Fluids & Plasmas, AA 363

Problem Set I: Kinetic Theory

1. **Length-scales:** Calculate, to an order of magnitude, the particle mean free path and the mean particle distance for the following gases/plasmas: (i) intergalactic medium with $n \sim 10^{-5}$ cm^{-3}, $T \sim 10^5$ K; (ii) warm interstellar medium with $n \sim 0.1$ cm^{-3}, $T \sim 10^4$ K; (iii) solar corona with $n \sim 10^8$ cm^{-3}, $T \sim 10^6$ K; (iv) core of the sun with density $\sim$1 g cm^{-3} and $T \sim 10^7$ K; (v) the air that we breathe; (vi) water (I won't give you the numbers for the last two; you must know this). For ionized plasmas also calculate the Debye length and the distance of closest approach. Caution: collisions in plasmas are mediated by long range Coulomb interactions, unlike in neutral gases where short range forces lead to billiard-ball collisions.

2. **The Maxwellian:** The distribution function $f(\vec{v})$ for uniform gas in collisional equilibrium is given by a Maxwellian

$$f(\vec{v}) = \frac{n}{(2\pi)^{3/2} v_t^3} \exp\left[-\frac{(\vec{v}-\vec{v}_0)^2}{2v_t^2}\right],$$

where $v_t = \sqrt{k_B T/m}$ is the isothermal sound speed, k_B is Boltzmann's constant, T is the temperature and m is the particle mass. Calculate the average speed of the particles and the rms speed in terms of v_t. What is the average fluid velocity?

3. **Collision Cross-section:** Calculate the differential scattering cross-section and the total cross-section in the center of mass frame for binary elastic collisions between identical billiard balls of size a.

4. **Conservation Laws:** Derive the following fluid conservation laws by taking appropriate moments of the Boltzmann equation:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0,$$

$$\frac{\partial (\rho \vec{u})}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}\vec{u}) = -\vec{\nabla} \cdot \overleftrightarrow{P} + \frac{\rho \vec{F}}{m},$$

$$\frac{\partial (\rho \epsilon)}{\partial t} + \vec{\nabla} \cdot (\rho \epsilon \vec{u}) + \vec{\nabla} \cdot \vec{q} + \overleftrightarrow{P} : \overleftrightarrow{\Lambda} = 0,$$

where $\rho = nm$ is the fluid mass density, $\vec{u}(\vec{x}, t)$ is the fluid velocity, $\overleftrightarrow{P} = \rho(\langle \vec{v}\vec{v} \rangle - \vec{u}\vec{u})$ is the pressure tensor, $\vec{F}(\vec{x})$ is the position-dependent force, $\epsilon = \langle |\vec{v} - \vec{u}|^2/2 \rangle$ is the specific thermal energy (internal energy per unit mass), $\vec{q} = \rho \langle (\vec{v} - \vec{u}) |\vec{v} - \vec{u}|^2 \rangle /2$ is the heat flux vector, and $\overleftrightarrow{\Lambda} = (\vec{\nabla}\vec{u} + \vec{\nabla}\vec{u}^T)/2$ (T stands for transpose) is strain tensor. This is an extension of the derivations I was doing in class but did not go into details. Its also outlined in the book. Further, show that these equations reduce to the standard Euler equations if the distribution function is a local Maxwellian.