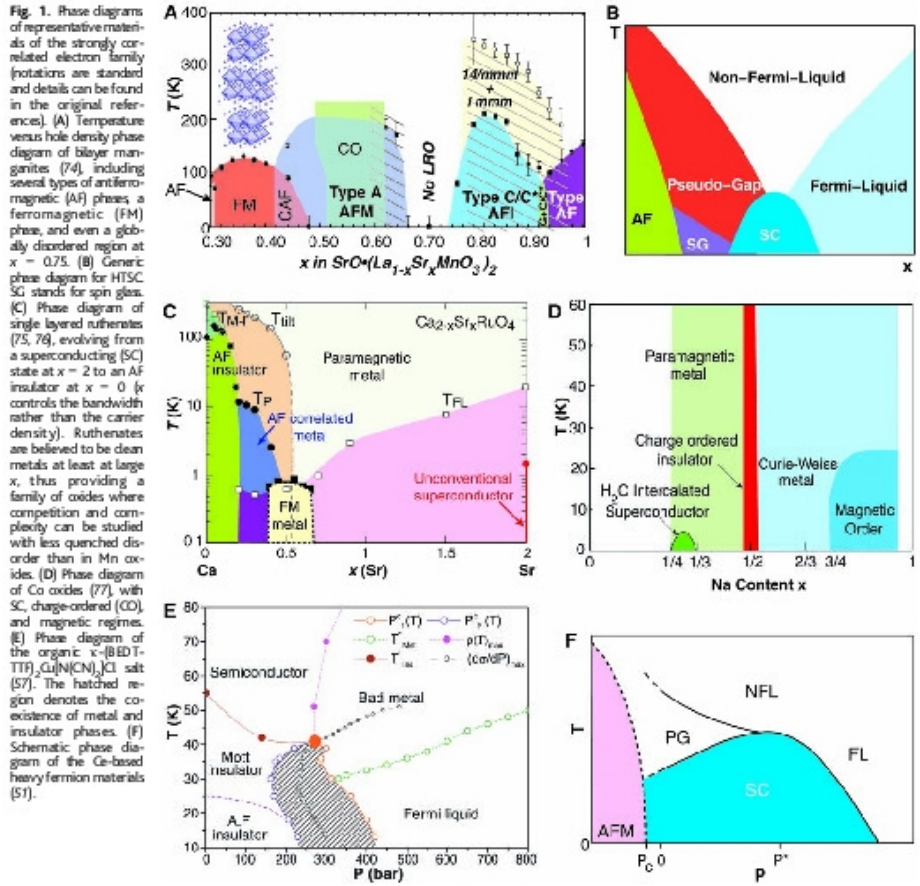


Zero Temperature!





(Internet)

Key Point

- Phase transitions can also occur at zero temperature – Quantum phase transitions

Phase Transitions

- Can be first order or continuous!

Phase Transitions

- Can be first order or continuous!
- Continuous transitions have "singularities"

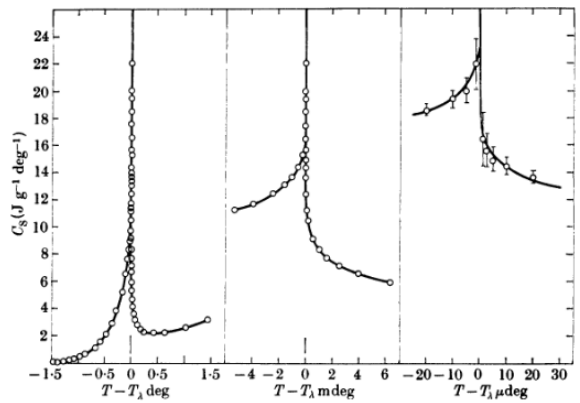


FIG. 1.16. Specific heat of ^{4}He as a function of $T - T_\lambda$ in K. Notice that the shape of the specific heat curve is rather like the Greek letter λ, whence the origin of the term 'λ-transition'. The fact that the specific heat is only about ten times its 'normal' value even at temperatures only a few microdegrees from T_λ is correlated with the fact that the critical-point exponent is extremely small (in fact, α is probably zero, corresponding to a logarithmic divergence). The width of the small vertical line just above the origin indicates the portion of the diagram that is expanded in width in the curve directly to the right. After Buckingham and Fairbank (1965).

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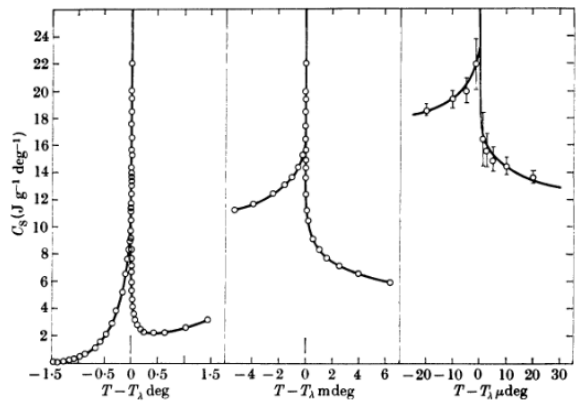

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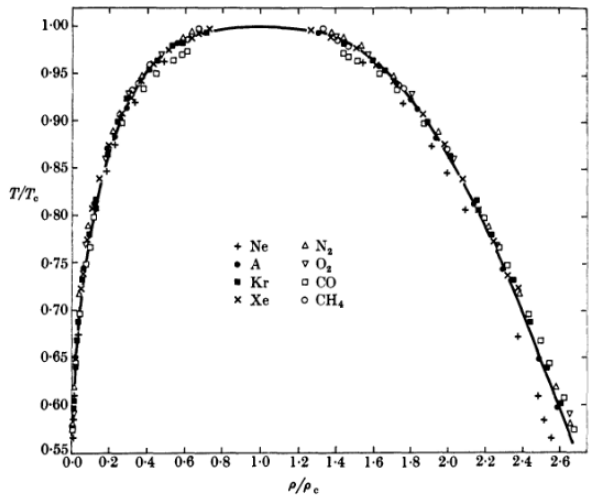

FIG. 1.8. Measurements on eight fluids of the coexistence curve (a reflection of the $P\rho T$ surface in the ρT plane analogous to Fig. 1.3). The solid curve corresponds to a fit to a cubic equation, i.e. to the choice $\beta = \frac{1}{3}$, where $\rho - \rho_c \sim (-\epsilon)^\beta$. From Guggenheim (1945).

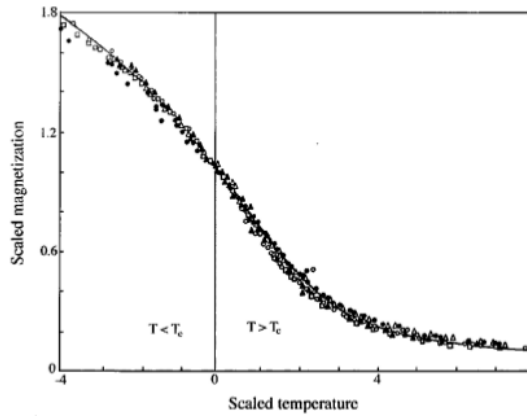

FIG. 1. Experimental MHT data on five different magn materials plotted in scaled form. The five materials are CrI EuO, Ni, YIG, and Pd_3Fe. None of these materials is an alized ferromagnet: $CrBr_3$ has considerable lattice anisotro EuO has significant second-neighbor interactions. Ni is itinerant-electron ferromagnet, YIG is a ferrimagnet, Pd_3Fe is a ferromagnetic alloy. Nonetheless, the data for materials collapse onto a single scaling function, which is calculated for the $d=3$ Heisenberg model [after Milošević Stanley (1976)].

(Stanley, 1971)

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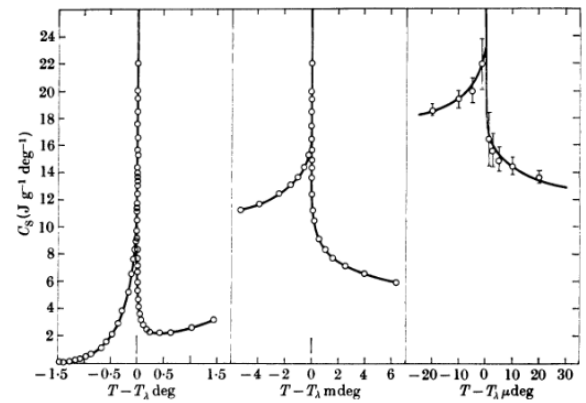

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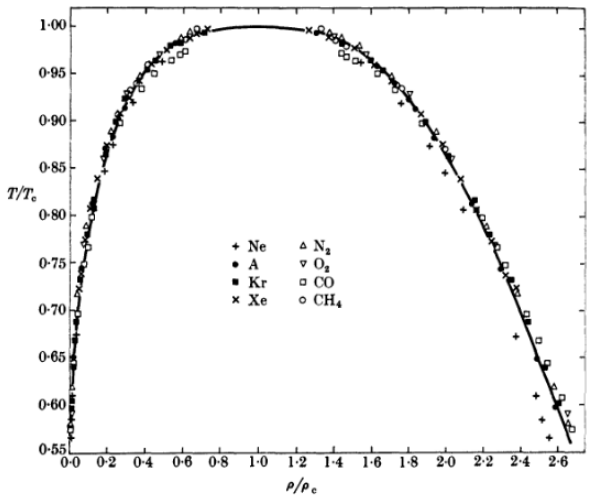

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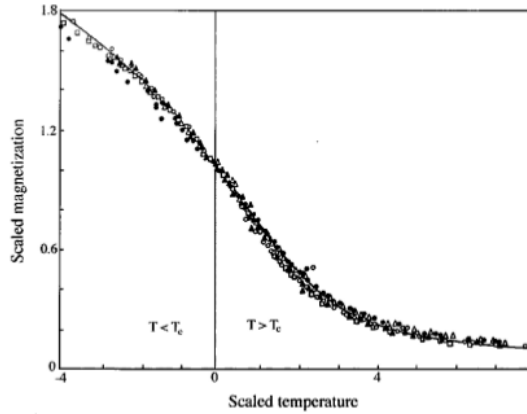

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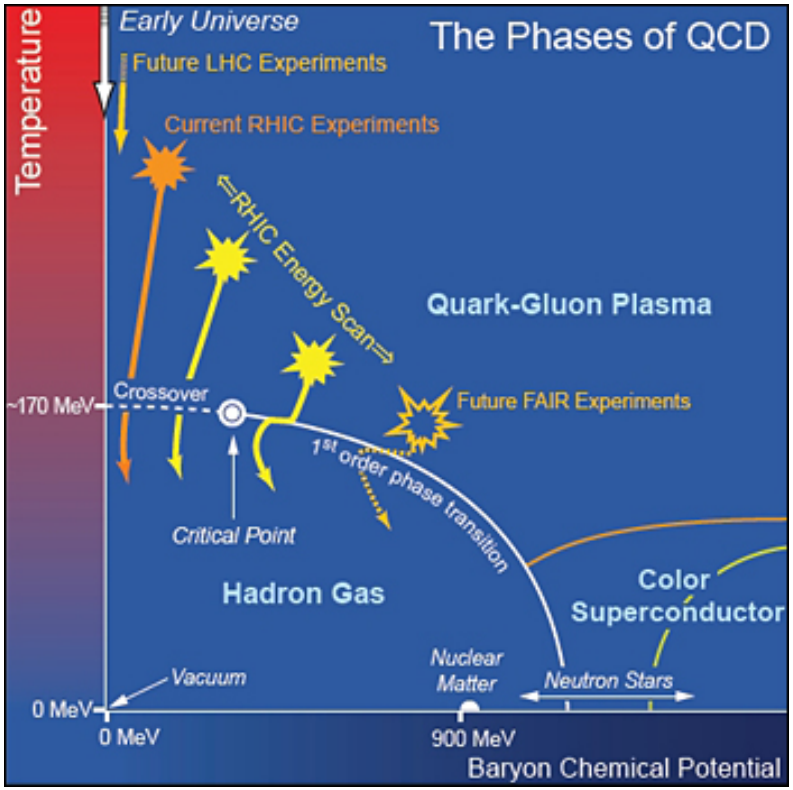
Key Point

- Phase transitions can be first order or continuous
- Continuous transitions have singularities, and universal features

...Not "Just" Condensed Matter



(Internet)

Key Point

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What's Coming...Course Outline

What You Should Expect...

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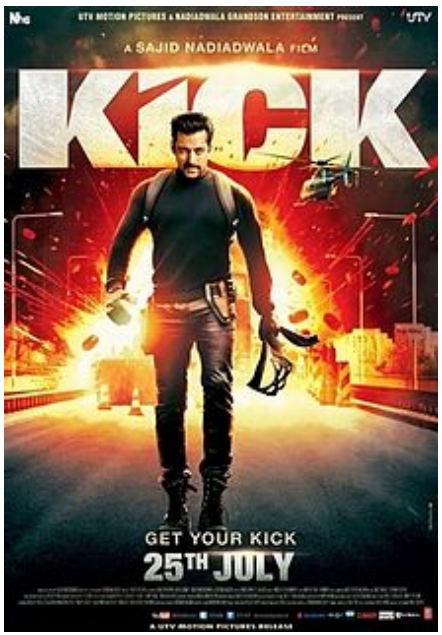

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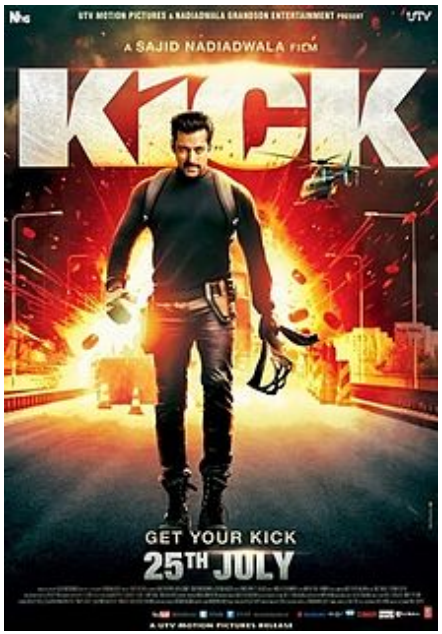

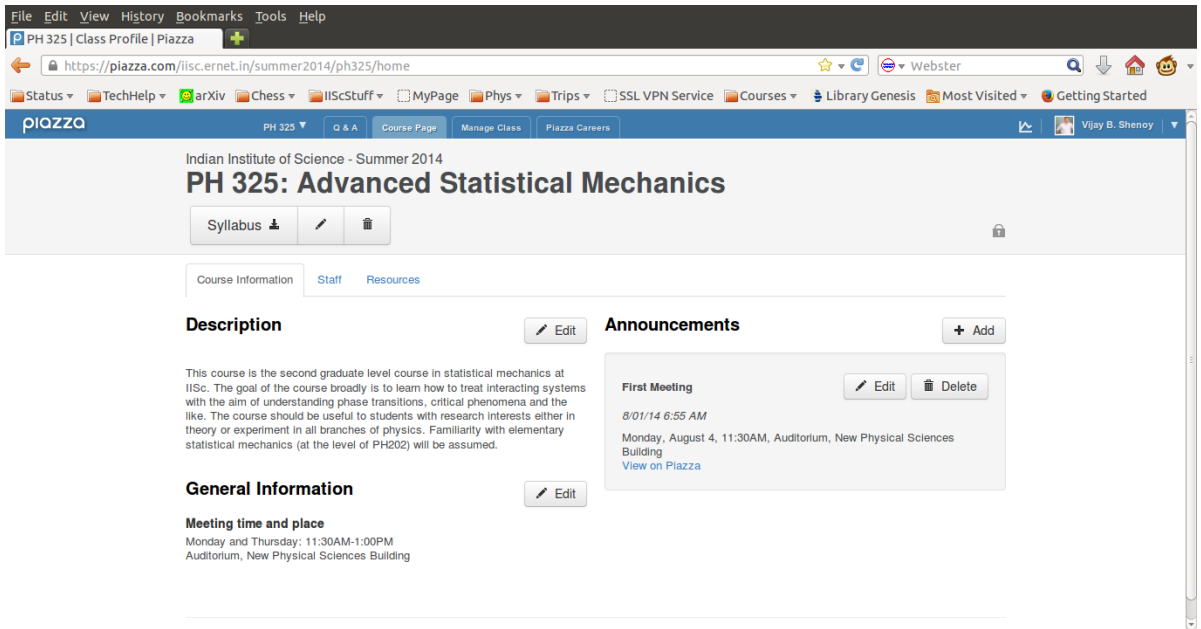

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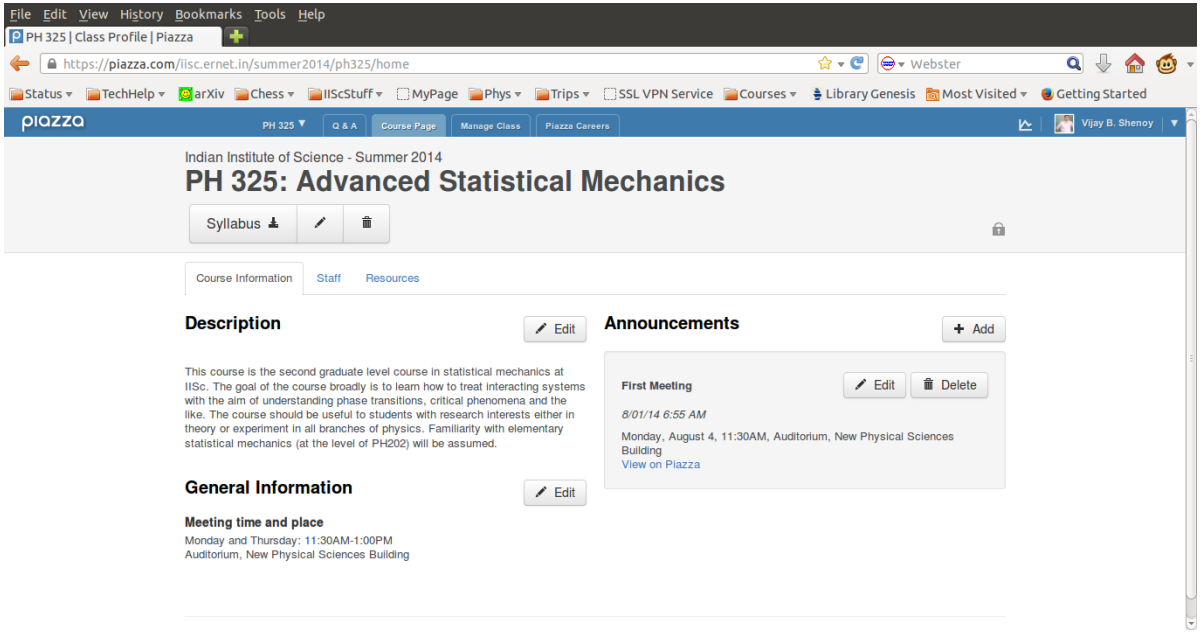
Logistics


File Edit View History Bookmarks Tools Help
PH 325 | Class Profile | Piazza
https://piazza.com/iisc.ernet.in/summer2014/ph325/home
Webster
Status TechHelp arXiv Chess IIScStuff MyPage Phys Trips SSL VPN Service Courses Library Genesis Most Visited Getting Started
piazza
PH 325
Q & A
Course Page
Manage Class
Piazza Careers
Vijay B. Shenoy
Indian Institute of Science - Summer 2014
PH 325: Advanced Statistical Mechanics
Syllabus
Course Information
Staff
Resources
Description
Edit
This course is the second graduate level course in statistical mechanics at IISc. The goal of the course broadly is to learn how to treat interacting systems with the aim of understanding phase transitions, critical phenomena and the like. The course should be useful to students with research interests either in theory or experiment in all branches of physics. Familiarity with elementary statistical mechanics (at the level of PH202) will be assumed.
General Information
Edit
Meeting time and place
Monday and Thursday: 11:30AM-1:00PM
Auditorium, New Physical Sciences Building
Announcements
Add
First Meeting
Edit
Delete
8/01/14 6:55 AM
Monday, August 4, 11:30AM, Auditorium, New Physical Sciences Building
View on Piazza