

# Digital VLSI design

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## Lecture 9: Logical Effort

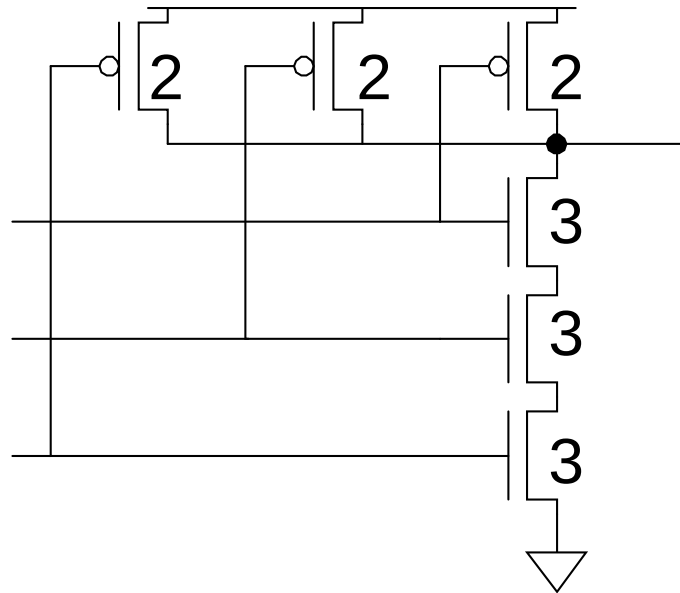


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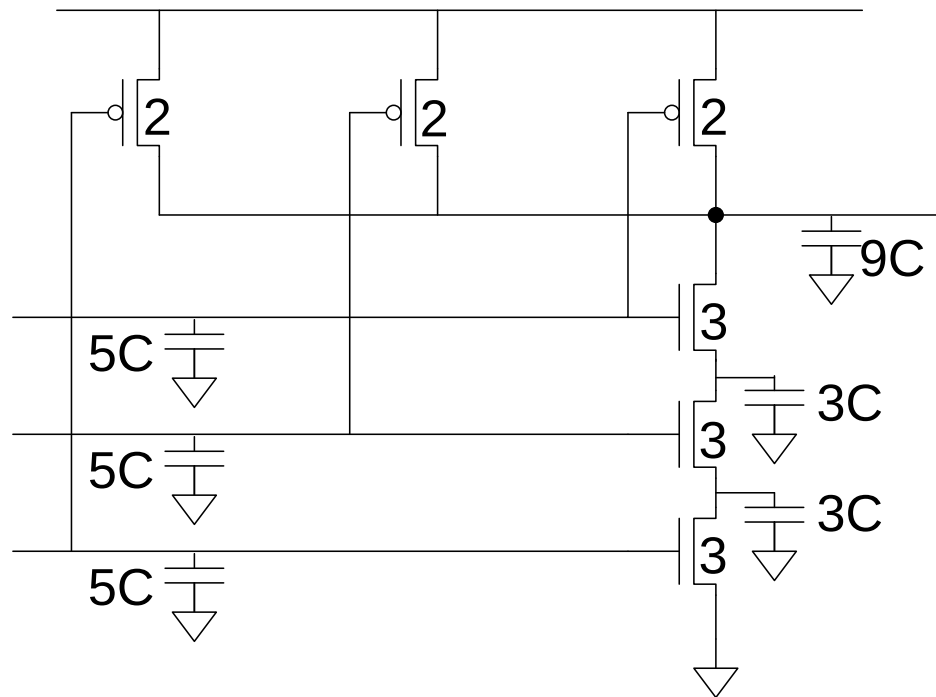
# Example: 3-input NAND

- Sketch a 3-input NAND with transistor widths chosen to achieve effective rise and fall resistances equal to a unit inverter (R).



# 3-input NAND Caps

- Annotate the 3-input NAND gate with gate and diffusion capacitance.



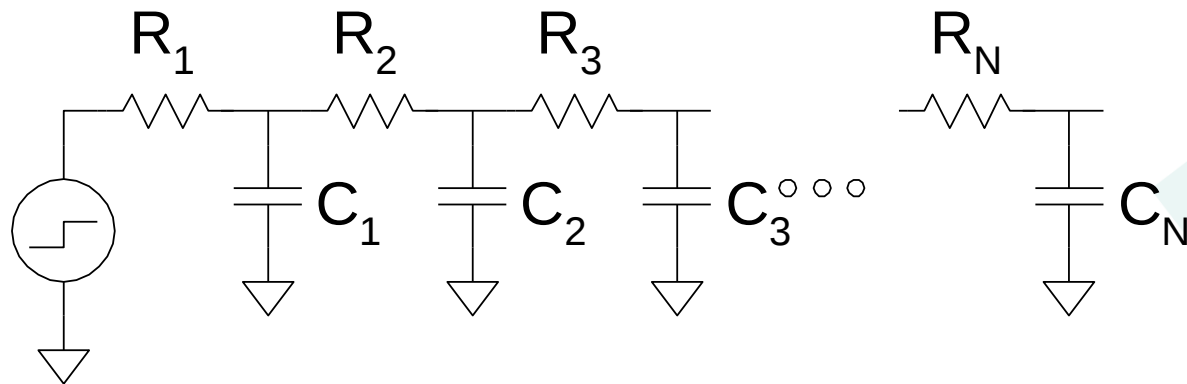
# Elmore Delay



- ON transistors look like resistors
- Pullup or pulldown network modeled as *RC ladder*
- Elmore delay of RC ladder

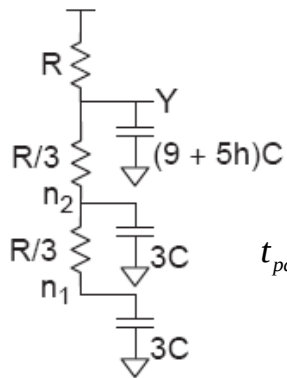
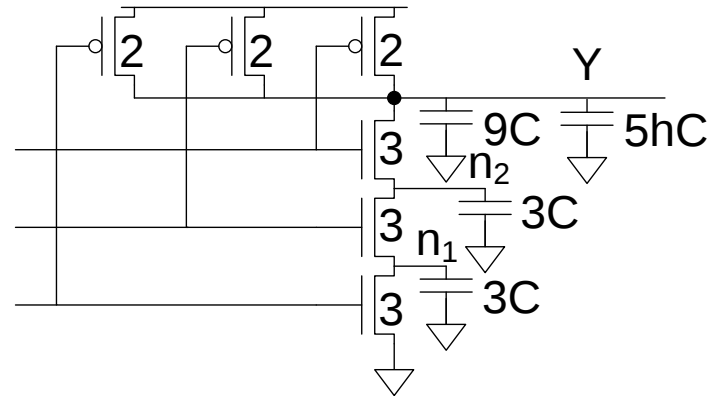
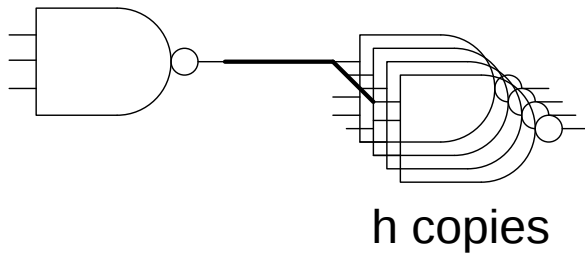
$$t_{pd} \approx \sum_{\text{nodes } i} R_{i\text{-to-source}} C_i$$

$$= R_1 C_1 + (R_1 + R_2) C_2 + \dots + (R_1 + R_2 + \dots + R_N) C_N$$

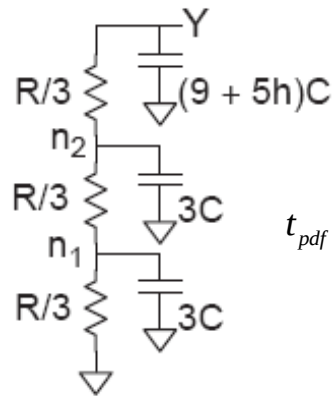


# Example: 3-input NAND

- Estimate worst-case rising and falling delay of 3-input NAND driving  $h$  identical gates.



$$t_{pdr} = (9 + 5h)RC$$



$$\begin{aligned} t_{pdf} &= (3C)\left(\frac{R}{3}\right) + (3C)\left(\frac{R}{3} + \frac{R}{3}\right) + [(9 + 5h)C]\left(\frac{R}{3} + \frac{R}{3} + \frac{R}{3}\right) \\ &= (12 + 5h)RC \end{aligned}$$

# Delay Components

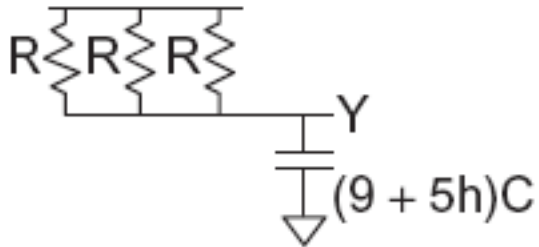
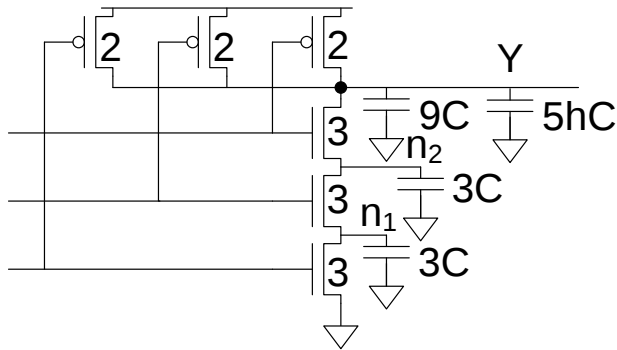
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- Delay has two parts
  - *Parasitic delay*
    - 9 or 12 RC
    - Independent of load
  - *Effort delay*
    - 5h RC
    - Proportional to load capacitance

# Contamination Delay

- Best-case (contamination) delay can be substantially less than propagation delay.
- Ex: If all three inputs fall simultaneously

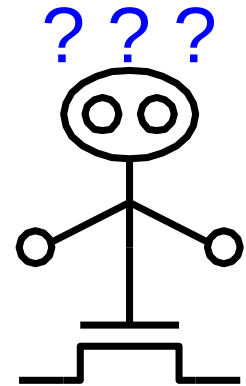


$$t_{cdr} = \left[ (9 + 5h)C \right] \left( \frac{R}{3} \right) = \left( 3 + \frac{5}{3}h \right) RC$$

# Introduction



- Chip designers face a bewildering array of choices
  - What is the best circuit topology for a function?
  - How many stages of logic give least delay?
  - How wide should the transistors be?
- Logical effort is a method to make these decisions
  - Uses a simple model of delay
  - Allows back-of-the-envelope calculations
  - Helps make rapid comparisons between alternatives
  - Emphasizes remarkable symmetries



# Example



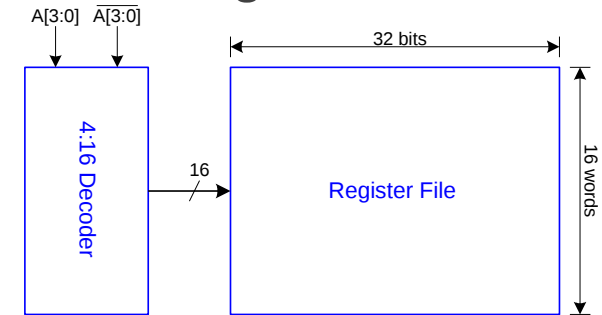
- Ben is the memory designer for the Motorola 68W86, an embedded automotive processor. Help Ben design the decoder for a register file.

- Decoder specifications:

- 16 word register file
- Each word is 32 bits wide
- Each bit presents load of 3 unit-sized transistors
- True and complementary address inputs  $A[3:0]$
- Each input may drive 10 unit-sized transistors

- Ben needs to decide:

- How many stages to use?
- How large should each gate be?
- How fast can decoder operate?



# Delay in a Logic Gate



- Express delays in process-independent unit
- Delay has two components:  $d = f + p$
- $f$ : *effort delay* =  $gh$  (a.k.a. stage effort)
  - Again has two components
- $g$ : *logical effort*
  - Measures relative ability of gate to deliver current
  - $g \equiv 1$  for inverter
- $h$ : *electrical effort* =  $C_{\text{out}} / C_{\text{in}}$ 
  - Ratio of output to input capacitance
  - Sometimes called fanout
- $p$ : *parasitic delay*
  - Represents delay of gate driving no load
  - Set by internal parasitic capacitance

$$d = \frac{d_{\text{abs}}}{\tau}$$

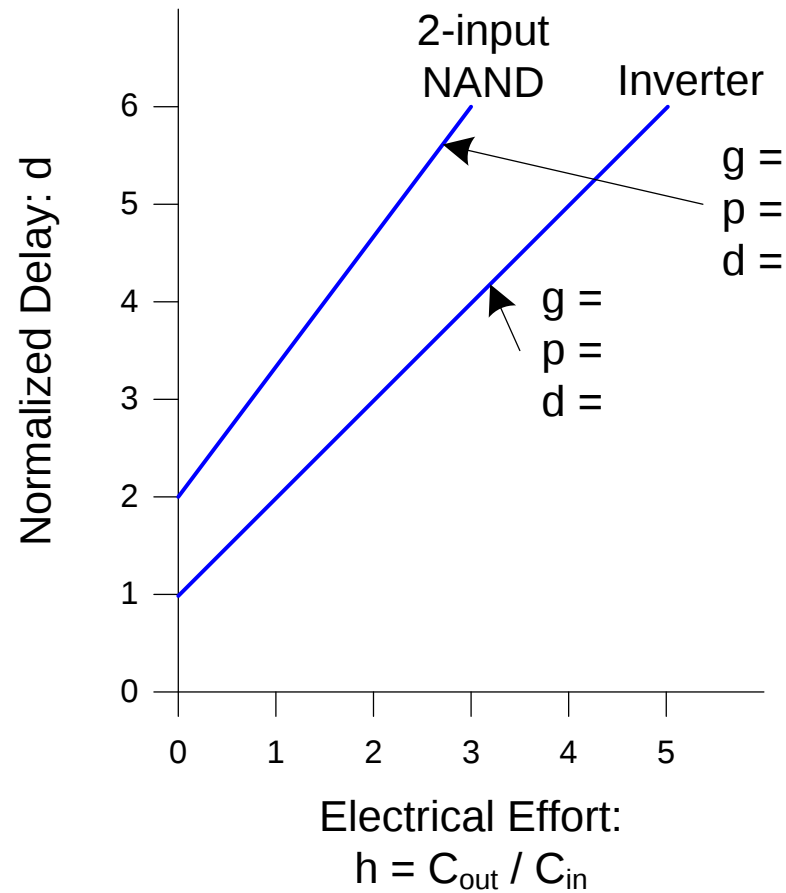
$$\tau = 3RC$$

$\approx$  3 ps in 65 nm process

60 ps in 0.6  $\mu\text{m}$  process

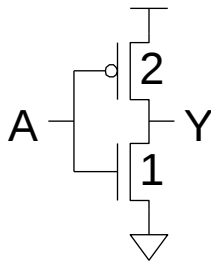
# Delay Plots

$$d = f + p$$
$$= gh + p$$



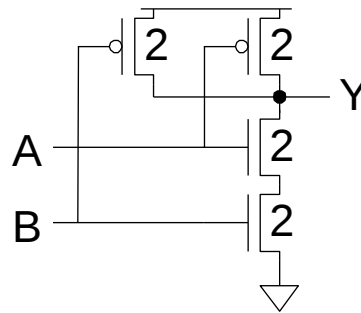
# Computing Logical Effort

- DEF: *Logical effort is the ratio of the input capacitance of a gate to the input capacitance of an inverter delivering the same output current.*
- Measure from delay vs. fanout plots
- Or estimate by counting transistor widths



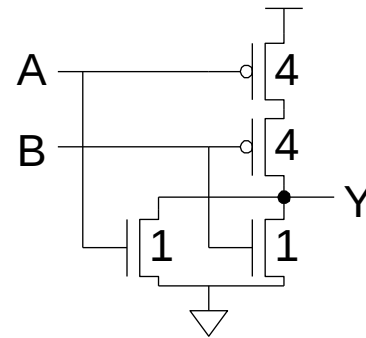
$$C_{in} = 3$$

$$g = 3/3$$



$$C_{in} = 4$$

$$g = 4/3$$



$$C_{in} = 5$$

$$g = 5/3$$