Submit your solution as a single PDF file using the script `submit_cse563` (or `submit_cse463`) on any CSE server. Use math fonts for a professional look, or use `sharelatex`. Do not submit scanned material.

Problem 1 (10 pts)

Consider the following facts about the Elm Street Bridge Club: Joe, Sally, Bill, and Ellen are the only members of the club. Joe is married to Sally. Bill is Ellen's brother. The spouse of every married person in the club is also in the club.

From these facts, most people would be able to determine that Ellen is not married.

To do:

1. Represent these facts as sentences in FOL, and show using appropriate interpretations that by themselves they do not entail that Ellen is not married.

2. Express in FOL some additional general knowledge that most people would be expected to have, and show that the augmented set of sentences now entails that Ellen is not married.

3. Add more facts to the result of the previous step, so that you obtain a set of formulas that also entails that Ellen is married. Is this set satisfiable?

Problem 2 (10 pts)

A binary relation R on a set S is:

- *symmetric*: if an element x is related to an element y through R, then y is related to x through R.

- *antisymmetric*: if an element x is related to an element y through R, and y is related to x through R, then $x = y$.

- *the equality relation*: an element x is related through R to itself and itself only.

All the above definitions assume that x and y could be any arbitrary elements of S.

To do:

1. Formulate the properties of symmetry and antisymmetry in first-order logic.

2. Show that the equality relation is both symmetric and antisymmetric. Find another (different) relation that is both symmetric and antisymmetric, if any exists.

3. Explain the relationship between symmetry+antisymmetry and equality in terms of entailment.

4. Show an example binary relation which is neither symmetric nor antisymmetric.

You can assume that constants are interpreted as themselves.