

Linear Algebra - Math 310

Ex] Find the equation of the parabola passing thru $(0, 16)$, $(2, 4)$, $(3, 7)$

$$y = ax^2 + bx + c$$

$$\begin{aligned} c &= 16 \\ 4a + 2b + c &= 4 \\ 9a + 3b + c &= 7 \end{aligned}$$

$$4a + 2b = -12$$

$$9a + 3b = -9$$

$$2a + b = -6$$

$$3a + b = -3$$

$$a = 3$$

$$b = -12$$

$$\therefore y = 3x^2 - 12x + 16$$

system
of linear
equations

Ex] Find where the lines ① $-x + y = 3$ intersect.
 $2x + y = 1$

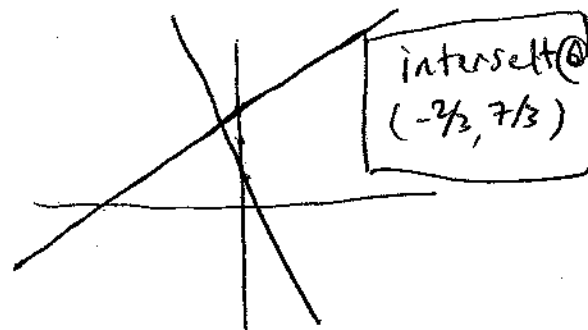
$$\begin{bmatrix} -x + y = 3 \\ 2x + y = 1 \end{bmatrix} \xrightarrow{+2I}$$

$$\begin{bmatrix} -x + y = 3 \\ 0x + 3y = 7 \end{bmatrix} \cdot \frac{1}{3}$$

$$\rightarrow \begin{bmatrix} -x + y = 3 \\ 0x + y = 7/3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} -x + 0y = 2/3 \\ 0x + y = 7/3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} x = -2/3 \\ y = 7/3 \end{bmatrix}$$



$$\textcircled{2} \quad \begin{aligned} x + y &= 5 \\ 2x + 2y &= 11 \end{aligned}$$

$$\text{II} - 2 \cdot \text{I} \rightarrow 0 = 1 \quad \text{!}$$

$$\textcircled{3} \quad \begin{aligned} x + y &= 5 \\ 2x + 2y &= 10 \end{aligned}$$

$$\text{II} = 2 \cdot \text{I}$$

$$x = 5 - y$$

$y = \text{anything}$
"free variable"

Systems
of Linear
Equations

3 possibilities:

- no solutions $\leftarrow$ inconsistent
- unique solution $\leftarrow$ consistent, no free variables
- infinitely many solutions $\leftarrow$ consistent, free variables

Ex 3 | Find all triples satisfying

$$2x + y + z = 5$$

$$\textcircled{1} \quad 4x - 6y = -2$$

$$-2x + 7y + 2z = 9$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$$

$\leftarrow$ coeff matrix

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{array} \right]$$

augmented
coeff matrix

maybe
not written

Gauss Elimination Steps

1. swap order of eqns
2. mult eqn by $\neq 0$ #
3. add mult of one eqn to another

- $\longleftrightarrow$
1. swap rows
 2. mult row by $\neq 0$ #
 3. add mult of one row to another

$$\left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{array} \right] \xrightarrow{-2I} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 8 & 3 & 14 \end{array} \right] \xrightarrow{+II}$$

$$\rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{+4/2} \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & 4 & 1 & 6 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$z = 2$$

$$4y + z = 6$$

$$4y = 4$$

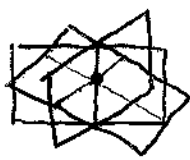
$$y = 1$$

$$2x + y + z = 8$$

$$2x + 3 = 8$$

$$2x = 5$$

$$x = 1$$



point