PH325: Advanced Statistical Mechanics

Department of Physics, IISc Bangalore
Semester I, 2013–2014

EXAMINATION[1]

Instructions:

- The maximum score possible is 30 points.
- The duration of the exam is 180 minutes, 90 minutes for work and 90 minutes for evaluation.
- **Please begin the answers to each question on a fresh page.**
- Note that even though all the questions carry equal points, they are of varying levels of difficulty.

1. (10 Points) A bright graduate student comes up with the following idea to produce magnetization at finite temperature in a 1D Ising system. She considers a 1-d chain with ferromagnetic interactions J_1 and J_2 on alternate bonds, with a Hamiltonian

$$\mathcal{H} = -\sum_i \left(J_1 S_{2i} S_{2i+1} + J_2 S_{2i+1} S_{2(i+1)} \right)$$

She argues that if she sends $J_2 \to \infty$, then the domain wall on the $(2i+1) \leftrightarrow 2(i+1)$ bonds will be highly expensive and this will stabilize the magnetic state at low enough temperatures. Do you agree with her? Justify your stand with physical arguments *and* mathematical substantiation of them.

2. (10 Points) Consider a (classical) Heisenberg magnet on a square lattice with single ion anisotropy. We have spins $\mathbf{S}_i = S_i^x \boldsymbol{e}_x + S_i^y \boldsymbol{e}_y + S_i^z \boldsymbol{e}_z$ at each site with $|\mathbf{S}_i| = 1$, and a Hamiltonian given by

$$\mathcal{H} = -J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_i (S_i^z)^2$$

where $J > 0$.

   (a) Other than arena symmetries, what is the symmetry group of the system?

   (b) What is the ground state of the system?

   (c) Prove or disprove the statement: For $K > 0$, long wavelength fluctuations will not allow for long range order of this system at any finite temperature. You *must* give *both* physical *and* mathematical justifications.

[1] Another opportunity to learn some physics!

(d) Does your previous result change for $K < 0$? Justify with physical/mathematical arguments.

3. (10 Points) The renormalization group flow equation for the temperature parameter t of the $O(n)$ non-linear σ-model[2] reads

$$\frac{dt}{d\ell} = -(d-2)t + a(n-2)t^2$$

where a is a number.

(a) Find the fixed points of the RG flow.

(b) Comment on the stability of the fixed points for various d and n.

(c) Find the critical exponents α and ν at every fixed point you find.

[2]No, you do not need to know what this is!