Fluids & Plasmas, AA 363

Problem Set V: Linear instabilities & Turbulence

1. **Rayleigh instability:** Consider an inviscid rotating fluid with a general rotation profile $\Omega(R)$. Using local analysis, show that the flow profile is linearly unstable to axisymmetric ($\partial/\partial\phi = 0$) perturbations if $d(R^2\Omega)/dR < 0$ (i.e., if specific angular momentum decreases with radius). Therefore, Keplerian accretion disks with $\Omega \propto R^{-3/2}$ are expected to be hydrodynamically stable to small perturbations. Having a look at the inertial waves problem in a previous problem set will be useful. One can also make a close analogy with inviscid convection.

2. **Reynolds numbers achievable in direct numerical simulations of turbulence:** Suppose we want to simulate 3-D incompressible Navier-Stokes turbulence in a box. Using K41 scaling arguments and the fact that the computational time step $\Delta t = \Delta x/u$ (where u is the maximum fluid velocity and Δx is grid spacing), show that the total number of computations required to evolve turbulence at Reynolds number Re scales as $\sim \mathrm{Re}^3 \log \mathrm{Re}$. Therefore, going beyond Re $\sim 10^4$ is difficult even for the fastest supercomputers.

3. **Kolmogorov viscous scales:** Show the following for the dissipation scale of turbulence at which viscosity is important: $l_\nu \sim \nu^{3/4}\epsilon^{-1/4}$, $u_\nu \sim \nu^{1/4}\epsilon^{1/4}$ and $t_\nu \sim \nu^{1/2}\epsilon^{-1/2}$, where ϵ is the energy dissipation rate per unit mass and ν is the kinematic viscosity. Show that in Kolmogorov turbulence velocity is dominated by larger eddies but the vorticity is dominated by the smaller ones and that the smaller eddies have a shorter lifetime.